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THE
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THE "ASTRONOMICAL ATOM" AND THE SPECTRAL SERIES OF HYDROGEN

By FERNANDO SANFORD

In a paper published in the *Astrophysical Journal* of November, 1916 (**44**, 201–209), the present writer was able to show that on the assumption that radiating electrons are revolving in elliptical or circular orbits about a central positive charge with velocities such that the photo-electric equation $\frac{1}{2}mv^2=h\nu$ applies to them, their orbital radii may be computed, and that these radii bear a linear relation to the corresponding atomic radii computed from other considerations.

In a paper on "The Nuclear Charges of Atoms," published in the *Physical Review* of May, 1917 (**9**, 383), it was shown that on the same assumptions regarding the electrons which emit X-rays it is possible to compute the nuclear charges of the radiating atoms by means of Moseley's equations for X-ray frequency, and that these charges are, as suggested by van den Broek, simple multiples of the unit electrical charge.

It is the purpose of the present paper to discuss the spectral series of hydrogen on the same hypothesis.

As a justification for the fundamental assumption adopted in this and in the earlier papers, viz., that electrons while emitting

radiation are revolving in elliptical or circular orbits, it may be permitted to call attention to the fact that it is this assumption which has made it possible to calculate the correct value of e/m from the Zeeman effect. The hypothesis here used differs from that used by Zeeman only in assuming that the central force by which the revolving electrons are held in their orbits is electrical, while the Zeeman effect may be explained equally well by assuming this to be an elastic force of any kind.

As regards the use of the Einstein photo-electric equation, it seems to be experimentally proved that the energy of the cathode rays which give rise to the X-ray spectrum as well as the energy of the electrons which are liberated by the action of X-rays on metals may be calculated from the vibration-frequency of the induced or inducing X-rays by using the equation $\frac{1}{2}mv^2 = h\nu$, where ν is the vibration-frequency of the X-rays and h is the Planck radiation factor, viz., $6.55 \cdot 10^{-27}$ ergs/sec.

The same relation has been shown to hold in the case of electrons liberated by the action of ultra-violet light upon metals, and in the case of electrons which induce the single-line spectra in numerous elements. It accordingly seems extremely probable that it holds in the case of the other lines of a spectral series which lie between the ultra-violet rays and the less refrangible rays of the single-line spectrum.

The only new assumption involved in the use of the equation in this and the earlier papers referred to is that the kinetic energy of the free electrons is transformed into or results from the kinetic energy of orbital revolution.

It is well known that since the important discovery by Balmer that the wave-lengths of a group of lines in the hydrogen spectrum could all be expressed mathematically in terms of a single fundamental number, much attention has been devoted to the discovery and computation of spectral series. As a result of this work, a number of empirical formulae have been proposed by means of which the wave-lengths of certain groups of lines may be computed with considerable accuracy. These formulae, with the exception of some proposed by Ritz, have no theoretical basis, and all are merely empirical rules for calculating the wave-lengths under

consideration. Konen,[1] in referring to this fact, says that no relation between any of the constants used in these computations and any physical properties of the atom have ever been discovered. He admits as the one possible exception to this statement the relation between the ionic charges of certain elements and the convergence-frequencies of their corresponding spectral series, to which attention was called by the present writer in the *Astrophysical Journal* of October, 1912 (**36**, 255–262), but he ends by rejecting the ionic charges as true physical constants.

In this connection it seems to the present writer that the recognition by Rydberg of the fundamental frequency factor which he calls N is the most significant step in the theory of spectral series which has been made since Balmer's original discovery in 1885.

The discoveries which have resulted largely from the work of Barkla and the Braggs have shown that most elements emit one or more characteristic groups of X-rays which resemble in their spectra the visible band spectra of many substances. In many of the elements two of these X-ray spectral bands are known; in elements of the highest atomic weight only one band is known, and in a few of the lighter elements no characteristic X-rays have certainly been observed.

The general position of these spectral bands is given for each element by Moseley's equation, $\nu = A(N-b)^2$, where ν is the vibration-frequency which gives rise to the spectral line under consideration, N is the serial number proposed by Rydberg and van den Broek (not the Rydberg frequency factor N), and A and b are constants for a particular type of radiation.

With the exception of those physicists, if any there be, who advocate the Bohr hypothesis, there seems to be a consensus of opinion that visible radiation, including infra-red and ultra-violet, as well as X-radiation, is due in some manner to oscillations of electrons in an atomic system. This being the case, it would seem that whatever system of oscillating electrons gives rise to X-rays must form at least a part of the system which emits visible radiation.

There have been several hypotheses regarding the necessary structure of an atomic system which shall be capable of giving off

[1] *Das Leuchten der Gase und Daempfe*, p. 195.

discontinuous spectra similar to those which have been observed. The proposers of these hypotheses, with the exception of Ritz, have assumed that the radiating electrons oscillate under the influence of electric forces. At the present time opinion seems to favor some hypothesis similar to the one proposed by Rutherford, and for which Lodge has proposed the name "Astronomical Atom"; viz., of one or more electrons revolving about a central positive mass in which most of the inertia of the system is located.

It is plain that an electron revolving about a positive charge will set up an oscillating electric field which will be transmitted by the ether. The orbital revolution is necessarily only a temporary condition and can be maintained only while energy is being supplied to the system. The condition of the electron when not in revolution is not at present under consideration. The only question now raised is, Will such a rotation account for any of the known laws of radiation? Or, putting it another way, Will this assumption combined with either of the known laws of radiation, e.g., Einstein's law or Moseley's law, enable us to calculate the other known law from it?

The article on nuclear atomic charges already referred to has answered this question in the affirmative, since it showed that the nuclear charges calculated by means of Einstein's equation and the law of circular motion have the same value as those calculated from Moseley's law and the unit electrical charge.

If we consider the atom as a mere planetary system, we can see no reason why the electrons should revolve in certain fixed periods which are closely related in the atoms of different elements, or why there should be such a thing as a spectral series. A satellite may revolve about its primary at any distance if it has the proper tangential velocity. In the case of a single electron revolving about a positive central charge, it is only necessary that $mv^2/R=Qe/R^2$, where Q is the central positive charge and e the negative electronic charge. The case is different if we locate a number of electrons outside the primary charge. In this case each revolving electron reduces the effective central charge for all electrons whose orbits lie outside its own, so that we have three variables, Q, v, and R, to take into consideration. It will accordingly be necessary to use some equation besides the general equation for circular motion to

account for the possible spacing of a number of electrons around a positive charge.

Both of the equations which have been found to apply to X-rays are of the discontinuous character suggested by what has been said. One of them has a discontinuity based upon the serial number of the element and the other has one based upon the Planck radiation factor, h. This latter equation shows that not all frequencies of radiation are possible, but only those which radiate a definite quantity of energy at each revolution.

Let us combine this equation for energy radiated with the equation for circular motion and see what limitations are thereby imposed upon the latter.

Our fundamental equations are then as follows:

$$Qe/R^2 = mv^2/R, \text{ and } Qe/R = mv^2 \qquad (1)$$

$$mv^2 = 2h\nu = 2hc/\lambda \qquad (2)$$

$$v = 2\pi R\nu. \qquad (3)$$

We have here three equations involving the variables Q, R, v, and λ, and by means of these we may express any one of these variables in terms of one of the others and of known quantities or of a numerical quantity. In computing these relations the following values of constants are used:

$$h = 6.55 \cdot 10^{-27}.$$

$$e/m = 1.765 \cdot 10^7 \text{ E.M.U.}$$

$$e = 4.774 \cdot 10^{-10} \text{ E.S.U.}$$

$$c = 2.9989 \cdot 10^{10}.$$

Then,

$$Q = hv/e\pi = 4.367\, v \cdot 10^{-18}. \qquad (4)$$

$$v^2 = \frac{2hc}{m\lambda} = \frac{4.357 \cdot 10^{10}}{\lambda} \text{ and } v = \frac{6.60 \cdot 10^5}{\sqrt{\lambda}}. \qquad (5)$$

$$\text{From (4) and (5), } Q = e\pi\sqrt{\frac{2h^3c}{m\lambda}} = \frac{2.882 \cdot 10^{-12}}{\sqrt{\lambda}}\,. \tag{6}$$

$$\text{From (1) and (4), } Rv = h/m\pi = 2.312. \tag{7}$$

$$\text{Also, } R = \frac{3.503 \cdot 10^{-6}}{\sqrt{\lambda}}\,. \tag{8}$$

Of course, many other similar relations may be shown.

The foregoing equations allow us to compute the kinetic energy of the revolving electrons from two distinct considerations. Thus, from equation (1), $\frac{1}{2}mv^2 = Qe/2R$; from the photo-electric equation, $\frac{1}{2}mv^2 = h\nu$. One of these equations involves the assumption of orbital motion and the other does not.

It is a frequent characteristic of the spectra of elements that there are several pairs of lines whose vibration-frequencies differ by a constant number. From the photo-electric equation the kinetic energies of the electrons which produce these lines must differ by a constant quantity, $\Delta h\nu$. Hence we may write $\Delta h\nu = \frac{e}{2}\left(\frac{Q_1}{R_1} - \frac{Q_2}{R_2}\right)$. I have computed these differences of energy by the two methods from seventeen pairs of lines in the thallium spectrum which differ by about 7791 vibrations per cm and both give the same value for the constant energy difference, viz., $1.54 \cdot 10^{-12}$ ergs/sec. The agreement is equally good for the two triplet series of mercury, the frequency-differences being alternately 4630 and 1766.

We may also test our equation for orbital motion in another manner. Thus the potential differences necessary to produce the single-line spectra of a number of elements have been determined, and the wave-lengths of the spectral lines are known. The velocity given to the inducing electrons by the potential difference required to set up the radiation may be calculated from the equation $v = 5.95\sqrt{E} \cdot 10^7$, where E is volts and v the electron velocity in cm/sec.

But v may be calculated in terms of the wave-length of the given spectral line from equation (5), viz., $v = 6.60 \cdot 10^5/\sqrt{\lambda}$. Equating these two values of v, we have $E = 1.23 \cdot 10^{-4}/\lambda$.

In Table I are given the observed and calculated voltages necessary to produce the single-line spectra of a number of elements. It will be seen that in every case the observed value is just a little greater than the minimum theoretical value.

TABLE I

Element	λ	E Obs.	E Calc.
Mercury	2537	4.89	4.85
Zinc	3076	4.04	4.00
Cadmium	3260	3.81	3.77
Magnesium	2852	4.35	4.31
Calcium	4227	2.94	2.91
Strontium	4608	2.69	2.67
Barium	5536	2.24	2.22

Passing to the consideration of the spectral series of hydrogen, it is well known that the most successful equation for calculating the wave-lengths of a spectral series from a fundamental wave-length is the one proposed by Balmer for the series which has since borne his name. In addition to the Balmer series three other series have been successfully calculated in hydrogen. One of these was first discovered in stellar spectra by E. C. Pickering; another, generally known as the principal series, was made out by Rydberg; and still another was predicted by Ritz in the extreme ultra-violet and was discovered by Lyman.

If, as has been shown in the case of X-rays, the atomic charge is the variable quantity upon which the wave-length of the spectral lines is based, it should be possible to find a characteristic hydrogen charge which may serve as a starting-point for calculating the effective charge to which each spectral line is due.

We have seen that upon the hypothesis of a planetary orbit for each radiating electron its effective charge may be computed from the equation $Q = 2.882 \cdot 10^{-12}/\sqrt{\lambda}$. It is accordingly possible to compute this hypothetical charge for every line in the Balmer series, and if our hypothesis is correct these charges should be capable of computation in terms of a single charge by a serial formula of the same character as the serial formula for wave-length. Since in our equation for the central charge $Q \propto 1/\sqrt{\lambda}$, it will be

convenient to use the Balmer equation in the form $1/\lambda = 1/h\left(\frac{m^2-4}{m^2}\right)$. It should then be possible to calculate Q from this equation by writing $Q = A\sqrt{\frac{m^2-4}{m^2}}$. In this equation A becomes a fundamental atomic charge from which all the other charges are computed. To determine A we have only to calculate the charge corresponding to each spectral line from the equation $Q = 2.882 \cdot 10^{-12}/\sqrt{\lambda}$ and to divide this charge by $\sqrt{\frac{m^2-4}{m^2}}$ for the corresponding line. The results of this computation for the first nine lines of the Balmer series and for the line for which $m = 31$ are given in Table II.

TABLE II

m	$\lambda \cdot 10^8$	$Q \cdot 10^{10}$	$A \cdot 10^{10}$
3	6563.07	3.558	4.776
4	4861.57	4.135	4.774
5	4340.53	4.373	4.774
6	4102.00	4.496	4.769
7	3970.33	4.574	4.774
8	3889.15	4.618	4.771
9	3835.51	4.656	4.775
10	3798.00	4.678	4.774
11	3770.73	4.694	4.773
31	3661.31	4.763	4.773

The foregoing values of the fundamental charge of hydrogen calculated from the Balmer series give a mean value of $4.773 \cdot 10^{-10}$ E.S.U., and we may write the equation for the effective charges corresponding to the lines of the Balmer series $Q = e\sqrt{\frac{m^2-4}{m^2}}$, where e is the unit electrical charge. This makes it possible to calculate the wave-lengths of all the lines in the Balmer series in terms of the unit electrical charge and the serial number of the line, thus,

$$\lambda = \frac{8.306 \cdot 10^{-24}}{e^2\left(\frac{m^2-4}{m^2}\right)}.$$

This equation also enables us to give a physical meaning to the Rydberg frequency-constant N. Thus the Balmer formula is frequently written $1/\lambda = N\left(\frac{1}{4} - \frac{1}{m^2}\right)$. The corresponding equation for the central charge then takes the form $Q = A'\sqrt{\frac{1}{4} - \frac{1}{m^2}}$.

If we compute A' from this equation it necessarily comes out twice as great as A in the former equation, and our equation for Q becomes $Q = 2e\sqrt{\frac{1}{4} - \frac{1}{m^2}}$. Since the value of Q increases with an increase of m until it becomes $Q = 2e\sqrt{\frac{1}{4}}$, it necessarily gives the same value for the atomic charge as does the former equation.

It follows from this that the Rydberg frequency, N, is the frequency which would be given by an electron revolving about a positive charge $Q = 2e$.

For the Pickering series in hydrogen Ritz gives the formula $1/\lambda = N\left[\frac{1}{4} - \frac{1}{(m+0.5)^2}\right]$. This equation reduces to the same value for N as does the Balmer equation, and for $m = \infty$, $Q = 2e\sqrt{\frac{1}{4}}$.

Since this is a series which seemingly cannot be accounted for by the Bohr equation, and which Bohr accordingly regards as a helium series, it may be worth while to compute the value of the fundamental atomic charge from the wave-lengths of some of the lines of this group.

In Table III are given the values of $2e$ computed from the first eight lines of the Pickering series, using the equation $2e = \dfrac{Q}{\sqrt{\frac{1}{4} - \frac{1}{(m+0.5)^2}}}$, Q being calculated as before from the equation $Q = 2.882 \cdot 10^{-12}/\sqrt{\lambda}$.

The lines of the Pickering series accordingly give a mean value for $2e$ of 9.544, making $e = 4.772$.

The principal series of hydrogen lies in the ultra-violet, with the exception of a single line. Rydberg has calculated the position of this series from the formulae for the Balmer series and the Pickering series, and a number of the lines have been observed in hot stars, though not in the vacuum-tube discharge. The Ritz

formula for this series is $1/\lambda = N\left[\frac{1}{(1.5)^2} - \frac{1}{(m+1)^2}\right]$. It should transform into $Q = 2e\sqrt{\frac{1}{(1.5)^2} - \frac{1}{(m+1)^2}}$.

TABLE III

m	$\lambda \cdot 10^8$	$Q \cdot 10^{10}$	$2e \cdot 10^{10}$
3	5413.6	3.916	9.550
4	4542.4	4.276	9.545
5	4200.7	4.447	9.543
6	4026.0	4.542	9.542
7	3924.0	4.600	9.543
8	3860.8	4.641	9.549
9	3815.7	4.663	9.536
10	3783.4	4.686	9.544

The values of $2e$ computed from this equation and a few of the lines of the series are given in Table IV.

TABLE IV

m	$\lambda \cdot 10^8$	$Q \cdot 10^{10}$	$2e \cdot 10^{10}$
1	4687.88	4.207	9.540
2	2734.55	5.510	9.551
3	2386.50	5.900	9.547
4	2253.74	6.067	9.540
5	2187.60	6.158	9.540

The mean value of $2e$ is, from the foregoing table, 9.554.

It will be observed that while the principal series of hydrogen enables us to calculate $2e$ with the same degree of accuracy as the other series, it converges to a positive charge greater than e, viz., $Q = 2e\sqrt{\frac{1}{(1.5)^2}} = 6.366 \cdot 10^{-10}$. Since we know of no way of increasing the positive charge of a hydrogen atom except by abstracting an electron, it would seem that it must be possible to have a hydrogen atom with a nuclear charge of $2e$. Such an atom should give off radiation of higher frequency than one with a charge only half as great, and its spectrum should be looked for in the ultra-violet.

Ritz concluded from other considerations that there should be a series in hydrogen with the formula $1/\lambda = N\left(1-\frac{1}{m^2}\right)$. This is a much simpler formula than the others, and it transforms into the equation $Q=2e\sqrt{1-\frac{1}{m^2}}$.

Three of the lines of this series were identified by Lyman, and their wave-lengths and the corresponding values of $2e$ are given in Table V.

TABLE V

m	$\lambda \cdot 10^8$	$Q \cdot 10^{10}$	$2e \cdot 10^{10}$
2	1216	8.258	9.302
3	1026	9.006	9.552
4	972	9.237	9.540

Since in this case the wave-lengths cannot be known with as great an accuracy as in the case of longer waves, the values of $2e$ do not agree as well as in the other series, but two of the three lines give values in close agreement with the values calculated from other series.

In this connection it may be interesting to recall that the α-particle, which is apparently the positive nucleus of the helium atom, is generally supposed to carry a positive charge $Q=2e$. Accordingly, this series might belong to helium as well as to hydrogen, as far as our evidence goes. As a matter of fact, Lyman is unable to say positively to which element it does belong, since all of the foregoing lines appeared when the tube contained either gas. Lyman says,[1] "In connection with Bohr's speculations it is important to observe that λ 1216, which forms the first member of the Ritz series, occupies exactly the same position when obtained from helium as when it is produced in hydrogen."

The convergence wave-length of this series would be $\lambda = 912$. As Lyman found no lines shorter than about $\lambda = 900$ which could be attributed to hydrogen, though he found lines as short as $\lambda = 600$, it seems extremely probable that there are no lines in the hydrogen spectrum which require a greater nuclear charge than $2e$.

[1] *Astrophysical Journal*, 43, 100, 1916.

The fact that the hydrogen atom may apparently exist in two electrical conditions, in one of which the central charge is e and in the other of which it is $2e$, raises the question as to which corresponds to the normal or so-called neutral condition of the atom. If the normal charge is e, the charge $2e$ must be caused by the abstraction of an electron from the normal atom, and the radiating system which gives rise to the principal series and the Ritz-Lyman series must be electro-positive. If, on the other hand, the normal atom has a central charge of $2e$, it would seem that the Balmer series and the Pickering series must be due to atoms which have captured an electron and have thus become electro-negative.

It is at least suggestive in this connection to recall that in his work on "Duration of Luminosity of Electric Discharge in Gases and Vapours"[1] Strutt found that the luminous hydrogen atoms which gave the Balmer series were universally electro-negative, and that when deflected to one side by an electric field they showed no traces of any other lines than those of the Balmer series.

There is so far no experimental evidence that either hydrogen or helium may give rise to X-rays. The Moseley equation for k-radiation would give for hydrogen $Q=2e(1-b)$ and for helium $Q=2e(2-b)$. If one accepts Moseley's value of $b=1$ for the k-radiation, λ_k for helium $=912$, and the Ritz-Lyman series in hydrogen and helium starts with the convergence wave-length of the X-ray spectrum of helium.

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May 20, 1918

[1] *Proceedings of the Royal Society*, **94**, 88, 1917.

THE VARIATION WITH TEMPERATURE OF THE ELECTRIC FURNACE SPECTRA OF CALCIUM, STRONTIUM, BARIUM, AND MAGNESIUM[1]

By ARTHUR S. KING

APPARATUS AND METHODS

The methods used in this investigation for observing the temperatures at which spectral lines appear and the rate at which they increase with rising temperature have been the same, in the main, as those employed for the examination of other spectra.[2] The electric furnace was operated *in vacuo*, except for occasional tests under other conditions, and the spectra were photographed in the first and second orders of a 15-ft. concave grating mounted in the vertical spectrograph in the Pasadena laboratory.[3] Spectrograms made with a 1-meter concave grating were used for a few lines in the ultra-violet and extreme red to supplement those of higher dispersion.

To obtain the various spectra the furnace was charged with calcium metal in small pieces, strontium chloride or bromide, barium chloride, and magnesium metal, both as powder and as fragments cut from rods.

A temperature of 1650° C., produced by a potential of 15 volts applied to the graphite tubes regularly used, yielded a considerable number of lines for each element. This accordingly was taken as the low-temperature stage. As intervals of 350° showed distinctive changes in the spectra, temperatures of approximately 2000° and 2350° were chosen as the medium and high temperatures, respectively. The spectrum of each of the substances was photographed at still higher temperatures, but the change beyond 2350° consisted chiefly in a broadening of lines already present, with more frequent

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 150.

[2] *Mt. Wilson Contr.*, Nos. 66, 76, 94, 108; *Astrophysical Journal*, **37**, 239, 1913; **39**, 139, 1914; **41**, 86, 1915; **42**, 344, 1915.

[3] *Mt. Wilson Contr.*, No. 84; *Astrophysical Journal*, **40**, 205, 1914.

reversals. At these temperatures calcium, strontium, and barium gave strong continuous spectra, which tended, through the equalization of emission and absorption, to conceal most of the sharp lines, while those usually showing reversal appeared as wide absorption lines, often with a width of several angstroms. This rendered impossible any comparison with the relative intensities at lower temperatures, so that the useful plates were limited to a temperature range within which the spectra always consisted of emission lines.

For the identification of the lines and a comparison of their intensities with those of the vacuum furnace, the arc in air was used, with the result that certain lines, especially in the spectra of calcium and barium, showed notable differences in structure in the two sources. Lines which in the air-arc are extremely diffuse, often without a distinct maximum and sometimes showing only as wide hazy patches, appear in the vacuum furnace as sharp lines which permit of close measurement, two or three such lines sometimes taking the place of the diffuse arc line. The reduced pressure in the furnace is in a measure responsible for this condition, but other conditions in the arc discharge contribute to the effect. A somewhat detailed study has been made of lines of this type in the spectrum of barium, including their changes under various conditions of temperature and pressure and the measurement of their furnace wave-lengths. This has yielded a considerable addition to the list of barium lines in the ultra-violet. Their peculiarities will be discussed later in the paper.

The line-classification adopted in previous papers has been adhered to as closely as possible, but in the extension into the ultra-violet a limit is reached beyond which no lines are emitted by the furnace at the given temperature, even with prolonged exposure. This limit proceeds steadily toward shorter wave-lengths as the temperature rises and furnishes the chief point of resemblance between the emission of furnace lines and that of the continuous spectrum of an incandescent solid. The effect on the classification is that lines beyond the point where the lowest temperature gives a spectrum go automatically into classes not lower than Class III, while lines too far in the ultra-violet to be given by the medium temperature chosen must be placed in Class IV or V. It may

happen as a result of this method that lines such as λ 2852 of magnesium or λ 3072 of barium are placed in Class III because they are too short in wave-length to be produced at the low furnace temperature, though they are very strong when the temperature is sufficient to show a spectrum in this region. In the visible region lines of this character would probably go into Class I. Lines belonging to a series which extends into the ultra-violet cannot be expected to go as a whole into the same temperature class, since the ultra-violet members may not be produced by a temperature which, for members of the same series in the visible region, results in a high intensity. While this condition must be considered as entering into the method of classification, the character of a line is indicated clearly enough for most purposes by noting the intensities for those temperatures at which the line in question is radiated.

EXPLANATION OF THE TABLES

The first column of each table contains the wave-lengths of lines as measured by Exner and Haschek[1] in the spectrum of the arc in air. In the second column are wave-lengths on the international system. For calcium the measurements of Crew and McCauley[2] with the vacuum arc are used, as the structure of lines in this source is similar to that of furnace lines and their list is more complete than that of Exner and Haschek, frequently giving values for groups of two or three lines which blend into a hazy line in the arc in air. For strontium the measurements of Hampe[3] on the international system are entered in the second column, and for magnesium those of Nacken,[4] the source in each case being the arc in air. For barium the international wave-lengths, beginning with λ 3640, are those for the arc in air given by Schmitz;[5] but to the violet of this point measures of the barium lines in the vacuum furnace made by the writer, with the assistance of Miss Brayton of the Computing Division, have been used, since the vacuum furnace lines are more numerous and include components whose wave-lengths differ appreciably from those of the corresponding diffuse blends of the arc in air.

[1] *Spektren der Elemente bei normalem Druck*, Leipzig, 1911.

[2] *Astrophysical Journal*, **39**, 29, 1914.

[3] *Zeitschrift für wissenschaftliche Photographie*, **13**, 348, 1914.

[4] *Ibid.*, **12**, 54, 1913.

[5] *Ibid.*, **11**, 209, 1912.

The sign † adjacent to a wave-length refers to a remark at the end of the table, which often includes important data regarding the character of the line.

Following the plan of previous papers the remaining columns in the tables give the intensity estimates for lines in the arc and for three furnace temperatures. The method of assigning lines to classes is also the same. Nebulous lines are indicated by "n" after the value of the intensity, very pronounced haziness being denoted by "N." The letters "r" and "R" indicate partial and complete self-reversal. Lines of Class I and Class II are strong at low temperature, those of Class II strengthening more rapidly as the tube becomes hotter, while Class I includes the lines for which the low-temperature furnace is especially favorable. Lines of Class III are absent or faint at low temperature, but appear at medium temperature, and are usually considerably stronger at high temperature. Class IV appears at the highest furnace temperatures, while Class V is usually absent in the furnace or, if present, the lines are faint compared with their arc intensities. "A" after the class number indicates that the line in question is relatively weak in the arc—usually not more than half as strong as in the high-temperature furnace.

For calcium and strontium a column headed "Series" is introduced, in which a line is placed in a proper pair, triplet, or "single-line" series according to the notation of Saunders.[1]

CALCIUM

Table I indicates the relative condition of the calcium lines for various furnace temperatures and for the arc in air. The number of lines in the furnace spectrum is practically the same as that observed by Crew and McCauley in the vacuum arc, the low pressure bringing out certain lines which in the arc in air are either quenched or blended with neighboring widened lines. Many calcium lines retain a high intensity at low temperature. In the red, certain lines are especially strong at medium temperature. Lines of Class V are found in the strong arc pairs λλ 3159, 3179 and λλ 3706, 3737, which have not been obtained in the furnace. The flame line λ 4227 dominates the furnace spectrum at all the temperatures

[1] *Astrophysical Journal*, 32, 153, 1910.

TABLE I

TEMPERATURE CLASSIFICATION OF CALCIUM LINES

λ Exner and Haschek	λ Crew and McCauley (I. A.)	Arc	Furnace: High Temperature	Furnace: Medium Temperature	Furnace: Low Temperature	Class	Series
2995.06....	2994.953	5	5	1	....	III	T
2997.41....	2997.309	5	5	1	....	III	p
2999.74....	2999.651	4	4	1	....	III	T
3000.96....	3000.865	5	5	1	....	III	
3006.97....	3006.864	6	6	2	....	III	p
3009.30....	3009.212	5	5	1	....	III	T
....	3107.388	1N	1	....	....	IV	T_2
....	3117.656	1N	2	....	....	IV A	T_2
....	3136.003	1N	4	1	....	IV A	T_1
....	3140.782	3N	6	1	....	IV A	T_1
....	3141.164		3	....	....	IV A	T_1
....	3150.747	4N	8	2	....	III A	T_1
....	3151.280		3	....	....	IV A	T_1
3159.01....	3158.877	10	....	....	....	V	P_1
....	3164.618	1N	1	....	....	IV	T_2
....	3169.854	1N	2	....	....	IV A	T_2
3179.50....	3179.340	15	....	....	....	V	P_1
....	3180.521	1N	4	1	....	III A	T_2
3181.43....	3181.283	4	....	....	....	V	P_1
....	3209.930	2n	6	4	....	III A	T_1
3215.0	3215.145	5n	8	5	....	III A	T_1
....	3215.334		4	2	....	III A	T_1
3225.6	3225.883	8n	10	6	....	III A	T_1
....	3226.129		5	3	....	III A	T_1
3269.37....	3269.090	1n	2	1	....	III A	T_2
3274.95....	3274.661	2	4	2	....	III A	T_2
3286.35....	3286.060	4	6	4	....	III	T_2
3344.52....	3344.508	8n	10	5	1	III	T_1
3350.25....	3350.198	25n	15	8	2	III	T_1
....	3350.361		10	5	1	III	T_1
3361.95....	3361.918	35n	20	12	3	III	T_1
....	3362.131		10	5	1	III	T_1
3468.70....	3468.484	4	5	2	1	II	T_2
3475.01....	3474.774	8	8	4	2	II	T_2
3487.82....	3487.611	12	10	6	3	II	T_2
3624.19....	3624.107	20	20	10	1	III	T_1
3630.87....	3630.749	30	30r	15	3	III	T_1
3631.07....	3630.973	15	15	5	1	III	T_1
3644.53....	3644.400	40	40R	15	5	III	T_1
3644.90....	3644.760	15	20	5	1	III	T_1
....	3644.990	2	3	1	....	III	T_1
....	3673.448	1	4	1	....	III A	t
....	3675.307	2	8	2	....	III A	t
....	3678.240	3	12	3	....	III A	t
3706.18....	3706.022	10	....	....	....	V	P_2
3737.06....	3736.903	12	....	....	....	V	P_2
....	3748.374†	....	6	....	....	IV A	t
....	3750.349†	....	12	1	....	IV A	t
....	3753.367†	....	15	1	....	III A	t

TABLE I—*Continued*

λ Exner and Haschek	λ Crew and McCauley (I. A.)	Arc	Furnace High Temperature	Furnace Medium Temperature	Furnace Low Temperature	Class	Series
....	3872.552†	?n	3	1	....	III A	t
....	3875.807†	?n	4	2	....	III A	t
....	3889.141†	....	2	1	....	III A	SL_3
3933.81....	3933.664	400R	60R	30	20	II	PH
3949.10....	3948.899	6	6	3	1	III	T_2
3957.22....	3957.054	10	10	6	2	III	T_2
3968.63....	3968.465	350R	50R	25	18	II	PH
....	3972.578†	....	3	1	....	III	SL_3
3973.91....	3973.716	12	12	8	3	III	T_2
....	4058.912	1n	1	....	....	IV	SL_2
4093.00....	4092.649	8	4	3	1	III	t
4095.30....	4094.944	12	8	6	2	III	t
4098.9	4098.552	15	10	8	3	III	t
....	4108.554†	10N	8	3	....	III	SL_3
4226.90....	4226.731	500R	1000R	500R	300R	I	SL
4240.61....	4240.455	6	6	2	tr	III	SL_2
4283.20....	4283.008	40	30R	30	30	I	p
4289.50....	4289.363	40	30R	30	30	I	T
4299.18....	4298.989	30	25R	25	25	I	T
4302.70....	4302.525	60r	50R	40	40	I	p
4307.90....	4307.738	45	35R	35	30	I	
4318.80....	4318.648	45	35R	35	30	I	T
4355.50....	4355.099	25	15	8	2	III	SL_3
4425.60....	4425.428	50	30R	40	30	I	T_1
4435.17....	4434.948	60r	50R	50	40	I	T_1
4435.88....	4435.673	40	25r	35	25	I	T_1
4455.00....	4454.765	80	70R	60	50	I	T_1
4456.10....	4455.875	40	25r	35	30	I	T_1
4456.84....	4456.612	10	10	10	5	II	T_1
....	4507.854	1	1	....	....	IV	
4509.8	4509.446	3	3	1	....	III	
4512.7	4512.281	5	5	2	....	III	
4527.35....	4526.944	30	15	8	3	III	SL_2
4578.88....	4578.570	30	10	10	6	II	t
4581.77....	4581.414	40	15	15	8	II	t
4586.22....	4585.868	50	20	20	12	II	t
4685.35....	4685.264	12	6	3	....	III	
4847.38....	4847.292	2	5	1	....	III A	
4878.38....	4878.132	50	25	20	4	III	SL_3
....	5019.981†	....	2	1	....	III A	vp
5041.83....	5041.613	40	15	5	2	III	SL_2
5189.00....	5188.846	50	25	5	2	III	
5260.58....	5260.375	2	2	1	....	III	
5261.87....	5261.701	20	20	10	2	III	
5262.40....	5262.238	25	25	12	2	III	
5264.41....	5264.237	20	20	10	2	III	
5265.73....	5265.559	40	40	30	4	III	
5270.44....	5270.272	60	60	40	6	III	
5349.69....	5349.470	25	20	8	2	III	
5513.16....	5512.978	20n	5	2	....	III	
5582.20....	5581.973	25	20	10	2	III	

TABLE I—*Continued*

λ Exner and Haschek	λ Crew and McCauley (I.A.)	Arc	Furnace High Temperature	Furnace Medium Temperature	Furnace Low Temperature	Class	Series
5588.94....	5588.746	80	50	50	8	III	
5590.30....	5590.109	20	15	10	2	III	
5594.70....	5594.464	60	40	40	6	III	
5598.60....	5598.484	50	50	30	4	III	
5601.50....	5601.283	30	30	10	2	III	
5603.09....	5602.829	25	20	10	2	III	
5857.69....	5857.476	100	80	15	2	III	
5867.89....	5867.578	1	1	....	....	IV	
6102.92....	6102.716	80	80	100	40	II	T_2
6122.49....	6122.216	100	80	150	60	II	T_2
6161.57....	6161.309	10	2	1	....	III	
6162.41....	6162.177	150	60	150	70	II	T_2
6164.05....	6163.749	10	5	2	....	III	
6166.70....	6166.443	15	10	5	....	III	
6169.30....	6169.034	25	15	8	1	III	
6169.82....	6169.576	40	25	15	2	III	
6439.35....	6439.086	150	100	125	50	II	
6450.01....	6449.811	50	40	20	8	II	p
6455.83....	6455.606	10	10	4	2	II	p
6462.80....	6462.576	125	80	100	40	II	
6471.90....	6471.659	40	30	15	6	II	
6494.02....	6493.789	80	60	60	25	II	p
6499.84....	6499.648	30	25	15	6	II	p
6573.00....	6572.783	8	30	150	100	I A	
6717.90....	6717.688	30	25	4	1	III	
....	7148.123	10	12	12	5	II	p
....	7202.161	3	4	4	2	II	p
....	7326.099	2	2	1	....	III	

Remarks

λ	
3748–3753.	Appear faintly in vacuum arc.
3872–3875.	In band at λ 3883.
3889.	Appears in vacuum arc.
3972.	Appears in vacuum arc.
4108.	Very hazy in arc; about 2 A wide.
5019.	Appears in vacuum arc.

used. Being very sensitive to widening influences, its intensity at a given temperature depends largely on the vapor density, as was noted in a previous investigation.[1] If a large quantity of vapor is present, as in the experiments which gave the intensities noted in Table I, the line changes from a narrowly reversed condition at low temperature to a great width at high temperature. A width of

[1] *Mt. Wilson Contr.*, No. 32; *Astrophysical Journal*, 28, 389, 1908.

reversal of 6.6 A has thus been obtained, the wings of the line blending with the continuous spectrum. Its progressive strengthening with temperature increase would place it in Class II; but it is essentially a low-temperature line and is not notably strong at high temperature unless much vapor is present. It may properly be classified, therefore, as a special type belonging in Class I.

The H and K lines, λ 3934 and λ 3969, appear at temperatures as low as 1650°; they strengthen rapidly with temperature increase but remain narrow, though it is possible to obtain reversals at high temperature. Their intensities in the arc are enormously greater than in the furnace and offer in this respect a strong contrast to λ 4227.

A large proportion of the calcium lines having been placed in series, it is of interest to note their behavior at different temperatures. As would be expected, the component lines of a series pair or triplet are affected alike; but lines belonging to series groups in different parts of the spectrum may appear in different classes on account of the rapid falling off of the low-temperature spectrum as the wave-length decreases. Thus the intensities of T_1 series lines in the group which has its strongest line at λ 4455.00 are nearly equal at the three temperatures and the lines are assigned to Class I (with the exception of the component λ 4456.84), while the ultra-violet members of the same series fall off rapidly with decrease of temperature and are placed in Classes III and IV. To determine whether the ultra-violet members of a series weaken more rapidly when the temperature is reduced than those of greater wave-length requires careful attention to photographic differences, but this effect has occasionally appeared when furnace spectra at different temperatures were recorded on the same plate. Of the three groups with strongest lines at λλ 4455.00, 3644.53, and 3361.95 the more rapid falling off of the violet groups at reduced temperature was very distinct. This corresponds to the result previously observed for the principal series of caesium.[1]

The alternating single-line series SL_2 and SL_3 show a varied behavior but must be regularly classed as high-temperature lines. In the blue they show a decided contrast with the T_1 series lines

[1] *Astrophysical Journal*, 21, 236, 1905.

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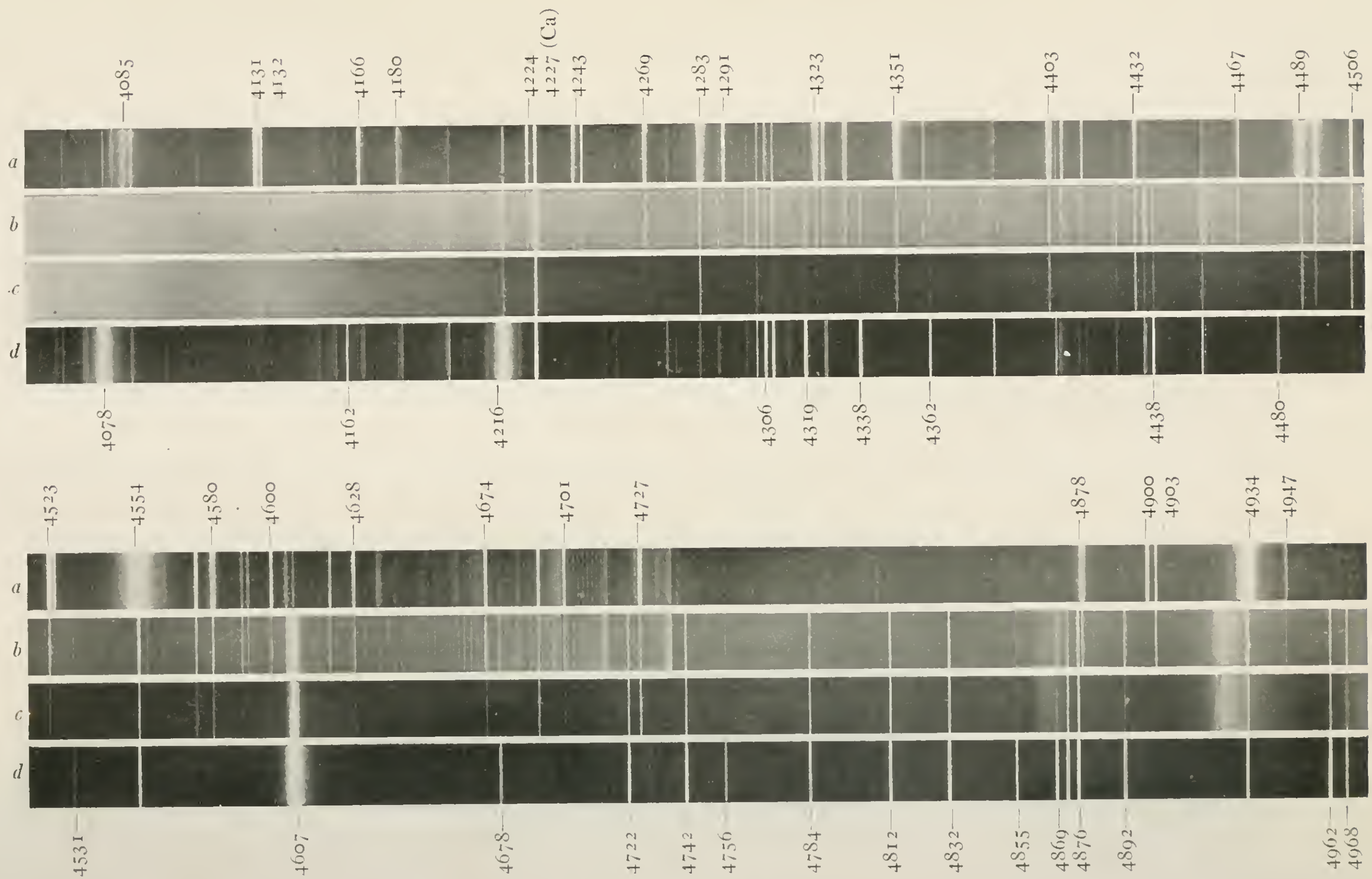
PLATE I



Furnace and Arc Spectra of Barium and Strontium

a. Arc spectrum of barium
b. Furnace spectrum of barium and strontium at 2050°
c. Furnace spectrum of barium and strontium at 1750°
d. Arc spectrum of strontium

and with the group of six near λ 4300. The several lines of this latter group, like those of the similar group near λ 3000, are affected alike at different temperatures.

In the region of greater wave-length the groups λ 5261–λ 5270 and λ 5582–λ 5603 behave in each case as a unit and their lines go into Class III. The lines farther to the red vary greatly in character, a notable feature being the high relative strength at the medium temperature in the case of several lines. λ 6573, whose high intensity in sun-spot spectra has been taken as evidence of the low temperature of these regions, is unique among the calcium lines, being faint in the arc and much weaker at high temperature than at the lower temperatures. Apparently it can be used with confidence as a low-temperature indicator.

STRONTIUM

The strontium lines are widely varied, but their notable features are for the most part sufficiently indicated by the data in Table II. The behavior of the strong lines λλ 4078, 4216, and 4607 in furnace and arc is very similar to that of their counterparts, H, K, and λ 4227 of the calcium spectrum. In the ultra-violet the arc gives a number of hazy, very unsymmetrical lines which reduce to sharp lines in the vacuum furnace, but without the complex structure sometimes observed for lines of this type in the case of barium. As in the case of calcium, the members of "single line" series are notably high-temperature lines. In the red there is a lack of the lines of Class I which appear in this region for both calcium and barium; excepting the strong lines λ 6408 and λ 6504 of Class II the dominant low-temperature lines are in the region of shorter wave-length.

Plate I shows the furnace spectra of a mixture of strontium and barium at temperatures of about 2050° and 1750° C., respectively, with the arc spectra of these substances above and below. The exposure times for the two temperatures were in the ratio of 11 to 1 and show lines of Class I, such as λ 4742 and λ 4812 of nearly the same intensity, while the larger gradations of lines in Classes II and III for these two temperatures are evident. The high intensity of λ 4607 in the furnace is shown, together with the relative faintness of λ 4078 and λ 4215.

TABLE II

Temperature Classification of Strontium Lines

λ Exner and Haschek	λ Hampe (I.A.)	Arc	Furnace High Temperature	Furnace Medium Temperature	Furnace Low Temperature	Class	Series
2569.55....	2569.502	20	10	....	....	IV	SL_1
2932.00....	2931.856	30	15	10	....	III	SL_1
3301.86....	3301.739	50	15	10	....	III	T
3307.6.....	3307.548	50n	15	10	....	III	p
3322.40....	3322.237	30	15	8	....	III	T
3330.20....	3330.011	30	8	8	....	III	
3351.45....	3351.258	150	50	20	....	III	p
3366.51....	3366.339	50	30	12	....	III	T
3371.10....	....	2	....	....	....	V	
3380.98....	3380.721	50	....	....	....	V	P_1
3390.09†...	....	4N	3	2	....	III	T_1
3400.39†...	....	8N	4	3	....	III	T_1
3411.7.....	....	12N	6	4	....	III	T_1
3456.8.....	3456.52	8n	2	1	....	III	T_2
3457.7.....	3457.54	15n	10	6	....	III	T_1
3464.68....	3464.470	50	....	....	....	V	P_1
3475.09....	3474.901	10	....	....	....	V	P_1
3477.3.....	3477.2	8n	8	5	....	III	T_1
3499.4†....	3499.61	20n	10	8	....	III	T_1
3504.9.....	3504.27	2n	1	....	....	IV	T_2
3547.8†....	3548.1	15n	10	6	1	III	T_1
3577.5†....	....	2n	?	?	....	III ?	T_2
3628.7.....	3628.37	3n	2	1	....	III	T_2
3629.1.....	3629.12	10n	6	3	....	III	T_1
3653.30....	3653.26	12n	6	4	....	III	T_1
3654.0.....	3653.91	3n	3	1	....	III	T_1
3706.0.....	3705.90	15n	8	5	1	III	T_1
3940.90....	3940.806	20	8	4	....	III	T_1
3969.40....	3969.270	30	8	5	1	III	T_1
3970.12....	3970.049	20	7	3	....	III	T_1
4030.55....	4030.386	40	15	8	1	III	T_1
4032.50....	4032.387	20	12	3	....	III	T_1
4033.25†...	4033.19	6	?	?	....	III ?	T_1
4061.3.....	4061.1	8n	4	2	....	III	t
4071.22†...	....	10n	6	3	....	III	t
4077.89....	4077.714	400r	40r	25	12	II	PH
4087.88....	4087.46	12n	....	....	....	V	t
4161.99†...	4161.812	30	?	?	....	IV ?	P_2
4215.70....	4215.515	300r	30	15	6	II	PH
4305.68....	4305.459	40	15	5	1	III	P_2
4308.5.....	4308.13	20n	10	8	2	III	t
4313.7.....	4313.23	3n	4	2	....	III	SL_2
4319.4.....	4319.090	25n	15	10	2	III	t
4326.64....	4326.444	8	10	6	2	II	T_2
4338.2.....	4337.704	30n	20	12	3	III	t
4361.88....	4361.710	20	20	15	4	III	T_2
4407.0.....	4406.11	10n	10	5	1	III	SL_3
4412.80....	4412.620	4	5	4	1	III	

TABLE II—*Continued*

λ Exner and Haschek	λ Hampe (I.A.)	Arc	Furnace: High Temperature	Furnace: Medium Temperature	Furnace: Low Temperature	Class	Series
4438.21....	4438.044	25	20	15	6	II	T_2
4451.99....	4451.803	2	3	2	....	III	
4480.9.....	4480.540	10n	10	5	....	III	SL_2
....	4531.340						
4531.52....	....	10	12	8	3	II	
....	4531.359						
4607.51....	4607.340	600R	600R	400R	300R	I	SL?
4678.4.....	4678.304	20n	10	8	2	III	SL_3
4701.2†....	4700.7	2	?	....	....	IV?	
4704.35†...	4704.0	2	?	1	....	III	
4707.5†....	4707.1	2	?	1	....	III	
4714.35†...	4714.0	3	?	2	....	III	
4722.48....	4722.272	30	25	25	20	I	p
4729.8.....	4729.48	4n	4	2	....	III	
4742.08....	4741.917	30	30	25	20	I	T
4755.60....	4755.467	12n	10	4	1	III	SL_2
4784.51....	4784.323	30	30	25	18	I	T
4812.03....	4811.867	40	40r	30	25	I	p
4832.27....	4832.075	50	50r	40	30	I	T_1
4855.20....	4855.078	20	20	12	3	III	t
4868.92....	4868.739	20	20	15	3	III	t
4869.45....	4869.194	4	5	2	....	III	t
4872.70....	4872.485	40	30	25	20	I	T_1
....	4876.062	15	20	15	8	II	T_1
4876.38....	4876.323	20	20	20	10	II	T
4892.20....	4892.009	25	25	20	6	II	t
4892.90....	4892.666	5	5	3	....	III	t
4962.43....	4962.244	40	40	30	20	I	T_1
4968.03....	4967.928	20	20	15	6	II	T_1
4971.79....	4971.649	2	5	3	2	II A	T_1
5156.38....	5156.068	8	8	6	1	III	SL_3
5213.22†...	5212.968	3	3	2?	....	III	
5222.50....	5222.200	20	10	8	3	II	
5225.35....	5225.110	20	10	8	3	II	
5229.51....	5229.266	20	10	7	3	II	
5238.82....	5238.548	30	15	12	6	II	
5257.10....	5256.897	50	20	20	10	II	
5330.11....	5329.816	8	8	5	1	III	SL_2
5451.20....	5450.818 5450.834	15	15	10	2	III	
5481.19....	5480.840	40	30	20	15	II	
5486.40....	5486.121	15	15	10	2	III	
5504.50....	5504.166	30	20	15	8	II	
5522.01....	5521.752	25	20	12	5	II	
5535.10....	5534.796	15	15	8	2	III	
5540.29....	5540.033	15	20	10	2	III	
5543.44....	5543.327	15n	12	3	....	III	
5970.6.....	5970.103	12n	10	3	....	III	
6159.15....	6158.967	1	1	....	....	IV	
6272.32†...	6272.052	2	3	?	....	IV?	

TABLE II—*Continued*

λ Exner and Haschek	λ Hampe (I.A.)	Arc	Furnace: High Temperature	Furnace: Medium Temperature	Furnace: Low Temperature	Class	Series
6346.05† . . .	6345.760	10	?	?	?	IV?	
6364.23	6363.932	8	8	5	1	III	
6370.23	6369.959	15	12	8	2	III	
6381.00	6380.740	30	20	15	3	III	
6386.84	6386.507	40	20	15	4	III	p
6388.50	6388.245	25	15	10	3	III	
6408.76	6408.465	100	60	30	20	II	
6446.89† . . .	6446.676	12	6	?	?	III	
6466.01† . . .	6465.788	10	4	?	?	III?	
6504.21	6503.990	80	50	30	20	II	
6547.00† . . .	6546.785	20	15	5	?	III?	
6550.59† . . .	6550.253	60	25	8	?	III?	p
6617.50† . . .	6617.268	50	20	10	?	III?	
6643.70† . . .	6643.545	20	15	5	?	III?	

Remarks

λ	
3390–3400.	Wide and hazy in arc. λ from Kayser and Runge.
3411.	Wide and hazy in arc.
3499.	Shaded to violet in arc.
3547.	Shaded to violet in arc.
3577.	Disturbed by band.
4033.	Disturbed by band. λ from Kayser and Runge.
4071.	Measured by writer.
4161.	Very faint if present in furnace.
4701–4714.	Disturbed by carbon band.
5213.	Disturbed by band.
6272.	Disturbed by band.
6346.	Concealed by band.
6446–6466.	Disturbed by band.
6547–6643.	Concealed by band at low temperature.

BARIUM

Table III gives the characteristics of the furnace spectrum of barium. Since the grouping of lines according to series is not so marked as with either calcium or strontium, the chief feature to be noted in this respect is that the behavior of the strong lines λ 4554 and λ 4934 is similar to that of H and K of calcium and λ 4078 and λ 4216 of strontium, while the low-temperature line λ 5536 corresponds to λ 4227 of calcium and λ 4607 of strontium. The furnace results thus bear out the resemblance observed between these lines in other light-sources.

The differences of structure shown by furnace spectra for many lines in the ultra-violet as compared with the arc in air warranted

TABLE III

Temperature Classification of Barium Lines

λ Exner and Haschek	λ King (I.A.)	Arc	Furnace High Temperature	Furnace Medium Temperature	Furnace Low Temperature	Class
2596.87		8n	5			IV
2647.42		2	2			IV
2702.70		8	3			IV
2739.38		3	1			IV
2785.36		15	8			IV
3071.72	3071.592	100R	100R	50R		III
3108.3	3108.21	10n	5	1		III
	3117.34	*	3n	1		III
	3117.638	*	3n	1		III
	3119.202	3N	2	1		III
	3132.602	*	8	5		III
	3135.72	*	8n	5n		III
	3137.700	*	10	5		III
	3155.336	*	10	5		III
	3155.673	*	10	2		III
	3158.046	*	12	4		III
	3158.54†	*	12N	6		III
	3165.598†	*	25	15		III
	3173.69	*	15n	10		III
	3183.156	*	30	15		III
3184.5	3183.96†	?N	15	8		III
	3193.912 †	*	25n	10		III
	3193.967 †			5		III
3204.5	3203.700†	25N	25	15		III
	3221.630 †	40N	30	20		III
	3222.188 †		40	25		III
3223.0	3222.441 †		8	3		III
3253.2	3253.067	5n	5	2		III
	3261.961 †	80N	50r	25		III
3263.0	3262.336 †		60R	25		III
3270.30	3270.115	4n	4	2		III
	3272.405	2n	2	1		III
	3281.503 †	70N	40r	20		III
3282.1	3281.772 †		15	6		III
3315.92	3315.753	8n	8	5		III
	3322.797 †	80N	50r	25		III
3323.3	3323.058 †		10	4		III
3357.15	3356.804†	80N	40r	25		III
3377.33	3376.975 †	80N	60R	35		III
	3377.391 †		20	10		III
	3413.835	3n	3	1		III
3420.70	3420.322 †	100N	70R	40		III
	3421.008 †		25	15		III
	3421.476 †		30n	15		III
	3426.453†	4n	3	?		III?
	3427.85	3n	3	1		III
3464.7 †	3463.741	?N	40	25		III
3501.31	3501.115	200R	150R	75r	20	II

* Not visible in barium arc in air, probably an account of extreme diffuseness.

TABLE III—*Continued*

λ Exner and Haschek	λ King Schmitz (I.A.)	Arc	Furnace High Temperature	Furnace Medium Temperature	Furnace Low Temperature	Class
3525.30†....	3524.975	80n	50r	25	2	III
3529.64.....	3529.480	15	10	6	....	III
3531.8†.....	3531.345	30N	40	20	1	III
3545.00†....	3544.663	80n	60R	30	3	III
3547.99.....	3547.696	20n	15	10	1	III
3562.19.....	3561.942	10	10	7	....	III
3566.83.....	3566.660	10	8	6	....	III
3576.24.....	3576.036	10	10	6	1	III
3577.81.....	3577.615	30	10	8	1	III
3579.91†....	3579.670	80n	50R	30	5	III
3586.69.....	3586.505	10	10	6	1	III
3588.32.....	3588.099	10	10	7	1	III
3590.15.....	3589.950	3	8	4	....	IIIA
3593.50.....	3593.204	15n	15	8	1	III
3599.62.....	3599.396	15	10	8	1	III
3611.20.....	3610.957	15n	15	10	1	III
3630.85.....	3630.641	40	30	20	3	III
3637.2 †....	3636.832	40N	40r	20	2	III
....	3639.715 Schmitz	2	2	1	....	III
3640.56.....	3640.391	10	8	5	....	III
3662.71.....	3662.523	15	8	6	....	III
3664.82.....	3664.598	3	3	1	....	III
3675.45.....	3675.268	2	1	1	....	III
3688.8	3688.473	20N	15	8	....	III
3701.90.....	3701.716	3	3	2	....	III
3794.88.....	3794.771	15	15	10	2	III
3862.07†....	3861.905	15	8?	8?	2	III
3889.43.....	3889.314	20	10	8	6	II
3891.90.....	3891.788	50	....	....	....	V
3892.90.....	3892.653	20	10	7	2	III
3900.56.....	....	2	2	2	....	III
3906.15.....	....	2	2	2	....	III
3910.03.....	3909.922	40	20	15	8	II
3917.38.....	....	4	4	3	....	III
3935.91.....	3935.715	50	15	10	6	II
3938.03.....	3937.876	20	10	8	2	III
3945.3 †....	3945.60	10N	15	7	....	III
3947.5 †....	3947.51	5N	8	4	....	III
3975.6	3975.362	4n	5	2	....	III
3983.06.....	....	2	1	1	....	III
3993.55.....	3993.395	80	20	12	6	II
3995.85.....	3995.663	30	15	10	2	III
3998.05.....	....	3	2	2	....	III
4026.55†....	....	2n	1?	....	....	IV?
4081.1	4081.347	6n	8	4	....	III
4085.0	4085.322	30N	30	12	2	III
4087.6	4087.371	8n	20	8	....	III A
4110.35.....	4109.884	2	1	1	....	III
4130.83.....	4130.683	80	....	....	....	V
4132.58.....	4132.444	20	20	20	15	I

TABLE III—*Continued*

λ Exner and Haschek	λ Schmitz (I.A.)	Arc	Furnace High Temperature	Furnace Medium Temperature	Furnace Low Temperature	Class
4166.17.....	4166.017	20	....	....	....	V
4179.58.....	4179.372	8n	20	10	....	III A
4224.20.....	4223.957	12	20	12	2	III
4239.85.....	4239.576	10n	15	10	1	III
4242.80.....	4242.619	10	15	10	1	III
4264.45.....	4264.386	15n	30	15	2	III A
4283.31.....	4283.111	100	40	40	20	II
4291.33.....	4291.165	12	15	10	2	III
4323.05.....	4323.004	20n	35	20	3	III
4325.34.....	4325.152	10	15	10	1	III
4332.97†....	4332.919	10n	10?	10	2	III
4350.60.....	4350.375	80	35	30	12	II
4359.76.....	4359.554	5	8	5	....	III
4402.80.....	4402.550	60	30	30	15	II
4407.07.....	4406.846	15	15	12	2	III
4413.89.....	4413.679	8	10	6	1	III
4432.10.....	4431.914	40	25	25	15	II
4467.30.....	4467.129	12	15	12	2	III
4489.10.....	4488.973	60n	50	35	6	III
4493.79.....	4493.641	50n	40	30	5	III
4506.13.....	4505.936	40	30	25	15	II
4523.48.....	4523.237	60n	40	25	12	II
4525.19.....	4524.946	35	....	....	....	V
4554.21.....	4554.038	1000R	100r	80	70	II
4574.02.....	4573.881	40	30	25	15	II
4579.82.....	4579.667	80	40	30	20	II
4589.83.....	4589.762	8n	15	12	2	III A
4591.92.....	4591.825	10n	15	12	2	III
4599.97.....	4599.751	30	20	15	3	III
4605.10.....	4605.012	8n	15	8	2	III A
4620.19.....	4619.978	20n	15	8	2	III
4628.45.....	4628.330	25n	25	20	3	III
4636.7 †....	4636.333	15N	?	....	....	IV?
4642.5 †....	4642.038	4N	?	....	....	IV?
4652.0 †....	....	5N	?	....	....	IV?
4663.0 †....	....	10N	?	....	....	IV?
4673.75.....	4673.621	30	25	20	12	II
4691.85.....	4691.630	35	30	25	15	II
....	4699.108†	15N	10	6	1	III
4700.67.....	4700.446	20	12	10	2	III
4724.96.....	4724.742	3	4	2	....	III
4726.68.....	4726.455	40	30	25	15	II
4732.0 †....	....	5N	?	?	....	III?
4739.5	....	5N	?	1	....	III
4807.8	....	2N	1	1	....	III
4877.8	4877.650	30N	25	12	2	III
4900.11†....	4899.971	35	3?	....	....	IV?
4903.05.....	4902.898	15	15	10	2	III
4934.26.....	4934.099	700R	70R	60	50	II
4947.50.....	4947.350	8	10	4	1	III
5055.14†....	5054.975	5	?	?	....	IV?

TABLE III—*Continued*

λ Exner and Haschek	λ Schmitz (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
5160.10†....	5159.919	20n	?	?	....	IV?
5175.72†....	5175.619	5	?	?	....	IV?
5177.52†....	5177.448	2	?	?	....	IV?
5267.18†....	5267.033	20	?	?	....	IV?
5277.75†....	5277.625	3	?	?	....	IV?
5303.00†....	5302.808	6	?	?	....	IV?
5305.92†....	5305.758	4	?	?	....	IV?
5309.10†....	5308.952	4	?	?	....	IV?
5405.20.....	5404.920	5n	10	5	....	III A
5424.85.....	5424.616	100	100R	50	10	II
5437.56.....	5437.393	4	10	5	....	III A
5473.90.....	5473.689	3	10	4	....	III A
5519.28.....	5519.115	200	150	70	20	II
5535.70.....	5535.534	1000R	1000R	500R	400	I
5593.50.....	5593.297	3	5	3	....	III
5620.2	....	4n	?	....	....	IV?
5620.6	....	20n	?	....	....	IV?
5680.45†....	5680.173	2	5	3	....	III A
		10	20	10	....	III A
5709.89.....	5709.546	2n	3	1	....	III
5713.80.....	5713.554	4n	6	2	....	III
5778.00.....	5777.695	400r	200R	70	20	II
5784.29.....	5784.105	4	6	2	....	III
5800.60.....	5800.299	100	80	50	4	III
5805.92†....	5805.712	20	30	40	8	II
5819.19.....	5818.906	5	20	12	....	III A
5826.56.....	5826.294	150	125r	60	30	II
5853.91.....	5853.699	200	40	20	....	III
5907.81.....	5907.656	15	30	20	10	II A
5962.7	5962.445	2	7	2	....	III A
5965.00.....	5964.787	5	5	2	....	III
5972.00.....	5971.715	100	80	60	50	II
5978.69.....	5978.496	4	10	6	....	III A
5997.39.....	5997.102	100	80	60	40	II
6019.70.....	6019.505	100	90	60	40	II
6063.44.....	6063.149	200	125r	80	60	II
6083.67.....	6083.441	5	15	5	....	III A
6111.10.....	6110.808	300r	200R	100	80	II
6129.50.....	6129.335	3	5	2	....	III
6141.95.....	6141.760	600r	50	30	5	III
6235.5 †....	....	30n	20	6?	....	III
6323.6 †....	....	3	5	?	....	III?
6341.93.....	6341.697	150	100	100	80	I
6411.95.....	....	4	20	10	....	III A
6451.11.....	6450.842	125	100	100	80	I
6483.21.....	6482.936	200	125	100	60	II
6497.21.....	6496.902	600r	75	75	8	III
6499.10.....	6498.776	300r	300r	200	125	II
6527.60.....	6527.324	250	150	150	100	I
....	6564.38 †	2	5	2?	....	III A
6581.00.....	6580.77 †	6	10	4	....	III

TABLE III—*Continued*

λ Exner and Haschek	λ Schmitz (I.A.)	Arc	Furnace			Class
			High Temperature	Medium Temperature	Low Temperature	
6595.65.....	6595.351	200	125	100	80	I
....	6642.40 †	In	3	?	....	III A
6654.27.....	6654.120	10	15	4	....	III
6675.50.....	6675.280	80	70	80	60	I
6694.08.....	6693.875	70	60	80	60	I

λ	Remarks
3158.	Very diffuse at high temperature.
3165.	Shaded to red at high temperature.
3184.	Appears as hazy patch in arc. Blend V in furnace.
3193.	Not resolved at high temperature.
3204.	Very hazy in arc.
3221.630.	Reduced in atmospheric furnace to one-third strength of λ3222.188.
3222.	Very hazy in arc.
3222.441.	Shows in atmospheric furnace as shading to λ 3222.188.
3261.961.	Reduced in atmospheric furnace to two-thirds strength of λ 3262.336.
3262.336.	Unsymmetrically reversed at high temperature.
3281.772.	Fades to trace in atmospheric furnace.
3322.797.	Unsymmetrically reversed at high temperature.
3323.058.	Not visible as separate line in atmospheric furnace.
3357.	Unsymmetrical to red at high temperature.
3377.391.	Fades to one-tenth strength of λ 3376.975 in atmospheric furnace.
3421.008.	Fades to one-tenth strength of λ 3420.322 in atmospheric furnace.
3421.476.	Very faint and diffuse in atmospheric furnace.
3426.	Disturbed by band.
3464.	Hazy patch in arc. Faint and diffuse in atmospheric furnace.
3525.	Unsymmetrically reversed at high temperature.
3531.	Very hazy in arc. Resembles λ 3464.
3545, 3579, 3637.	Unsymmetrically reversed at high temperature.
3862.	Disturbed by band.
3945, 3947.	Very hazy in arc. Measured in vacuum furnace.
4026.	Disturbed by band.
4332.	Blend with V at high temperature. Probable intensity of V line subtracted.
4636–4663.	Disturbed by carbon.
4699.	Measured in vacuum furnace.
4732.	Disturbed by carbon.
4900.	High-temperature line may belong to carbon.
5055–5309.	Disturbed by band. Faint, if present in furnace.
5680.	Double, not fully resolved.
5805.	Very strong at medium temperature.
6235–6323.	Disturbed by band.
6564–6580.	Measured in vacuum furnace.
6642.	Shaded to red in arc. Measured in vacuum furnace.

a special measurement of these lines as produced by the furnace. These differences consist not merely in a sharpening of the lines, but, in several cases, of a resolution of diffuse arc lines into two

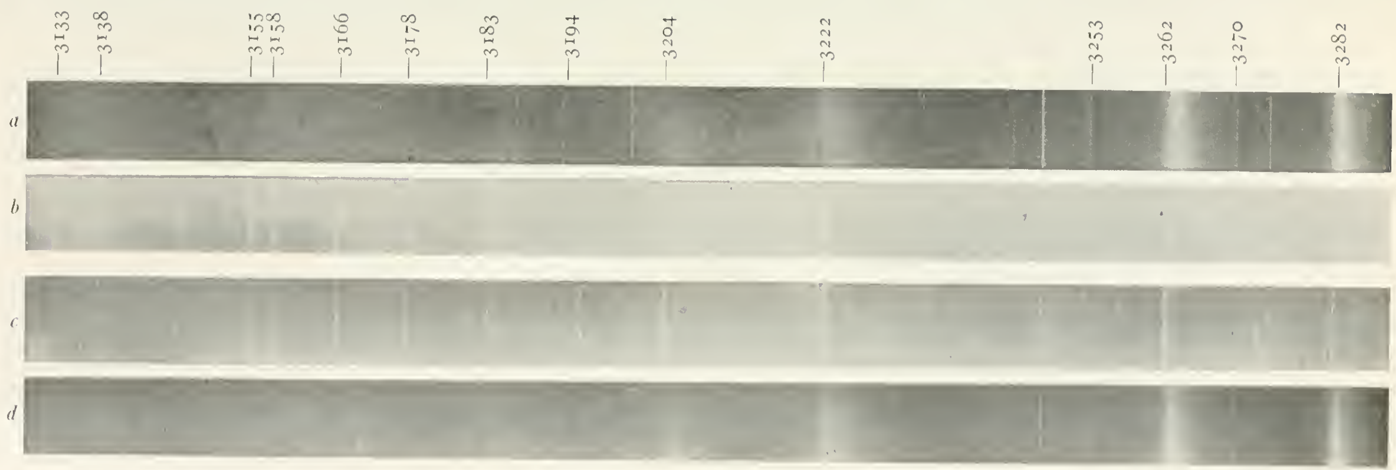
or three components in the furnace. For lines remaining single in both furnace and arc the dissymmetry almost disappears in the furnace spectrum, so that wave-lengths very nearly free from widening influence are obtained.

Photographs for the measurement of furnace lines were made in the second order of the 15-ft. concave grating spectrograph, the dispersion being 1 mm = 1.86 A. The furnace was operated at about 2000°, which gave very sharp lines, without the reversals combined with incipient dissymmetry which appeared at higher temperatures. Standard lines were obtained by mixing a small amount of iron with the barium in charging the furnace. As far as λ 3640, the wave-lengths on the international system in the second column of Table III were deduced from measurements of these plates, the mean of three complete sets being taken. For the iron lines the values of Burns[1] were used. Since the data on pressure-shift are scanty for this region, his wave-lengths, which are for the iron arc at atmospheric pressure, were used for the vacuum source without correction.

The different appearance of the furnace lines as compared with those of the arc in air is shown in Plates II and III. The arc spectra above and below, in each plate, were photographed alongside the furnace spectra at 2350° and 2000° respectively (spectra *b* and *c*), and the close coincidence of symmetrically reversed lines such as λ 3071 and λ 3501 in arc and furnace showed a freedom from instrumental displacement. The arc lines become more and more diffuse as the wave-length decreases until the lines to the violet of λ 3204, while they cannot be said to be absent, are so hazy as to be indistinguishable even on strong photographs. In the furnace they are well defined and fairly sharp and have been measured on the furnace plates. The other barium lines shown are so wide and unsymmetrical in the arc in air that only rough measurements are possible. Measurements of the furnace lines give not only a close value for the position of the unwidened line, but, in some cases, the wave-lengths of component lines whose presence is not indicated in the arc. This results from the fact that lines are frequently resolved into components which in the arc in air are

[1] *Lick Observatory Bulletin*, 8, 27, 1913.

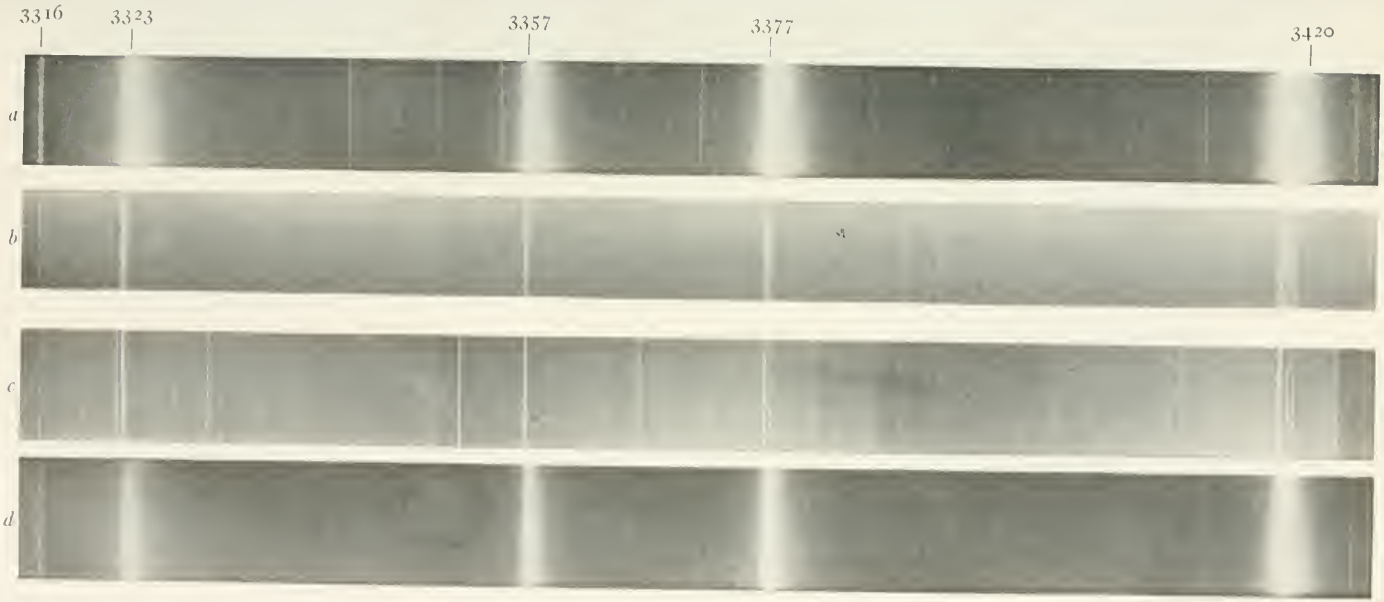
PLATE II



FURNACE AND ARC SPECTRA OF BARIUM

a, *d*. Arc spectrum at atmospheric pressure
b. Furnace spectrum at 2350°
c. Furnace spectrum at 2000°

PLATE III



Furnace and Arc Spectra of Barium

a, *d*. Arc spectrum at atmospheric pressure
b. Furnace spectrum at 2350°
c. Furnace spectrum at 2000°

merely wide, unsymmetrical lines with no indication of structure. These components sometimes occur in pairs, as for λλ 3262, 3282, 3377, and sometimes with a third component as in the case of λ 3222 and λ 3420. The position of the chief component is at the extreme violet edge of the arc line; this is also the case when there is but one component, as, for example, with λ 3357. The extra components, when they occur toward the red, might be considered as blending into a shaded line in the arc, but both λ 3222 and λ 3262 show a component to the violet, where nothing appears in the arc. A violet component of λ 3323 is shown in Plate III*c*, but this is due to a strontium impurity.

The question arises as to whether this resolution into components is due solely to the low pressure in the furnace or in part to the absence of the discharge conditions of the arc. To test this, photographs were made at approximately the same temperature (2300°) with the furnace in vacuum and at atmospheric pressure, the results being compared with the spectrum of the arc in air. The results showed that the furnace at atmospheric pressure gives an intermediate spectrum to the extent that the extra lines characteristic of the vacuum furnace are weakened and an unsymmetrical widening of the principal components has commenced; but the use of atmospheric pressure has by no means changed the furnace structure into that of the arc. The distinctive features of the furnace lines, such as the violet components of λ 3222 and λ 3262, are still present.

Another method of making the furnace lines approach the structure given by the arc is to increase the temperature of the vacuum furnace with plenty of barium present. This, as may be seen from Plate III*b*, leaves the relative intensity of the furnace components almost unchanged from the condition at lower temperature (Plate III*c*), but produces an unsymmetrical reversal of the stronger lines. This dissymmetry evidently is a beginning of the strong one-sidedness shown by the arc lines.

The type of excitation occurring in the arc is thus chiefly responsible for the fading out of certain components and the strong dissymmetry shown by the arc lines in this region of the barium spectrum. Increase of pressure reinforces the widening action but

is only slightly effective in changing the intensity of the components of furnace lines.

Occurrence of barium lines in the solar spectrum.—That barium occurs in the solar atmosphere is rendered fairly certain by the strong arc pair λ 4554 and λ 4934, which correspond in each case with a close doublet in the solar spectrum. λ 5853.91 and λ 6141.95 also occur as strong solar lines. All of these lines are much weaker in the furnace than in the arc. Lines strong in both arc and furnace are faint in the sun, and often unidentified with barium by Rowland. Thus λ 5535.70, which dominates the furnace spectrum, if present in the sun, has an intensity of only 0. Other strong furnace lines are still fainter in the solar spectrum, the tendency being toward a weakening of those lines for whose appearance the furnace is especially favorable. Most of the ultra-violet lines of barium, strongly widened in the arc and reduced to sharp lines in the furnace, are of a type which should be present in the solar spectrum, though probably faint. It is of interest to examine some of the cases in which a wide arc line is replaced by two or three narrow lines in the furnace. Seven groups were taken, from λ 3183 to λ 3421, and their wave-lengths on the Rowland system were deduced from neighboring iron lines. In some cases the position of the barium line coincides with that of a stronger line of another substance, but when this is not the case a solar line of intensity from 00 to 0000 invariably appears close enough to the position of the furnace line to be within the error of measurement. There is thus considerable probability that the solar conditions are such as to produce a resolution of these lines into the components which occur in furnace spectra. Such conditions, according to present evidence, appear to be a moderately high temperature combined with low pressure in the region occupied by the barium vapor.

MAGNESIUM

The magnesium lines listed in Table IV show a great variety of types. The low-temperature spectrum consists of the triplet near λ 3830, the single line λ 4571, and the b group λλ 5167–84. λ 2852, while one of the strongest lines observed during the furnace investigations, is beyond the ultra-violet limit reached by photographs

of the low-temperature spectrum. λ 4571 is a low-temperature line of extreme type, in that it is decidedly weakened both in the furnace

TABLE IV

TEMPERATURE CLASSIFICATION OF MAGNESIUM LINES

λ Exner and Haschek	λ Nacken (I.A.)	Arc	Furnace High Temperature	Furnace Medium Temperature	Furnace Low Temperature	Class
2733.7	2733.55	5n	1	....	....	IV
2736.8	2736.60	10n	2	....	....	IV
2776.82	2776.704	10	4	....	....	IV
2778.40	2778.289	10	4	....	....	IV
2779.95	2779.853	12	5	....	....	IV
2781.51	2781.431	10	4	....	....	IV
2783.08	2782.989	10	4	....	....	IV
2795.64	2795.545	60	10	....	....	IV
2802.82	2802.718	40	8	....	....	IV
2852.25†	2852.128	300R	1000R	400R	....	III
2937.00	2936.754	2	2	....	....	IV
2938.70	2938.487	4	4	1	....	IV
2942.22	2942.016	8	8	2	....	III
3091.20	3091.093	20	20	5	....	III
3093.14	3093.011	40	40	12	....	III
3097.08	3096.914	50	50	15	....	III
3330.09	3329.934	10	10	5	....	III
3332.31	3332.163	15	20	15	....	III
3336.83	3336.688	20	30	20	....	III
3829.51†	3829.364	40	?	20	15	II
3832.49†	3832.306	80r	?	40	30	II
3838.45†	3838.283	100r	?	50	40	II
4059.15	4057.81	5n	3	1	....	III
4167.8 †	4167.65	10n	5?	?	....	III?
4352.35	4351.940	30	?	1	....	IV
4571.31†	4571.114	5	?	80	80	I A
4703.40†	4703.069	40	?	....	....	V
5167.50	5167.303	40	20	10	5	II
5172.87†	5172.673	80	40	25	15	II
5183.78†	5183.600	125	60	40	20	II
5528.70	5528.466	10	....	....	....	V
5711.38	5711.127	1	....	....	....	V

REMARKS

λ	
2852.	Line may attain enormous intensity in furnace. Reversals up to 30 A wide have been observed.
3829–3838.	Difficult at high temperature on account of λ 3883 band. Apparently not stronger than at medium temperature.
4167.	Disturbed by band.
4352.	Concealed by band at high temperature.
4571.	Very faint, if present at high temperature. Weakens as emission line; appears at 2500° as narrow absorption line.
4703.	Disturbed by band; very faint if present.
5172–5183.	Reversed at 2500°.

at high temperature and in the arc. It has not been possible to obtain this line strongly widened, as the conditions which give widening diminish its strength. At high temperature it is likely to appear as a narrow absorption line if the continuous background is strong enough. Its characteristics are very similar to those of λ 6573 of calcium, and, like that line, it is a good indicator of low temperature.

The furnace shows only faintly λ 4352 and λ 4703, both of which are strong in the arc and not enhanced in the spark. The enhanced line λ 4481 is not listed in Table IV, as it does not appear in the ordinary carbon arc containing magnesium. As would be expected, no trace of it is found in the furnace spectrum.

In the blue and green a series of spectrograms made in a previous investigation,[1] when hydrogen at pressure up to 20 cm was used in the furnace, were compared with spectra given by the vacuum furnace, but the hydrogen appeared to have no effect on the line spectrum.

SUMMARY

The foregoing pages summarize an investigation of the furnace spectra of calcium, strontium, barium, and magnesium from the standpoint of the lines appearing at each of three stages of temperature. The lines are classified according to the temperature at which they appear and their rate of change with increase of temperature. Special note is made of connection with series relations, of change of characteristic features of the furnace spectrum with the wave-length, and of differences of structure in furnace lines as compared with those of the arc. The differences of structure, in the case of barium, have led to the measurement of a number of ultraviolet lines not given in tables of arc wave-lengths.

Mount Wilson Solar Observatory
February 1918

[1] *Mt. Wilson Contr.*, No. 114; *Astrophysical Journal*, 43, 341, 1916.

ON STELLAR EVOLUTION

By WILLIAM DUNCAN MacMILLAN

There are two questions at the present time which are of fundamental interest to astronomers and physicists. The first question is, What becomes of the enormous flood of energy which is poured forth so lavishly by the sun and by the stars? Does it travel unendingly through the depths of space until it strikes some material object, or does it not?

The second question is, What is the source of the enormous subatomic energies which have been revealed in recent years by the radioactive elements, and which by implication exist in all of the other elements?

In the first question we ask, What becomes of this energy? In the second, Where does this energy come from? Surely such a situation is not so embarrassing as it would be if we had but one of these questions, for an infinite source or an infinite sinkhole of energy is scarcely to be thought of. The two questions seem mutually to answer one another, and it seems reasonable to conjecture that the energy which disappears from the sun and stars into space reappears sooner or later in the subatomic energies of the atoms.

One may suppose that the physical universe is finite or that it is infinite, for it is not possible to verify either supposition. The idea that the physical universe is finite is doubtless repugnant to most minds that have dwelt upon the subject, and we therefore reject this supposition. The distribution of matter in space may be roughly uniform or it may be distinctly non-uniform. Again we are at liberty to make either supposition, for neither can be verified. But if we assume the universe to be infinite, then unless the distribution of stars is non-uniform of a special type the entire sky should glow with a brightness equal to that of the sun's disk. Certainly this would be true if radiant energy is not extinguished in its course through space.

It is quite possible to distribute infinitely many stars in such a manner that the total quantity of light received from them should be anything we please. For example, imagine a series of concentric spheres of radius 1, 2, 3, ... n, ... ; and on the surface of each sphere is placed a number of stars, the number being equal to the integral part of the square root of the radius of the sphere. If the amount of light received from the star on the first sphere be taken as unity, then the entire amount of light received from all of the stars would be less than $1+\frac{1}{2^{3/2}}+\frac{1}{3^{3/2}}+\frac{1}{4^{3/2}}+\frac{1}{5^{3/2}}+\ldots$ which is finite, but the number of stars in the system would be infinite. In any such distribution, however, the average stellar density approaches zero as the distance becomes sufficiently great. While such distributions of stars are possible, they seem so highly improbable that we reject them and seek some other explanation of the blackness of the night sky.

There is no recourse save in the hypothesis that radiant energy is extinguished in its course through space. If we assume that there is a uniform distribution of stars and that the stars are all alike, there should be four times as many stars in any given magnitude as in the magnitude next brighter. The actual star-counts, however, show that while this ratio is maintained between stars of magnitude one and magnitude two it falls off steadily until between magnitudes sixteen and seventeen the ratio is only 1.8 instead of 4. Is the decline in the number of stars due to the extinction of light in traversing these enormous distances? It is a simple matter to assume that a certain percentage of radiant energy is lost in traveling through space and to test the hypothesis by an appeal to the star counts. Obviously the stars do not all emit the same amount of light; that is, they are not all of the same absolute brightness. Thus the star AOe(N) 17,415 is only 0.004 times as bright as the sun, while Canopus cannot be less than 10,000 times as bright as the sun (absolute magnitudes, of course, being understood). Thus between the faintest known star and the brightest known star there is a ratio of 2,500,000 or sixteen magnitudes (absolute). Assuming that the stars are distributed over fourteen magnitudes (absolute) in accordance with the law of probability, and that 1 per cent of light

is extinguished in traveling 4.11 parsecs (13.6 light-years), the following table has been computed showing the number of stars of the various relative magnitudes on the hypothesis of uniform distribution of the stars in space. The actual star-counts of Chapman and Melotte of the Royal Observatory at Greenwich are given for comparison.

Mag.	Star-Counts	Computed	Mag.	Star-Counts	Computed
6........	2,026	2,201	12..........	961,000	960,200
7........	7,095	7,135	13..........	2,023,000	2,080,000
8........	22,550	21,770	14..........	3,964,000	4,219,000
9........	65,040	62,230	15..........	7,824,000	8,034,000
10........	172,400	166,200	16..........	14,040,000	14,420,000
11........	426,200	413,400	17..........	25,390,000	24,510,000

Certainly there is nothing in these figures to forbid us from supposing that the blackness of the sky is due to the extinction of light in its journey through space; and the amount of the loss (1 per cent in 13.6 years) does not seem excessive.

But what becomes of the energy which is lost? Is it permissible to suppose that the light is intercepted by dark material scattered through space? It is clear that the effectiveness of dark material in cutting off light is increased by supposing it in a finely divided state. If it is supposed that the dust of space consists of particles one one-hundredth of an inch in diameter it is found that one such particle to every 560 cubic miles of space would be sufficient to account for the 1 per cent of loss mentioned above. This does not seem to be an excessive amount of dust particles, and yet a continuation of the computation shows that in the 40 cubic parsecs which, according to the foregoing figures, is the sun's share of space, there is $6\frac{3}{4}$ times as much material as there is in the sun itself, and if the particles average one-tenth of an inch in diameter there is 67.5 times as much material as in the sun.

It may indeed be true that such dark material exists in space, but nevertheless it cannot account for the blackness of the sky, because the energy which it intercepts is either retained or radiated. If it is radiated, then there is no change in the total amount of radiation; at most merely a change of wave-length, since the

amount radiated is the same as the amount intercepted. So far as the total quantity of energy is concerned the result is the same as though the dark material were transparent. If the energy is retained, then the dark material would eventually become hot and would itself be bright. One concludes, therefore, that dark material in space cannot account for the blackness of the sky.

The accepted notion that radiant energy suffers no loss in transmission through a dust-free ether is not analogous to other physical processes, for in the physical world "perfection" does not seem to be attained. Perfection is an intellectual ideal, comfortable only so long as it represents the known facts with an approximation sufficient for our purposes. If we confine ourselves to a sufficiently small portion of the earth's surface we may be well satisfied with the hypothesis that the earth's surface is a plane, for the facts encountered are in close agreement with our hypothesis; but in a larger field of operations the curvature of the earth's surface is thrust upon us and cannot be ignored. So with the transmission of radiant energy it may be quite accurate enough to assume that there is no loss in such distances as are encountered in the solar system, but appreciably wrong when the distances encountered are of interstellar dimensions. According to Kapteyn the average distance of the first magnitude stars is 75 light-years. We have a right to be cautious in extending our hypothesis of "perfection" in the transmission of radiant energy into regions in which 75 light-years is the unit of distance.

If dark material seems inadequate to diminish the total amount of radiation, we may have recourse to the absorption of energy in the ether. But the energy cannot be absorbed without doing work, and in casting about for some sort of work which this lost energy might do there occurs the possibility that it is here that the foundations of the atoms are laid, and perhaps also the completed structure.

Let us assume that absorption does occur and attempt to construct a model to illustrate how the kinetic energy of the ether-waves might be converted into the potential energy of an organized system.[1]

[1] It is not essential, perhaps, to suppose that there is an ether. Some other process would answer our purpose; but it seems preferable to use the current concepts of physics.

Imagine a number of spheres floating on the surface of the ocean. Imagine further that on these spheres there are springs, and that at the bottom of each spring there is a hook. As these spheres are tossed about by the waves there will be frequent collisions, followed in general by an immediate separation. Occasionally, however, two spheres will collide in such a way that when the springs are compressed the hooks are engaged, and separation does not follow. The two spheres are locked in tight embrace, and we have the beginning of an organized system. The energy of the compressed springs was absorbed from the energy of the ocean waves, though the amount of energy absorbed was perhaps relatively small. The two spheres thus joined would, in the course of time, unite with other spheres, and thus an organized system would be built up and the internal energy of the system would have been derived from the ocean waves. It is not necessary, indeed, to dwell upon the details of such a process. Through the agency of chlorophyll it is known that the radiant energy of the sun is absorbed and locked up in the organized systems of the vegetable world, though the mechanical details of the process are quite unknown. In a manner analogous to the organic molecule, and by a process the details of which are quite unknown, we may suppose that the ordinary atom comes into being and that the familiar properties of inertia and gravitation are due to the energies locked up within. Disrupt the atom and set its energies completely free and the properties of mass and gravitation at once disappear.

Important consequences follow the admission that atoms are built up in this manner. It would follow that space contains much material of atomic or even molecular dimensions, and that regions long undisturbed by stellar objects would tend to become more or less crowded with atoms and molecules on account of the ceaseless passage of radiant energy through it. In this manner we see the genesis of a nebula with its enormous gravitational and subatomic energies. A sufficiently large mass in its journey through space would gather in this atomic and molecular material and feed upon its substance and energies. It would be a nucleus around which material would gather. If this nucleus were relatively small and dark, such as the earth, its growth would be slow; the

subatomic energies would persist as subatomic energies, and the mass would increase. In the course of time the internal pressure, density, and temperature would increase, and one can imagine that a critical situation would eventually be reached in which the subatomic energies can no longer wholly persist as subatomic. The atoms begin to break down and give up their stores of energy. In the event of a complete dissolution of the atom we would expect the complete disappearance of its mass and a complete restitution of the energy by which it was organized. If the dissolution were but partial we would have the familiar radioactive phenomena and a partial restitution of the subatomic energies. The energies thus released would raise the temperature of the nucleus and presumably hasten the process of disintegration. The density would decrease until, if the mass were large enough, the increased molecular energies would convert the once solid nucleus into a gaseous sphere. If the mass continued to grow after a completely gaseous state had been reached, the increased gravitational pressure would cause the density again to increase, and this increase with a growing mass would continue until eventually again a critical state would be reached of heat and pressure, and the release of subatomic energies would be so great that the gaseous mass would begin to glow. A further increase of mass would again hasten the process of dissolution accompanied by a rise in temperature and a second decrease in density, and the process could continue, as far as we can see, until the tenuity of a nebula was attained. If the various bodies in our own solar system, with whose masses and densities we are familiar, be arranged according to their masses, it is found that they do conform to these ideas, as is shown in the diagram (Fig. 1). Thus all of the planets and satellites which are smaller than the earth are in a solid state and their densities increase with an increase of mass. Somewhere between the mass of the earth and the mass of Uranus, which is fourteen times the mass of the earth, there would exist a mass of maximum density beyond which a solid mass cannot persist as wholly solid, and there begins a transitional state between the solid and the wholly gaseous condition, and in this transitional state we find the planets Uranus and Neptune. Saturn, with a mass equal to 94 times the mass of the earth, seems to

have attained a wholly gaseous condition and has the smallest density of any object in the solar system. The mass of Jupiter is 3.3 times the mass of Saturn and its density is about twice that of Saturn. There are no bodies in our solar system intermediate between Jupiter and the sun, but it is not difficult to imagine that somewhere in between the increased mass and increased density would again produce an internal condition in which the production of heat would be so great that the mass would begin to glow and that relief from this state of excessive energy would be found in a decreased density. Thus the sun, which is in the very midst of stellar conditions, has a density but little in excess of that of Jupiter notwithstanding its enormous mass.

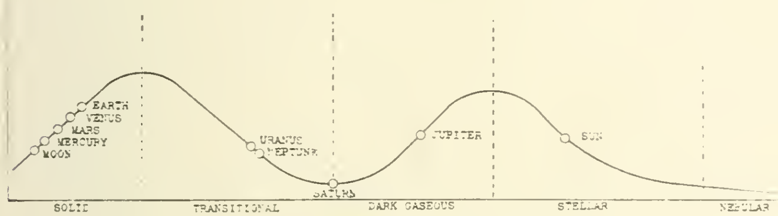


FIG. 1.—Density as a function of mass

When the stellar condition had been reached by a growing gaseous mass, the radiation of its energies into space would afford relief to the imprisoned atoms, tending to check the disintegrating process. Eventually there would be an equilibrium between the energies furnished by the process of dissolution and the energies expended in radiation and gaseous expansion against gravitation. If the process of gathering up atomic and other material from space were discontinued, the mass of the star would diminish, its volume would shrink, and its density would increase. The temperature eventually would fall and the critical state would be passed again, but this time in the direction of a return to the pre-star state. There would be relief from the excessive pressure and temperature which had brought about the release of subatomic energies, and a return to the dark state would be possible.

It is not necessary, however, to suppose that the ingathering process is stopped. If it continued at a suitable constant rate an equilibrium would be attained between the income and outgo of energy, the energies of radiation would be continued, the mass would remain constant, and it would endure as a star forever. If in its wanderings the star passed through a region unusually rich in material, its mass again would grow and its temperature would rise until it attained the white brilliancy of star of spectral type A or B or in the extreme case pass into the nebular state, in which the internal energies are gravitational-potential rather than the kinetic energies of heat. If it is assumed, as is natural, that in such a vigorous process of dissolution there would be a large residue of hydrogen and helium, we can account for the peculiar character of the spectra of stars of these types, for, owing to the lightness of these gases, they would rise to the surface and form an extensive envelope surrounding the brilliant star itself.

In this connection W. W. Campbell has made the following observations:[1]

> The class B stars and the stars containing bright lines are where the planetary and irregular nebulae exist. Going further into detail: wherever there is a great nebulous region either in or near, or outside of the Milky Way you will find the class B and earlier types of stars abnormally plentiful; and the chances are fairly strong that some of the stellar spectra will contain bright lines. This is true of great regions in the Milky Way; it is true of the Orion and Pleiades regions, which we see at some distance outside of the Milky Way structure, though they are doubtless within our system. If you see a wisp of nebulosity near a bright star, look up the star's spectrum and you will probably find it an early class B, as in the case of Gamma Cassiopeiae, a second-magnitude star, with nebulous structure near it whose spectrum contains both dark and bright lines of hydrogen and helium. If you see an isolated bright star enmeshed in an isolated patch of nebulosity, such as the one shown in Fig. 37, and the books say the star (BD$-10°4713$) is yellow, or of class G, communicate your suspicions that the books are mistaken about the star's spectrum to Professor Pickering, and he will probably reply that the star is in reality a very blue one of early class B. That is what happened a fortnight ago about this particular nebula and the star near its apparent center. If you find a red or yellow star of normal type do not look for a nebula in apparent contact with it. Nebulae and red stars do not coexist. You will find about

[1] "Address of the Retiring President of the American Association for the Advancement of Science," *Science*, 45, 545, 1917.

the same number of red stars in the Milky Way that are visible in similar areas far from the Milky Way. You will find an occasional red star in the region of the Orion nebula and of other large nebulae, but the red stars will not appear there in greater numbers than their approximately uniform distribution over the sky requires.

The connection between the nebulae and the bright line stars and between nebulae and the early class B stars is close, both as to their types of spectra and as to their geometric distribution.

Just as the kinetic theory of gases shows that there is a lower limit to the mass of a planet which can retain an atmosphere, so the present considerations suggest a lower limit of mass to a star which can emit light, and possibly also an upper limit to the mass of a star beyond which the star passes into a nebula. An exact correlation between the mass of a star and the intensity of its radiation would be expected only for those stars in which there was equilibrium between the energies released and the energies radiated. A star growing in mass with relative rapidity might lag in temperature, owing to a possible time element in the release of subatomic energies; and likewise a star decreasing in mass might remain for a long period relatively too hot. But on the whole one would expect an increase in temperature with an increase of mass until the kinetic energy of the molecules became so great that the star tended to pass from the gaseous to the nebulous stage. This would mean a decline in internal pressure and in radiation, although the internal energies, increasingly of the potential form, were exceedingly great. The increase in the mean free path of the molecules and the decline in internal pressure would tend to check the release of subatomic energy, and in the extreme nebular state this release may virtually cease. In this manner one is led to imagine a maximum mass beyond which a star, as such, cannot exist. If nebular radiation be left out of consideration, a maximum of stellar mass would imply that there exists a maximum of stellar radiation. Although different stars vary enormously in the amount of their *radiations* their variations in *mass* are not excessive, at least if we may judge from the few masses which are definitely known.

If in its early stellar stage a star is of red color, one would expect to find a class of red stars of relatively small mass, viz., those masses which have been slowly growing toward starhood. On the other

hand, stars which are condensing from a nebula of large mass would present a class of red stars of large mass. The kinetic theory of gases would lead us to doubt the possibility of a nebula of small mass, or a nebula of very low density, even though the mass be large, ever condensing into a star through its own gravitational attraction. Thus, *if all the mass* in the solar system were a spherical nebula 10 times as large as the orbit of Neptune, its velocity of escape would be less than the velocity of escape on the moon, and it is well known that the moon cannot retain an atmosphere. It would seem that such a nebula would dissipate rather than concentrate. A third class of red stars would be those which were approaching extinction or passing into the dark gaseous state. If the main source of stellar radiation is the subatomic energy,[1] and if these energies are completely given up so that the atomic mass disappears, then one would expect those stars which are approaching extinction to be of small mass. If the various chemical elements have different critical conditions of temperature and pressure for the release of their subatomic energies, the mass of a fading star would depend upon its chemical constitution, and so also would the mass of a young star. While this variation of composition would permit a variation in the mass of such stars, on the whole one would expect them to be small and of high density. One would expect, further, red stars of small mass to show considerable variability in their luminosity, owing to the cataclysmic nature of the process of changing from one physical state to another. These anticipations with respect to the red stars are quite in harmony with our present knowledge of this class of stars.

It is natural to suppose that atoms which are formed by the flow of energy through space would have little or no velocity at the time of their formation, and that the recently formed, irregular nebulae would have low velocities. On the other hand, nuclei of stellar types which have long been of stationary or of decreasing mass would have relatively high velocities, owing to the differential

[1] According to the hypothesis of the present paper the energy, or heat, obtained from gravitational contraction is merely energy which has been absorbed from the star itself on some previous occasion during a process of expansion against gravitation. In the long run no energy is obtained from this source, though it serves admirably as a reservoir of energy which can be drawn upon during times of famine.

gravitation of all the stars exerted over enormous periods of time. During the process of its growth, however, a star would be increasing its mass without increasing its momentum, since the momentum of the added material would be approximately zero, and therefore its velocity would be decreasing. If the surmise that a growing mass means an increase in the rate of the release of subatomic energy is correct, then the growing star would push its way through the various spectral types toward class B with an ever-decreasing speed, which is quite in harmony with our knowledge that stars of class B have low velocities and that higher velocities are associated with the stars of deeper color. It harmonizes also with the knowledge that the stars of class B are on the whole the massive stars.

So also a star of class B which was produced from a recently formed nebula would have a low velocity, and this velocity would increase through the ages, owing to the gravitational action of other stars. But in the meantime, as it radiated away its energies and decreased in mass its spectral type would change in the direction from B toward M, so that again there would be an association of the deeper colors with higher velocities. Obviously, the same star may at one time be increasing in mass and decreasing in speed, and at another be decreasing in mass and increasing in speed, the spectral type changing correspondingly, and these changes may be repeated indefinitely. In view of these possibilities, then, we cannot assign an upper limit to the duration of the life of a star; nor indeed could we say that a star has but one life, for it is quite conceivable that its life may be extinguished and renewed many times.

One can imagine that a wandering star finds its way into a nebula of sufficiently enormous expanse and has its velocity so decreased that it is unable to escape. Another star, and still another, is entangled in its filmy substance, and finally a whole group of stars are brought to rest within its borders. If these stars come from all directions at random, the moment of momentum of the group would be small. Since the moment of momentum of the nebula itself would be small, there would be little tendency for the system as a whole to rotate, but under their mutual attractions these stars would take the form of a globular cluster. In

the course of time they would sweep up the nebulous material which had bound them together; and this material for many ages would furnish the energy for their lavish radiation. But eventually it would be exhausted and the stars would decline in mass. As they did so, the gravitational control of the group on the individual members would be relaxed. The cluster would expand, and finally, one by one, the stars would escape and pursue their lonely journeys in search of new adventures.

An unusual epoch in the existence of a star will occur when it happens to pass through the immediate neighborhood of another star, an event which is almost certain to occur in a sufficiently extended period of time. The results of such an encounter are studied in the well-known researches of Chamberlin and Moulton on the planetesimal hypothesis of the development of our own planetary system. In this hypothesis the fundamental assumption is that at some remote epoch in the past our sun, even at that time a star, passed close by another star and that our planetary system has grown up and developed from the material which was torn from the sun by the tidal and disruptive action of the second sun. If the life of our sun is limited to some such period as a thousand millions of years it must be admitted that the chance for its encounter with another sun during this relatively short interval is small, and this objection has been urged. But if the time limit on the duration of the sun's life be removed, such an encounter becomes very probable, indeed almost a certainty, and our confidence in the planetesimal hypothesis is strengthened, or, perhaps better, our skepticism is somewhat weakened.

But if the sun is living upon material which is drawn in from surrounding space, the planets, too, at a smaller rate, must be adding to their masses and therefore growing, since they have not yet reached a stellar condition. In the course of time they too will grow to the full stature of a sun unless in the meantime the mutual perturbations of the planets shall have brought about the destruction of one or more of them through collisions among themselves or with the sun. Obviously the growth of the planets in such a manner would result in greatly contracting the dimensions of the planetary orbits, not only through the gravitative effect of increased

mass, but also because the ingathering process would have much the same effect as friction, since the process would add nothing to the momentum of the planets and would therefore decrease their speeds. It seems quite likely that the road to stellar condition would not be traveled very far before the terrestrial planets would be swallowed up by the sun, and, if the sun also were growing, by the time it had arrived at spectral type B the sun and Jupiter only would be left to form a binary star of short period—a typical type B binary. At the present time Jupiter is too small in mass to be a star. If the sun is now passing through a period in which its mass is decreasing, the mass of Jupiter will be growing, or at the very worst will be stationary, and therefore gaining relatively to the sun. One can fancy the sun decreasing toward extinction while Jupiter is slowly growing toward stellar conditions. In this manner Jupiter and the sun might approach equality in mass. At a later time, if the solar system passed through a region rich in matter the sun and Jupiter would grow equally, and our solar system, if not reduced to two members, at least would present the interesting phenomenon of a system of two dominating suns of approximate equality. In the early stages of this rejuvenation the sun and Jupiter would form a binary star of long period and reddish color. As they grew in mass gathered from surrounding space their colors would brighten, they would draw closer together, and their period would shorten. Since the effect of a resisting medium is to make the orbits circular, it can be shown that the product of the period and the fifth power of the sum of their masses would remain constant during this process of growth, and so also would the product of the mean distance and the cube of the sum of the masses remain constant. In other words, the period would vary inversely as the fifth power of the sum of the masses, and the mean distance inversely as the cube of the sum of the masses. If the masses of the sun and Jupiter were increased by gathering in atomic material to five times their present masses, the distance of Jupiter would be reduced from 500,000,000 miles to 4,000,000 miles, and its period would be reduced from about twelve years to approximately thirty-three hours. If the masses were of approximate equality their spectra would change in the direction from type M toward type B, and if

their masses were sufficiently great to have a spectrum of type B we should certainly have a short-period binary with the circular orbit which is characteristic of this class of spectroscopic binaries. On the other hand, if we follow such a binary in our imaginations as through the ages it decreases in mass, we see the reverse process taking place, the distance between the stars increasing and the eccentricity increasing likewise, but the temperature decreasing and the color tending through the yellow toward the red until we see finally a typical visual binary.

An indefinite prolongation of a star's life undoubtedly would vastly increase the significance to be attached to a close approach of two stars, since such a close approach is merely a matter of sufficient time. With the relatively short span of life hitherto assigned to a star such an event is highly improbable until many aeons after the star has become cold and dead. The approach of two cold and solid stars would certainly have to be extremely close to have any further effect on the stars than to change their speeds and their paths, and it is very doubtful if anything short of actual collision would disrupt them. A quite different state of affairs would arise, however, if the stars were massive, and very hot and active. These are the conditions postulated in the planetesimal hypothesis, and the consequences of such conditions must play a very common rôle in the life-history of the stars. From the consequences in the sun-Jupiter system, in which it has been shown how Jupiter might become as massive as the sun and the sun become a binary star, a possible mode of genesis of these interesting objects is obtained. An extension of the planetesimal hypothesis seems quite competent to account for the existence of many binary stars, from the typical yellow, visual binaries of high eccentricity and long period to the typically short-period, white binaries of spectral type B with their almost perfectly circular orbits. The coalescence of the many members of a planetary system into a system of two or three members would furnish many occasions for the flashing up of a star into an intense but temporary brilliance, such as is exhibited in the relatively frequent temporary stars. It is not necessary, however, to suppose that all binaries are formed in the same manner, for it is quite conceivable that if a nebula has two centers of con-

densation a binary of long period and high eccentricity should result through the process of condensation. It is quite possible too that there exist other processes which have not yet been formulated.

Two processes are here recognized by which a star comes into existence. The first is a *possible* condensation from a nebula, though this does not seem to be as inevitable a process as it has generally been regarded. The second is the growth of a nucleus—a fragment perhaps from some disrupted mass, a witness of some titanic cataclysm—by the accretion of atoms and small particles to such a mass that the release of subatomic energies transforms it into a radiating star. By collision, or *very* close approach, only can we account for a star passing out of existence, in the first of which two masses are united into a single one, and in the second a single solid mass is disrupted into many fragments. But during the continuance of its existence a star is essentially a singular point in an infinite field of energy. Through these singular points the energy ebbs and flows. When the flow exceeds the ebb the star grows in mass and radiating power and character of spectrum. When the ebb exceeds the flow the star declines in mass and radiation, at times even to the point of extinction. But even during the period when its radiation fails, the singular point persists, and through it again flows the tide of energy when the conditions are suitable. Just as the atom and the molecule are permanent forms of physical existence, so also is the star a permanent form of physical existence, notwithstanding that the individual may pass from birth to its dissolution. There is no necessary limit to its age, and though the star itself may rise and fall, the universe as a whole is not essentially altered. The singular points may change their positions and their brilliancy, but it is not necessary to suppose that the universe as a whole has ever been or ever will be essentially different from what it is today.

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ON SOME PHENOMENA OBSERVED IN THE FOUCAULT TEST

By SUDHANSUKUMAR BANERJI

The "knife-edge" test introduced by Foucault is one of the best-known methods for examining the performance of an optical surface.[1] Lord Rayleigh has recently published a paper dealing with the theory of this test, taking into account the part played by the diffraction of the rays reaching the focal plane.[2] In the present note I propose to describe some new phenomena that I have observed in the Foucault test, and to explain the results obtained in the light of the theory given by Lord Rayleigh. The case most readily admitting of mathematical treatment and which is the one considered in Lord Rayleigh's paper is that in which the surface under test is bounded by parallel straight edges and the knife-edge used is also placed parallel to these edges at or near the focal plane, the source of light being a fine slit parallel to the boundaries of the surface and to the knife-edge. My own observations have also been made with a similar arrangement.

It has long been known that when the illumination of the surface under observation is cut off by the advancing knife-edge in the focal plane, the edges of the surface remain bright and in fact shine out with enhanced brilliancy. This effect has long been known to be due to diffraction[3] and has been discussed in Lord Rayleigh's paper. There is another effect due to diffraction observed in Foucault's test which does not appear to have been previously recorded or explained. I have found that as the knife-edge is gradually advanced in the focal plane, the surface lying between the boundaries does not continuously decrease in brightness but

[1] See for instance the memoirs by Draper and Ritchey on the construction of a silvered glass telescope, *Smithsonian Contributions to Knowledge*, **34**, 1904.

[2] "On Methods for Detecting Small Optical Retardations, and on the Theory of Foucault's Test," *Phil. Mag.*, **33**, 161, 1917. See also a note by the present author in *Nature*, **99**, 206, 1917.

[3] See p. 32 in Ritchey's memoir quoted above.

that the illumination of the entire surface undergoes large fluctuations, becoming alternately greater or less, finally tending, however, to zero when the knife-edge is advanced considerably. With white light some very remarkable color-effects may be noticed. It is found that the boundaries of the surface appear luminous and white, but the region inside the boundaries shows color, this being practically of the same tint throughout, but most marked midway between the boundaries. The whole of the field between the boundaries passes through an interesting succession of colors as the knife-edge is slowly moved in the focal plane. The field outside the boundaries also shows a color (though much less vividly) which is in general complementary to that observed between the boundaries. These color-effects are obviously due to the fluctuations of the intensity of the entire field between the boundaries, not being in the same phase for different parts of the spectrum.

The theory of Foucault's test as developed by Lord Rayleigh is found to be capable of explaining these remarkable color-phenomena. The expression given by Lord Rayleigh for the intensity of the field as viewed in the direction ϕ is

$$I=\left[\mathrm{Si}\left\{\frac{2\pi}{\lambda}\theta\left(1+\frac{\phi}{\theta}\right)\xi_2\right\}-\mathrm{Si}\left\{\frac{2\pi}{\lambda}\theta\left(1+\frac{\phi}{\theta}\right)\xi_1\right\}\right.$$
$$\left.+\mathrm{Si}\left\{\frac{2\pi}{\lambda}\theta\left(1-\frac{\phi}{\theta}\right)\xi_2\right\}-\mathrm{Si}\left\{\frac{2\pi}{\lambda}\theta\left(1-\frac{\phi}{\theta}\right)\xi_1\right\}\right]^2$$
$$+\left[\mathrm{Ci}\left\{\frac{2\pi}{\lambda}\theta\left(1-\frac{\phi}{\theta}\right)\xi_2\right\}-\mathrm{Ci}\left\{\frac{2\pi}{\lambda}\theta\left(1-\frac{\phi}{\theta}\right)\xi_1\right\}\right.$$
$$\left.-\mathrm{Ci}\left\{\frac{2\pi}{\lambda}\theta\left(1+\frac{\phi}{\theta}\right)\xi_2\right\}+\mathrm{Ci}\left\{\frac{2\pi}{\lambda}\theta\left(1+\frac{\phi}{\theta}\right)\xi_1\right\}\right]^2,$$

where ξ_1, ξ_2 are the limits of the aperture in the focal plane and θ is the angular semi-aperture of the lens under test. In practice ξ_2 is large. To illustrate the fact that the luminosity of the field between the boundaries undergoes fluctuations as ξ_1 is gradually increased, I have prepared Table I showing the intensity of different points of the field as calculated from the expression given above, taking $\xi_2=\frac{300\lambda}{2\pi\theta}$ and $\xi_1=\frac{3\lambda}{2\pi\theta}x$, where x has the values 1, 1.63, 2,

2.62, and 3 in succession, these being approximately the values for which the intensity at the center of the field is greatest or least.

It will be seen from the figures in Table I that the intensity of the entire field between the limits $\phi/\theta = \pm 1$ becomes alternately greater and less as ξ_1 is increased. This fact is not brought out in Rayleigh's paper. The region outside the boundaries does not show such a marked variation. In fact, observation shows that when ξ_2 is large a variation of ξ_1 produces a relatively insignificant effect on the intensity of the field outside the boundaries. If, however, ξ_2 be not large, experiment and theory agree in showing that a variation of either ξ_1 alone or of both ξ_1 and ξ_2 ($\xi_2 - \xi_1$ remaining constant) results in a marked fluctuation of the intensity of the

TABLE I

x	Values of ϕ/θ												
	0	±.1	±.2	±.3	±.5	±.8	±.9	±1	±1.1	±1.2	±1.5	±1.8	±2
1.....	0.31	0.34	0.320	0.350	0.472	1.015	1.310	20.23	1.685	0.755	0.174	0.071	0.038
1.63...	0	.0001	.005	.020	.063	.299	1.330	16.82	1.601	.621	.197	.087	.060
2.....	.09	.07	.083	.095	.147	.395	1.009	15.30	.837	.328	.052	.019	.035
2.62...	0	.0009	.002	.007	.006	.153	.604	13.73	.727	.235	.042	.013	.015
3.....	.01	.033	.032	.038	.019	.178	.544	12.35	.544	.178	.040	.032	.012

field outside the boundaries, and the colors observed in this region with white light become more prominent. It should be remarked here that when ξ_2 is not large the regions inside and outside the boundaries (observed in white light) are not generally of uniform tint throughout, as diffraction fringes appear showing a regular succession of colors, although a preponderance of a particular color within the boundaries and of a complementary color in the region outside may be noticed. As the knife-edge is moved in, the relative intensities of the different colors change to an enormous extent, and the positions of the different fringes shift to and fro. The changes in the position and the color of the bands within the boundaries are most interesting to watch when their number is small. If it is arranged to alter both ξ_1 and ξ_2 ($\xi_2 - \xi_1$ remaining constant and small), a fluctuation in the number of the fringes

between the boundaries from one to two, or two to three, and vice versa, may be observed, these changes being accompanied by a fluctuation in the intensity of the bands. As an illustration of this I have calculated the distribution of intensity from Lord Rayleigh's expression for two sets of values of ξ_2, ξ_1, these being $\xi_2 = 0.7$ mm and $\xi_1 = c.5$ mm, and $\xi_2 = 0.8$ mm and $\xi_1 = 0.6$ mm, and $\frac{2\pi\theta}{\lambda}$ was taken to be $30x(\text{mm})^{-1}$. The intensity-curves have been plotted in Figs. 1 and 2.

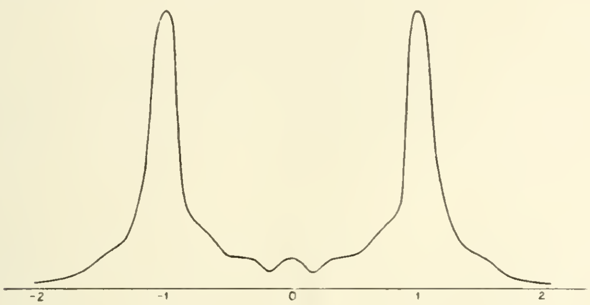

FIG. 1

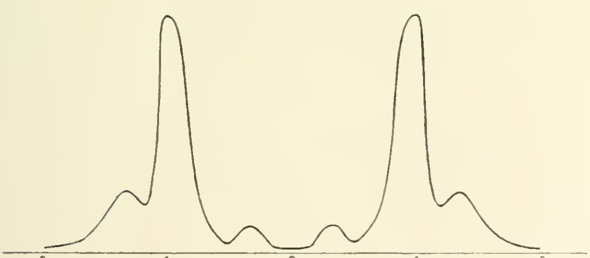

FIG. 2

PHENOMENA OBSERVED WHEN THE KNIFE-EDGE IS NOT IN THE FOCAL PLANE

Lord Rayleigh has confined himself in his paper to the case in which the knife-edge is put exactly in the focal plane. In the practical application of the Foucault test, the position of the focal plane has to be found by actual trial, and it is therefore of interest

to consider also the effects observed when the knife-edge is put in a plane a little in advance of or behind the focal plane. Actual experimental observation in the case of a rectilinear boundary under these conditions shows that when the knife-edge is sufficiently advanced so as to cut off the general illumination of the surface, the two edges of the field still appear luminous *but differ markedly in their brilliancy.* This difference becomes greater and greater as the knife-edge is put in a plane farther and farther from the focus. The effect is reversed if the knife-edge is put in front of instead of behind the focal plane, and has no doubt been frequently noticed by those who have applied Foucault's test in practice. It should be remarked also that when the knife-edge is considerably advanced in any given plane the inequality of the luminosity of the two edges diminishes. The mathematical theory of these effects is given below.

Let A represent the lens with its rectangular aperture which brings parallel rays to a focus O. Let a screen containing an aperture parallel to the aperture of the lens be placed at a distance a in advance of the focus. A is the center of the first aperture and Q any point on it, so that $AQ=s$. If P be any point on the second aperture and B the center of the screen, then putting $BP=\xi$ and $AO=f$, we get

$$\begin{aligned} PQ^2 &= QO^2+OP^2-2QO\cdot OP\cos POQ \\ &= f^2+a^2+\xi^2+2f\cdot\sqrt{a^2+\xi^2}\cos\left(\frac{s}{f}+\tan^{-1}\frac{\xi}{a}\right) \\ &= (f+\sqrt{a^2+\xi^2})^2-4f\cdot\sqrt{a^2+\xi^2}\sin^2\tfrac{1}{2}\left(\frac{s}{f}+\tan^{-1}\frac{\xi}{a}\right) \end{aligned}$$

Since $\sin^2\frac{1}{2}\left(\frac{s}{f}+\tan^{-1}\frac{\xi}{a}\right)$ is small, provided ξ/a is small, we have, extracting the square root,

$$PQ=f+\sqrt{a^2+\xi^2}-\frac{2f\cdot\sqrt{a^2+\xi^2}}{f+\sqrt{a^2+\xi^2}}\sin^2\tfrac{1}{2}\left(\frac{s}{f}+\tan^{-1}\frac{\xi}{a}\right).$$

Now, since ξ/a is supposed to be small, we get

$$PQ=f+\sqrt{a^2+\xi^2}-\frac{f\sqrt{a^2+\xi^2}\cdot\xi^2}{2(f+\sqrt{a^2+\xi^2})a^2}-\frac{\xi\sqrt{a^2+\xi^2}}{a(f+\sqrt{a^2+\xi^2})}\cdot s-\frac{\sqrt{a^2+\xi^2}}{2f(f+\sqrt{a^2+\xi^2})}\cdot s^2.$$

Thus we see that

$$PQ=f+A+Bs+Cs^2$$

where

$$A=\sqrt{a^2+\xi^2}-\frac{\sqrt{a^2+\xi^2}\cdot f\xi^2}{2(f+\sqrt{a^2+\xi^2})a^2},$$

$$B=-\frac{\xi}{a}\cdot\frac{\sqrt{a^2+\xi^2}}{f+\sqrt{a^2+\xi^2}},$$

$$C=-\frac{\sqrt{a^2+\xi^2}}{2f\cdot(f+\sqrt{a^2+\xi^2})}.$$

Therefore the disturbance at the point ξ of the second aperture will be represented by

$$\int_{-s}^{s}\cos 2\pi\left(\frac{t}{T}-\frac{f+A+Bs+Cs^2}{\lambda}\right)ds\,.$$

If T be written for $\left(\frac{t}{T}-\frac{f}{\lambda}\right)\lambda$, the foregoing integral can be written in the form

$$\sin 2\frac{\pi}{\lambda}T\int_{-s}^{s}\sin 2\pi\frac{A+Bs+Cs^2}{\lambda}ds+\cos\frac{2\pi}{\lambda}T\int_{-s}^{s}\cos 2\pi\left(\frac{A+Bs+Cs^2}{\lambda}\right)ds\,.$$

The rays from the various points of the second aperture may be regarded as a parallel pencil inclined to the axis at a small angle ϕ.

When we proceed to inquire what is to be observed at an angle ϕ, we have to consider the expression

$$\cos kT \int_{\xi_1}^{\xi_2} \int_{-s}^{s} \cos k(A+Bs+Cs^2-\phi\xi)\,ds\,d\xi$$
$$+\sin kT \int_{\xi_1}^{\xi_2} \int_{-s}^{s} \sin k(A+Bs+Cs^2-\phi\xi)\,ds\,d\xi,$$

where $K=\frac{2\pi}{\lambda}$ and ξ_1, ξ_2 define the limits of the second aperture.

The intensity I represented as the sum of the squares of the integrals is given by

$$I=\left[\int_{\xi_1}^{\xi_2} \int_{-s}^{s} \cos k(A+Bs+Cs^2-\phi\xi)\,ds\,d\xi\right]^2$$
$$+\left[\int_{\xi_1}^{\xi_2} \int_{-s}^{s} \sin k(A+Bs+Cs^2-\phi\xi)\,ds\,d\xi\right]^2.$$

The integration can be carried out with respect to s on putting $s=z-\alpha$ and choosing α so that the term containing the first power of z in the expression $A+B(z-\alpha)+C(z-\alpha)^2-\phi\xi$ vanishes. The integrals are thus reduced to integrals of the Fresnel class and can be integrated in semi-convergent series. Since K is a large quantity, we retain only a few terms of the series, and the subsequent integration with respect to ξ is effected by integrating by parts.

We thus find that the intensity is proportional to the sum of the squares of the expressions (I) and (II) given below.

(I)

$$\frac{c_2}{2C_2k}\Big[-\cos k(A_2+B_2s+C_2s^2-\phi\xi_2)\cdot s\Big]_{-s}^{s}$$
$$+\left\{a_2-\left(b_2-\frac{B_2c_2}{2C_2}\right)\frac{B_2}{2C_2}\right\}\left[-\frac{\cos k(A_2+B_2s+C_2s^2-\phi\xi_2)}{k(B_2+2C_2s)}\right]_{-s}^{s}$$
$$+\left(b_2-\frac{B_2c_2}{2C_2}\right)\frac{1}{C_2k}\Big[-\cos k(A_2+B_2s+C_2s^2-\phi\xi_2)\Big]_{-s}^{s}$$

$$-\frac{c_1}{2C_1k}\Big[-\cos k(A_1+B_1s+C_1s^2-\phi\xi_1)\cdot s\Big]_{-s}^{s}$$

$$-\left\{a_1-\left(b_1-\frac{B_1c_1}{2C_1}\right)\frac{B_1}{2C_1}\right\}\left[-\frac{\cos k(A_1+B_1s+C_1s^2-\phi\xi_1)}{k(B_1+2C_1s)}\right]_{-s}^{s}$$

$$-\left(b_1-\frac{B_1c_1}{2C_1}\right)\frac{1}{2C_1k}\Big[-\cos k(A_1+B_1s+C_1s^2-\phi\xi_1)\Big]_{-s}^{s}$$

(II)

$$-\frac{c_2}{2C_2k}\Big[\sin k(A_2+B_2s+C_2s^2-\phi\xi_2)\cdot s\Big]_{-s}^{s}$$

$$-\left\{a_2-\left(b_2-\frac{B_2c_2}{2C_2}\right)\frac{B_2}{2C_2}\right\}\left[\frac{\sin k(A_2+B_2s+C_2s^2-\phi\xi_2)}{k(B_2+2C_2s)}\right]_{-s}^{s}$$

$$-\left(b_2-\frac{B_2c_2}{2C_2}\right)\frac{1}{2C_2k}\Big[\sin k(A_2+B_2s+C_2s^2-\phi\xi_2)\Big]_{-s}^{s}$$

$$+\frac{C_1}{2C_1k}\Big[\sin k(A_1+B_1s+C_1s^2-\phi\xi_1)\Big]_{-s}^{s}$$

$$+\left\{a_1-\left(b_1-\frac{B_1c_1}{2C_1}\right)\frac{B_1}{2C_1}\right\}\left[\frac{\sin k(A_1+B_1s+C_1s^2-\phi\xi_1)}{k(B_1+2C_1s)}\right]_{-s}^{s}$$

$$+\left(b_1-\frac{B_1c_1}{2C_1}\right)\frac{1}{C_1k}\Big[\sin k(A_1+B_1s+C_1s^2-\phi\xi_1)\Big]_{-s}^{s},$$

where

$$A_2=a+\tfrac{1}{2}\frac{1}{f+a}\xi_2^2, \qquad A_1=a+\tfrac{1}{2}\frac{1}{f+a}\xi_1^2,$$

$$B_2=-\frac{1}{f+a}\xi_2, \qquad B_1=-\frac{1}{f+a}\xi_1,$$

$$C_2=-\frac{a}{2f(f+a)}\left[1+\frac{f}{2a^2(f+a)}\xi_2^2\right], \qquad C_1=-\frac{a}{2f(f+a)}\left[1+\frac{f}{2a^2(f+a)}\xi_1^2\right],$$

$$a_2=\frac{f+a}{\xi_2-\phi(f+a)}, \qquad a_1=\frac{f+a}{\xi_1-\phi(f+a)},$$

$$b_2=\frac{f+a}{\{\xi_2-\phi(f+a)\}^2}, \qquad b_1=\frac{f+a}{\{\xi_1-\phi(f+a)\}^2},$$

$$c_2=\frac{f+a}{\{\xi_2-\phi(f+a)\}^3}+\frac{\xi_2(f+a)}{2a(f+a)\{\xi_2-\phi(f+a)\}^2},$$

$$c_1=\frac{f+a}{\{\xi_1-\phi(f+a)\}^3}+\frac{\xi_1(f+a)}{2a(f+a)\{\xi_1-\phi(f+a)\}^2}.$$

This expression has been used to determine the ratio of the intensities of the two edges for different positions of the advancing edge at different distances from the focal plane. The ratio of the brightness of the two edges was also determined experimentally by photometric comparison. For this purpose a double-image prism was used to obtain a pair of images of the luminous edges polarized in perpendicular planes which were then observed through a nicol. The results are shown in Table II. The agreement between theory and experiment is fairly satisfactory.

TABLE II

$2s=3.47$ mm, $f=43.3$ cm, $\lambda=0.0006$ mm

a	ξ_1	ξ_2	Observed Ratio of the Intensity of the Two Edges	Calculated Ratio of the Intensity of the Two Edges
cm	mm	mm		
−4.0	0.85	5.65	0.25	0.21
−4.0	2.65	5.65	0.41	0.38
−2.5	0.72	5.38	0.32	0.30
−2.5	2.43	5.38	0.53	0.48
−1.5	1.00	5.95	0.50	0.45
−1.5	2.87	5.82	0.65	0.59
0	....	....	1.00	1.00
+2.5	0.76	5.76	1.32	1.28
+2.5	2.76	5.76	1.14	1.11
+4.5	0.61	5.61	2.37	2.29
+4.5	2.60	5.61	1.42	1.37

CALCUTTA UNIVERSITY
March 5, 1918

Minor Contributions and Notes

ON CHANGES OF THE WAVE-LENGTHS OF LINES IN STELLAR SPECTRA WITH CHANGE OF TYPE

In the March number of the *Astrophysical Journal*, Voûte has revived the research which was initiated in 1906, and continued in 1911, by Albrecht on the progressive changes of wave-length of stellar lines, depending on the stellar type. In some respects the results obtained by Albrecht were surprising. In the case of blended lines, especially if only two components were involved—and these perhaps of different types of line, such as enhanced and unenhanced—one would naturally expect a progressive change of position, depending on the alteration in relative intensity of the components as one proceeded along the various types; and, knowing the composition of the blends, it was fairly easy to predict which direction the changes in wave-length would take. Some of Albrecht's lines, however, were free from any suspicion of blending, and as these also showed progressive changes, obviously some other cause must be involved other than that recognized for the blended lines. Examples of this class of "pure" lines are Sc 4246.99, Cr 4254.51, Fe 4260.64, and p Ti 4468.66.

In other cases more than two lines compose the blends, and here the chances seem all against the changes taking place in one direction only, the cause being assumed to be the shift of the center of gravity of the blend, owing to relative changes in intensity among the various component lines. And yet Albrecht's results showed the same progressive changes for these as for the others. As examples of this class we may mention 4288.1, involving the solar lines Ti 4288.038, Ni 4288.149, and Ti-Fe 4288.310; 4315.1, composed of Ti 4314.964, p Ti 4315.138, and Fe 4315.262; and 4352.0, formed apparently by the two solar lines 4351.930 (due partly to p Fe line 4351.93 and partly to a Cr line of identical wave-length) and Mg 4352.083. For the last of these blends one would have

expected the lower wave-lengths to have prevailed toward the F end of the spectral types, as the enhanced lines are more prominent there, whereas the unenhanced lines of Cr and Mg are more pronounced in the solar and later types. Albrecht's values show an exactly contrary result, the wave-length diminishing in going from F to M. Although the line 4351.93, one of the components of this blend, is ascribed by Rowland to Cr only, there can be no doubt that in the solar line the enhanced line of Fe at 4351.93 is involved. In fact, if one goes farther back to the A and late B types, the evidence is almost overwhelming that in the former the stellar line in this position is due chiefly to p Fe, and in the latter wholly so.

Assuming Albrecht's results for the progressive changes in wave-length to be correct, no satisfactory explanation has been adduced in the case of unblended lines and complex blends involving more than two lines.

Voûte affirms that his resulting changes in wave-length agree on the whole with Albrecht's. There are, however, some very striking differences among the pure or unblended lines. Thus, for Sc 4247.0 Albrecht gets increased wave-lengths from F to K; Voûte's show a progressional decrease. For Fe 4260.6 Albrecht gets a continuous increase from F to K; Voûte's show an increase A to F, a big drop (0.16 A) F to G, then a bigger rise (0.24 A) G to K. For Cr 4274.96 Albrecht's decrease, Voûte's first decrease, then rise. For Hγ Albrecht's are nearly constant; Voûte's vary 0.1 A. In fact, of eight cases of unblended lines where the results of the two observers are comparable, only three show similar behavior. These are p Ti 4399.94, Fe 4469.55, and p Ti 4468.66.

In Voûte's own results for unblended lines there are one or two apparent inconsistencies in the wave-length changes for Fe lines which are always associated in laboratory spectra as being of similar behavior. Thus, for the two strong neighboring iron lines 4271.33, 4271.93, the former decreases from type A to F, then rapidly increases through type G to K, the variation being 0.18 A. The latter increases from A to F, falls from F to G, then rises rapidly to K, the whole range being 0.11 A. For the well-known Fe triplet 4383.72, 4404.93, 4415.29, whereas the two former have

nearly constant wave-lengths in the various stellar types. the last shows an increase continuously from A to K, and has a range of 0.13 A. It seems very probable that some of these results do not represent real changes, but are, in the main, due to errors in estimating the wave-lengths.

This kind of research is so important that it is to be hoped other observers with available material of the necessary refinement will undertake it, and thus afford comparison with Albrecht's consistent, though in some ways remarkable, results. Albrecht, in his paper of 1911, stated that he intended to continue the research, extending both the region of spectrum investigated and the range of stellar types, but, as far as the writer knows, he has not published anything further.

F. E. Baxandall

Cambridge, England
May 23, 1918

SUGGESTION TO OBSERVERS OF NOVA AQUILAE

I desire to call the attention of observers who have powerful reflectors at their disposal to the desirability of making spectroscopic and polariscopic observations of the nebula which, judging from Nova Persei, may be expected to make its appearance about the new star in Aquila.

Attempts were made by the writer[1] to obtain such observations of the nebulosity about Nova Persei, but owing to the faintness of the nebulosity during the later stages when the attempts were made the results were not so decisive as could be desired, especially with regard to polarization.

It seems entirely possible with our present means to obtain decisive information on both of these points, if observations are made as soon as such nebulosity is well visible and separated from the star. In the case of Nova Persei, impressions of two nebulous rings were obtained about five weeks after maximum brightness. If Nova Aquilae behaves similarly, such nebulosity should be far enough away for separate observation in from two to three months

[1] *Lick Observatory Bulletins*, 1, 180, 1902; 2, 32, 1903.

from the date of the outburst, or in the month of August or September of the present year.

Such observations should throw light on the very important question whether the nebulosity is a product of the outburst or whether it existed in the region and may have been a factor in causing the outburst.

C. D. Perrine

Observatorio Nacional Argentino, Córdoba
June 17, 1918

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THE VISIBILITY OF RADIATION

By EDWARD P. HYDE, W. E. FORSYTHE, AND F. E. CADY

It would almost appear redundant, in view of the numerous and careful determinations of the visibility of radiation, such as those of Ives,[1] Nutting,[2] and Coblentz and Emerson,[3] to undertake a new investigation of this function. And yet a casual survey of the literature reveals striking differences among the results of the several investigators, and in the second place the old and much-mooted question of the reliability of the highly developed method of flicker photometry in giving results consistent with the older and more commonly used method of direct comparison is apparently still unsettled.

In connection with a new experimental investigation of the relative brightness of a black body as a function of its temperature, in which the measurements of brightness were made by the method of direct comparison, it was desired to compare the relative brightnesses as determined experimentally against the corresponding relative values computed from the energy-curves as given by Planck's equation and visibility data, after the method of Eisler.[4]

[1] *Phil. Mag.* (6), **24**, 149, 1912, and *Phys. Rev.*, **6**, 329, 1915.

[2] *Trans. Illum. Eng. Soc.* (U.S.), **9**, 633, 1914, and **13**, 108, 1918.

[3] *Scientific Papers of the Bureau of Standards*, No. 303; *Bulletin of the Bureau of Standards*, **14**, 167, 1918.

[4] *Elek. Zeit.*, **25**, 188, 1904.

To this end it seemed advisable to undertake a new determination of the visibility-curve by the method of direct comparison and under experimental conditions which should be so chosen as to be comparable with those involved in the measurements of brightness rather than to accept any of the published flicker values, or to impose upon the new determination any of the various specified conditions suggested by the flicker investigations but at variance with the conditions obtaining in the measurements of brightness of the black body, e.g., the restriction of the size of the field such as has become common practice in flicker photometry.

It is not the intention in this paper to enter upon a historical discussion of the subject; for this the reader is referred to the papers of other investigators, notably to the recent extended monograph on the subject by Coblentz and Emerson. It is of great importance, however, for the present purpose to consider briefly the question of the relative merits of the method of flicker photometry and of the older method of direct comparison in equating illuminations of different color.

It is a matter of opinion on what basis two illuminations of different color should be adjudged equal, and yet it is probable that by consensus of opinion equality of brightness as given in an ordinary photometer would be selected as the criterion, granted, of course, that this judgment can be formed. Herein lies the whole difficulty, and because it was found that illuminations of gross difference in color could only approximately and with great difficulty be equated in intensity by the method of direct comparison, attention was turned to the flicker method, which permits of comparatively easy measurements, even though individual peculiarities still exhibit themselves. And when it was thought to have been found that the visibility of radiation as determined by the flicker method was sensibly the same as that indicated by the less accurate method of direct comparison, exponents of the flicker method arose who wished to standardize this method as the accepted one for all heterochromatic measurements.

The two questions which the authors wish to raise are: (1) whether the inaccuracies of the direct-comparison method in all ordinary practical problems of heterochromatic photometry

are so large as to demand the introduction of a new method; and (2) whether the findings of the new method, under the prescribed experimental conditions, are the same as those of the older and commonly accepted method within the errors of measurement.

In answer to the first question the authors would refer to the paper by Middlekauff and Skogland[1] on "An Interlaboratory Photometric Comparison of Glass Screens and of Tungsten Lamps, Involving Color Differences." This paper reports the results of measurements made at several laboratories on the transmission of various blue-glass screens and on the relative candle-powers of several tungsten lamps each operated at a number of widely different voltages. The report shows that in the determination of the relative candle-powers of tungsten lamps at 72 volts and 132 volts, involving a color-difference of the order of magnitude of that existing between an old-type 4 w.p.c. carbon lamp and a 0.85 w.p.c. vacuum tungsten lamp, three laboratories using the Lummer-Brodhun photometer obtained results agreeing among themselves within a maximum difference of less than 2 per cent. And, as will be pointed out later, at least a part of this difference is probably to be ascribed to the small number of observers at each laboratory and the consequent undue weight assigned to individual idiosyncrasies of vision. If a determination involving so large a color-difference can be made by the older method with an error of probably less than 1 per cent from the mean, there would seem to be little need of introducing a new method, particularly if the foundation upon which it rests is insecure. In this same comparison a determination at another laboratory with the flicker photometer gave results markedly different from the results of direct comparison, and a similar difference in the same direction and of even greater magnitude was indicated by the results of Crittenden and Richtmyer,[2] again using the flicker photometer.

The second question is partially answered in the discussion above. The results obtained in the intercomparison among the

[1] *Trans. Illum. Eng. Soc.* (U.S.), **11**, 164, 1916; *Bulletin of the Bureau of Standards*, **13**, 287, 1918.

[2] *Scientific Papers of the Bureau of Standards*, No. 299; *Bulletin of the Bureau of Standards*, **14**, 87, 1918.

various laboratories and also the results of a comparative study by Crittenden and Richtmyer show that ratios given by the flicker photometer, under the prescribed conditions of size of field, etc., are distinctly different from those found by the more commonly used method of direct comparison. For the color-difference referred to in the preceding paragraph the largest deviation of any laboratory using the direct-comparison method from the mean value for the three laboratories was 1 per cent, and, as already stated, the deviations from the mean would probably be lessened if the readings of a larger number of observers in each laboratory should be taken. On the other hand, the values obtained with the flicker photometer in two different laboratories and with no strictures on account of the limited number of observers are, on the average, about 2.5 per cent different from the mean value found by the other method. The direction of the difference is such as to indicate that the less refrangible end of the spectrum is given relatively more weight in the flicker method, so that the relative candle-power of a lamp at any temperature in terms of its candle-power at a lower temperature is found to be smaller than that obtained by the method of direct comparison.

It is true that with the method of flicker photometry employed certain limitations with regard to size of field were prescribed which, it might be argued, account, at least in part, for the observed difference, but other data are available which would seem to vitiate this explanation as a complete one. Luckiesh[1] performed an experiment in which two fields, one red and the other blue-green, were compared by both the flicker and the direct-comparison methods, using the same apparatus. He found for his eye that the ratio blue-green to red was very much larger (50 to 100 per cent) with the direct-comparison method as compared with that obtained with the flicker method.

The recent elaborate investigation by Coblentz and Emerson[2] on visibility also indicates relatively greater blue sensibility in the method of direct comparison, but strangely the results of Ives[3] and of Coblentz and Emerson are at variance in one very important aspect. Whereas the data of Coblentz and Emerson show that the

[1] *Electrical World*, **61**, 620 and 835, 1913. [2] *Loc. cit.* [3] *Loc. cit.*

visibility-curve obtained with the method of direct comparison is somewhat broader and flatter than the corresponding curve obtained with the flicker method, the data of Ives point to the opposite conclusion—a result markedly indicated by the experimental data to be presented later in this paper. The importance of this difference lies in its effect on the resultant computed value for the mechanical equivalent of light, since a difference in the area of the visibility-curve affects directly the value of this important constant.

In view of the foregoing considerations it would seem to the authors that somewhat different answers must be given to the two questions raised above from those which the advocates of the method of flicker photometry are urging. Whatever may be the uncertainties in determining the average visibility-curve by the method of direct comparison (a question on which the authors will later adduce evidence), there is much reason to believe that for those color-differences which are commonly encountered in practical photometry the method of direct comparison may be used with reasonable confidence. And secondly, the evidence available seems to show that the flicker method, as commonly employed, does not yield results consistent with those obtained by the older method, and that the differences between the two are sufficiently pronounced to manifest themselves both in the ordinary heterochromatic measurements of practical photometry and in the curves of visibility obtained by the two methods.

Before describing the method and presenting the results of the present investigation the authors wish to point out that the dominant thought in the investigation was to reproduce in the determination of visibility the conditions obtaining in ordinary photometry and to pay no special attention to some of the minor conditions which might have been imposed as a result of the recent investigations on the subject. Thus no attempt was made to keep the illumination constant, and so the brightness at the ends of the spectrum was much lower than that in the more luminous regions. At 0.5μ the brightness of the Lummer-Brodhun cube was approximately 0.001 candles per cm^2, at 0.56μ it was 0.005 candles per cm^2, and at 0.65μ it was 0.003 candles per cm^2. These

brightnesses correspond approximately to illuminations of a perfectly reflecting, perfectly diffusing surface of 30, 150, and 90 meter candles. Since an artificial pupil of 0.6 mm^2 was employed, the illumination intensities of the retina would be somewhat smaller than those corresponding to the same objective brightnesses in practical photometry. It is seen that everywhere over the range of wave-length investigated the brightness was reasonably high and probably beyond that of the Purkinjé region except possibly at the extreme blue end of the spectrum. Moreover, the authors do not think the evidence at present available sufficient to justify the conclusion that the visibility-curve obtained under the conditions of equal brightness is to be preferred to that obtained under the normal conditions of a dispersed spectrum, granted the two are different.

APPARATUS AND METHOD

The distinguishing characteristics of the present investigation, apart from the employment of the method of direct comparison under conditions with respect to size of field, etc., obtaining in ordinary photometric practice, are to be found in the use of the step-by-step method and in the determination of the distribution of energy in the spectrum. The step-by-step method has been employed before, as in an experiment by Ives,[1] but to the best of the authors' knowledge this method has not been used in any extended investigation with a large number of observers. The steps were chosen so small (varying from 0.0052μ in the red [$\lambda = 0.66\mu$] to 0.0022μ in the blue [$\lambda = 0.5\mu$]) that on the basis of Steindler's[2] data the interval everywhere throughout the spectrum would be less than that corresponding to the limen of hue-discrimination. It was subsequently found, when the apparatus had been constructed and the experiment begun, that the step was still too large to eliminate all hue-differences, though these differences were relatively small in magnitude. As it was, some fifty steps were required to span the range of spectrum studied, though the method of investigation was such as to avoid the necessity of actually making so many measurements.

[1] *Loc. cit.* [2] *Wiener Sitzungsberichte* (II*a*), **115**, 1, 1906.

The evaluation of the spectral energy was founded on the determination of the color-temperature of the source. The color-temperature is the temperature of a black body having the same distribution of energy in the visible spectrum as the source employed. By means of Planck's equation the energy-distribution was computed and, allowing for dispersion and absorption by the optical system and for scattering, the relative energy entering the eye in the different parts of the spectrum was readily determined. In the opinion of the authors this method has much to commend it

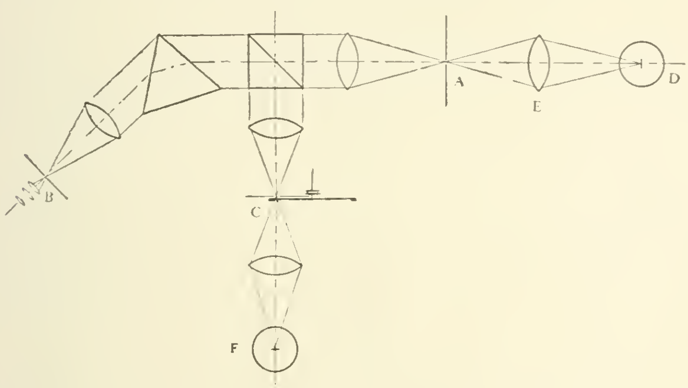

FIG. 1.—Diagram of apparatus

over the very difficult and uncertain method of attempting to measure directly the energy at the eye-slit.

The apparatus employed (Fig. 1) consisted of a tungsten lamp *D*, whose broad, flat filament was focused by means of the projection lens *E* on the slit *A* of a Lummer-Brodhun spectrophotometer, having the absorption strips removed from the Lummer-Brodhun prism so that the settings were made on the basis of equal brightness rather than on that of equal contrast. The comparison field was obtained from a second tungsten lamp *F*, the settings being made by means of the special variable rotating sectored disk *C*.[1] A low-power eyepiece *B* was employed in order to facilitate fixation upon the diagonal surface of the Lummer-Brodhun cube where are located the two fields to be compared.

[1] *Astrophysical Journal*, **35**, 237, 1912.

The slit A was specially designed so that it could be moved sidewise a definite fixed amount, 0.15 mm, thus providing the means of securing a small shift of one spectrum with respect to the other. The amount of this shift, expressed in wave-lengths and differing slightly in magnitude from one part of the spectrum to another, was determined very carefully in several different ways.

The test lamp D was operated at a color-temperature of 2045° K maintained constant throughout the experiments. It was so mounted that it could be moved with the slit. In this way the

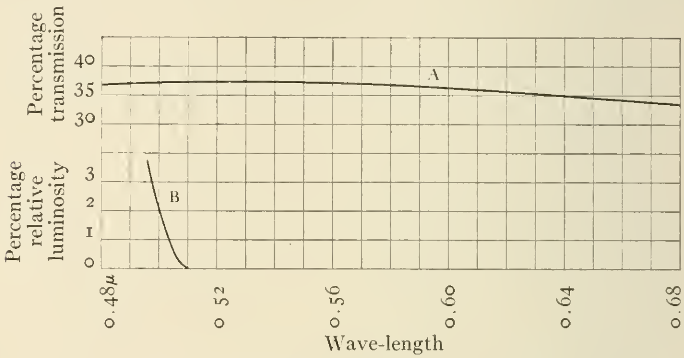

FIG. 2.—Corrections for stray light and selective transmission of optical system. A, selective transmission of optical system; B, luminosity of stray light at wave-lengths shown, in percentage of total luminosity at the corresponding wave-lengths.

same portion of the broad filament was always focused on the slit, and consequently there was no alteration in radiation passing through it. The width of the slit A was 0.1 mm and that of the ocular slit 0.2 mm, being thus so small as to produce an error of only a fraction of 1 per cent owing to the impurity of the spectrum.[1] Of course it was necessary to allow for the dispersion of the prism in computing the energy-distribution at the eyepiece, and to correct for stray light and for the selective absorption of the complete optical system.

These corrections are shown in Fig. 2. The stray light was evaluated through the use of color-screens placed in front of the eyepiece slit, and was determined directly in units of luminosity rather than of energy. It is seen to be a relatively small correc-

[1] *Astrophysical Journal*, 35, 237, 1912.

tion factor. The selective absorption of the optical system was obtained through the aid of a spectral pyrometer and was determined in two different ways yielding consistent results.

In carrying out the main experiment the method employed consisted in placing the slit A in its middle or normal position and varying the voltage of the comparison source F until the relative brightnesses of the two photometric fields were approximately the same throughout the spectrum. Then a set of measurements was made of the relative brightnesses at eighteen different wave-lengths distributed at approximately equal wave-length intervals from $0.5\,\mu$ to $0.66\,\mu$, the range of wave-lengths studied. Then the slit A was displaced laterally by the predetermined fixed amount and the same set of measurements repeated. If

$$R_1 = \frac{L'_\lambda}{L_\lambda}$$

is the observed ratio of the luminosity (L'_λ) of the comparison field at the wave-length λ to that (L_λ) of the test field at the same wave-length (as determined by the first experiment), and

$$R_2 = \frac{L'_\lambda}{L_{\lambda+\Delta\lambda}}$$

is the observed ratio of the luminosity (L'_λ) of the comparison field at the wave-length λ to that ($L_{\lambda+\Delta\lambda}$) of the test field at the wave-length $\lambda+\Delta\lambda$ (as determined by the second experiment, when the slit A has been shifted an amount corresponding to $\Delta\lambda$), then the ratio

$$R_\lambda = \frac{L_\lambda}{L_{\lambda+\Delta\lambda}}$$

of the two luminosities of the test field at the wave-lengths λ and $\lambda+\Delta\lambda$ is seen to equal R_2/R_1, and so is determinable from the two sets of measurements.

If now these experimentally determined values of R_λ at the eighteen points throughout the spectrum are plotted against the corresponding wave-lengths, a curve may be drawn giving the value of R_λ for the interval $\Delta\lambda$ for every wave-length. Then starting at one end of the spectrum and proceeding by successive

intervals $\Delta\lambda$ (differing slightly in different parts of the spectrum, as previously determined), the relative luminosity-curve of the test field for the observer making the measurements is computed by multiplying the observed ratio R_{λ_1} corresponding to the interval $\Delta_1\lambda$ by the ratio R_{λ_2} corresponding to the next interval $\Delta_2\lambda$, and this product in turn by R_{λ_3} corresponding to the next interval, and so on until the other end of the spectrum has been reached, plotting the value of the product at each successive step as the relative luminosity of the test field at the corresponding wave-length.

It is evident that since the number of these steps is determined by the magnitude of the displacement of the slit A this quantity must be known with accuracy. The uncertainty in the value of this quantity is so small as to produce a probable error in the final luminosity-curve of not more than 3 or 4 per cent, which is less than uncertainties arising from other sources.

Each of twenty-nine observers, most of whom were experienced in photometric measurements, made at least two independent sets of determinations by this method, and the average of the several sets of any observer was taken as giving his luminosity-curve for the energy-distribution employed. Some observers obtained remarkably consistent results (within 3 or 4 per cent) in their independent sets of measurements, while others showed differences several times as large.

The twenty-nine luminosity-curves thus obtained were then averaged, employing a method of averaging somewhat different from either of the two methods that have been used in other recent investigations. The luminosity-curves were all reduced to the same area and then the ordinates averaged at each wave-length, taken in steps of $0.01\,\mu$. This method would seem, in the judgment of the authors, to have a better theoretical basis than that of averaging the ordinates of the individual visibility-curves reduced to the same area, or that of averaging the ordinates of the individual visibility-curves reduced so that the maximum ordinate of each is unity.

The reason for the adoption of this method is as follows: The integral luminous flux from a source at any given color-temperature should be assumed to be the same for all observers, and the weight

assigned to the luminosity in any region of the spectrum for any observer should be determined on the basis of equal total luminous flux for that color-temperature. This weighting would of course be different for different energy-distributions corresponding to various color-temperatures of the source, but if some average temperature is taken the resultant average visibility-curves will be entirely correct for that temperature and only slightly in error for other temperatures, since the range of color-temperature encountered in practical photometry is relatively small. Theoretically it is preferable to choose as the standard color-temperature that of the carbon lamps adopted as the representative standards of luminous intensity, but since the color-temperature of the source employed in the present investigation is so nearly that of the standard carbon lamps (2077° K), the results are practically the same as those which would have been obtained had correction been made to the latter. The color-temperature which we have taken is 2045° K, that of the test lamp *D*, or more accurately a slightly higher temperature corresponding to the actual distribution of energy at the ocular slit, which is slightly different from that of the source, owing to scattered light and selective absorption of the optical system. The same assumption probably underlies the method of averaging the individual visibility-curves reduced to the same area, but this corresponds to an equal energy-distribution throughout the spectrum and so lies entirely outside the range of experience.

The method of reducing the various individual visibility-curves to the same value of maximum ordinate would seem to have no theoretical foundation and must be judged by its results. For the twenty-nine curves obtained in the present investigation it was found that this method of averaging yielded an average visibility-curve differing by many per cent at some wave-lengths from the curve obtained by the more rigorous method employed. As a matter of interest the observations of Coblentz and Emerson, who reduced the visibility-curves to equal maximum ordinates, were worked up by the more rigorous method, and the results showed an average visibility-curve sensibly the same as that derived by the other method. Coblentz and Emerson justified their method on

the ground of the large number of observers, and their conclusions were evidently warranted, but comparisons of individuals or of small groups taken from their one hundred and twenty-five observers would be subject to possible error unless the more rigorous method of averaging were employed.

EXPERIMENTAL RESULTS

The relative visibility data for the twenty-nine individual observers are given in Table I. These data are reduced on the basis of equal areas of the luminosity-curves for the chosen color-temperature (approximately 2045° K). The relative average visibility data for the twenty-nine observers, obtained on this basis, and also the average values obtained on the basis of equal value (unity) for the maximum ordinate of each visibility-curve are included. The former are also given in Table II and Fig. 3, where for purposes of comparison the published results of the recent investigations of Ives, Nutting, Coblentz and Emerson, and Reeves,[1] all obtained by the flicker method, are also included. With the exception of the data of Reeves,[2] which for some unknown reason differ largely from the other data obtained by the flicker method, it is seen that the visibility-curve given by the authors and obtained by the method of direct comparison is relatively narrower and more suppressed in the red end of the spectrum than the curves obtained with the flicker photometer. Nutting's curve most nearly agrees with that of the authors, and if his original published data had been used instead of his modified data, based on a more recent determination of the energy-distribution in the spectrum of his acetylene-flame source, the agreement would have been even better and well within the experimental errors.

From Fig. 3 the wave-length of maximum visibility may be taken to be $0.556\ \mu$ in the present investigation as compared with $0.557\ \mu$ found by Coblentz and Emerson, but the authors feel that

[1] *Trans. Illum. Eng. Soc.* (U.S.), 13, 101, 1918.

[2] The data of Reeves are reduced to the same basis of energy-distribution for the acetylene flame as that employed by Nutting in his final corrected values (kindly furnished by Dr. Nutting), since this acetylene-flame source was the same as that used by Nutting. The energy-distribution for this flame was determined by Coblentz and found to be the same as that published by Coblentz in his paper of 1916 on the subject.

this value is uncertain by 0.003 μ, owing to the limitations involved in drawing the curve. The question naturally arises whether the twenty-nine observers of the present investigation represent a

TABLE I

RELATIVE-VISIBILITY DATA FOR TWENTY-NINE OBSERVERS REDUCED TO EQUAL AREAS OF THE WAVE-LENGTH LUMINOSITY-CURVES FOR THE CHOSEN COLOR-TEMPERATURE (APPROXIMATELY 2045° K)

Wave-Length	Relative-Visibility Data								
	E.P.H.	F.E.C.	R.G.B.	C.F.S.	I.W.	M.L.	W.W	W.E.F.	A.G.W.
μ									
0.50...	316	236	393	242	324	413	237	193	132
0.51...	526	417	641	406	502	642	418	353	236
0.52...	758	619	921	581	658	846	600	533	355
0.53...	911	805	1143	722	787	1004	736	699	469
0.54...	977	973	1315	830	907	1121	840	831	580
0.55...	975	1054	1341	870	972	1145	880	889	651
0.56...	952	1085	1276	894	1004	1124	898	922	710
0.57...	895	1030	1103	882	980	1032	881	908	741
0.58...	810	914	877	840	907	887	834	856	748
0.59...	705	760	652	768	782	708	759	770	727
0.60...	591	596	464	666	625	534	652	659	678
0.61...	472	446	314	541	467	384	528	531	607
0.62...	356	318	207	407	331	265	399	399	514
0.63...	254	217	132	285	224	175	283	283	409
0.64...	166	138	80	181	143	108	185	184	302
0.65...	103	84	47	108	88	64	114	114	205
0.66...	57	46	25	57	50	34	64	63	119
Wave-Length	**Relative-Visibility Data**								
	P.W.C.	W.W.K.	C.N.	P.F.S.	E.J.E.	H.H.K.	G.H.M.	H.O.	N.L.
μ									
0.50...	225	407	435	407	297	396	324	488	242
0.51...	351	659	670	623	473	615	463	731	377
0.52...	478	922	866	826	660	819	594	931	512
0.53...	597	1126	1021	993	814	981	706	1061	624
0.54...	712	1255	1134	1092	932	1105	808	1118	721
0.55...	784	1244	1137	1073	974	1128	852	1090	772
0.56...	843	1162	1086	1007	992	1093	880	1033	805
0.57...	856	1022	986	909	961	1004	874	939	803
0.58...	822	857	854	803	877	877	835	828	774
0.59...	750	681	704	691	752	721	761	697	720
0.60...	660	514	550	568	607	558	647	553	643
0.61...	559	362	405	439	466	404	515	412	555
0.62...	448	240	279	319	344	278	383	287	455
0.63...	333	152	182	220	241	183	270	190	348
0.64...	226	89	109	138	156	112	177	115	247
0.65...	143	51	62	83	97	66	111	67	164
0.66...	81	26	32	44	54	36	63	35	97

TABLE I—*Continued*

Wave-Length	Relative-Visibility Data								
	T.P.	C.F.K.	L.L.M.	W.L.E.	E.T.F.	K.H.M.	A.F.B.	I.A.V.	G.B.
μ									
0.50...	206	471	324	366	223	407	187	561	435
0.51...	364	708	526	560	378	610	298	757	599
0.52...	532	936	729	727	553	808	428	914	726
0.53...	654	1081	910	863	729	980	561	1011	844
0.54...	738	1192	1040	958	884	1093	694	1078	965
0.55...	772	1218	1056	978	956	1126	778	1063	1016
0.56...	798	1196	1027	975	990	1127	851	1015	1032
0.57...	799	1093	956	931	965	1055	881	930	994
0.58...	774	933	857	849	892	917	862	825	909
0.59...	723	729	731	729	775	735	795	700	770
0.60...	649	523	591	591	637	550	689	561	606
0.61...	560	344	447	456	497	386	566	421	444
0.62...	456	212	319	337	366	260	440	297	306
0.63...	347	124	217	238	254	169	322	197	202
0.64...	243	67	137	158	162	103	216	121	124
0.65...	159	35	82	101	97	62	135	71	73
0.66...	90	16	45	60	51	34	74	37	40

Wave-Length	Relative-Visibility Data				
	O.B.	L.W.H.	Average	Same in Terms of Maximum Taken as Unity	Average on a Different Basis (See Text)
μ					
0.50...........	424	206	328	328	322
0.51...........	675	333	514	515	507
0.52...........	895	484	697	698	690
0.53...........	1030	667	846	847	839
0.54...........	1091	861	967	968	954
0.55...........	1078	964	995	996	992
0.56...........	1034	1007	994	995	995
0.57...........	953	979	943	944	953
0.58...........	848	901	854	855	868
0.59...........	716	784	734	735	751
0.60...........	568	645	599	600	617
0.61...........	420	502	464	464	483
0.62...........	291	370	341	341	358
0.63...........	192	258	238	238	252
0.64...........	117	166	154	154	164
0.65...........	68	101	95	95	102
0.66...........	36	54	52	52	56

fairly average eye. It is possible through the medium of the results of the interlaboratory comparison of Middlekauff and Skogland, referred to previously, to compare the twenty-nine observers here with the one hundred and twenty-five observers employed in the investigation of Coblentz and Emerson. The

five observers of this laboratory who determined the ratio of candle-power of the tungsten lamps at 132 and 72 volts, respectively, were all included in the present investigation, and computations show that for the foregoing candle-power ratio (i.e., for the corresponding color-difference) the twenty-nine observers would obtain a ratio 0.5 per cent less than that obtained by the five

TABLE II

Comparative Relative-Visibility Data of Various Investigators

Wave-Length	Relative-Visibility Data				
	Hyde Forsythe Cady	Ives Kingsbury	Nutting	Coblentz Emerson	Reeves
μ					
0.50	0.328	0.318	0.314	0.316	0.275
.51	.515	.473	.456	.503	.474
.52	.698	.637	.646	.710	.686
.53	.847	.801	.815	.862	.841
.54	.968	.915	.925	.954	.935
.55	.996	.988	.986	.994	.993
.56	.995	.996	.995	.998	.985
.57	.944	.947	.949	.968	.935
.58	.855	.859	.871	.898	.836
.59	.735	.758	.762	.800	.710
.60	.600	.653	.634	.687	.580
.61	.464	.534	.498	.557	.446
.62	.341	.396	.368	.427	.319
.63	.238	.283	.268	.302	.214
.64	.154	.183	.166	.194	.140
.65	.095	.110	.105	.115	....
.66	.052	.068	.058	.0645	....

observers. Similarly, taking the observers at the Bureau of Standards who participated in both investigations, computations show that the one hundred and twenty-five observers would obtain a ratio possibly 0.2 or 0.3 per cent greater than that obtained by the eight observers who were employed in the measurements of Middlekauff and Skogland. Since, according to the report of this interlaboratory comparison, Nela Research Laboratory obtained a ratio 1.9 per cent greater than that obtained at the Bureau, it would follow that the twenty-nine observers here would have obtained a ratio only 1.2 or 1.1 per cent greater than the one hundred and twenty-five observers used in the investigation of Coblentz and Emerson.

This difference is so small, considering the unsatisfactory way in which the comparison was carried out, that it would be unsafe to draw any conclusion except that, so far as can be ascertained, the twenty-nine observers employed in the present investigation do

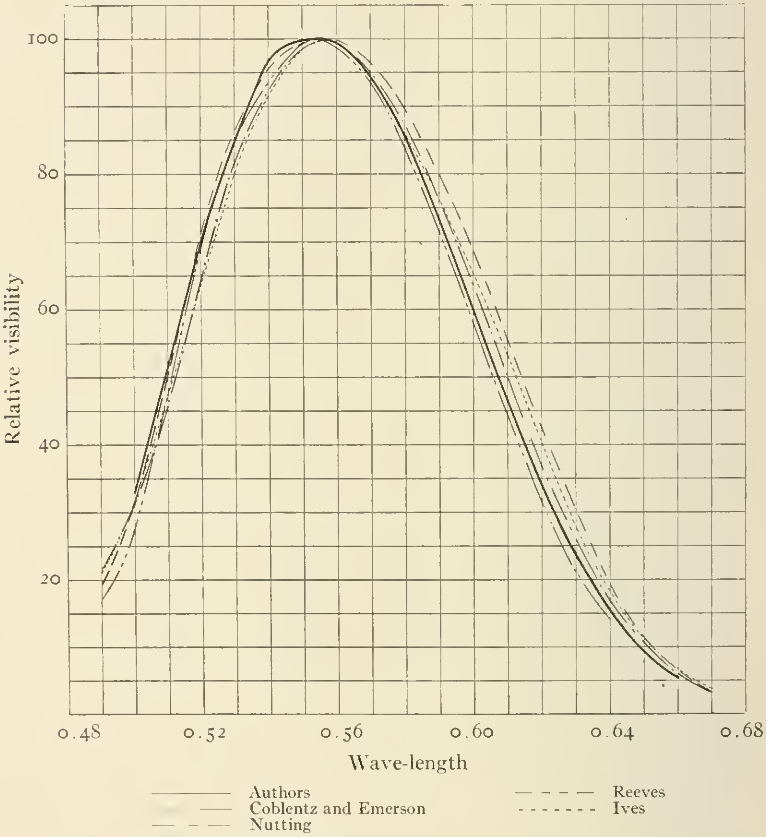

Fig. 3.—Relative average visibility data obtained in present investigation by the direct-comparison method as compared with published data of other investigators using the method of flicker photometry.

not differ materially in the average from the one hundred and twenty-five observers used in the investigation of Coblentz and Emerson. The comparison would seem to indicate, however, as intimated in an earlier paragraph, that the differences found by

Middlekauff and Skogland among the various laboratories would probably have been somewhat smaller had a larger number of observers been employed in each laboratory.

The significance of the difference between the visibility-curve obtained in the present investigation using the method of direct comparison and those obtained with the flicker method lies to some extent in the greater consistency of the results following the application of the former in computing relative candle-powers of a source at two widely different temperatures, but in large part in the distinctly smaller value of the mechanical equivalent of light which results from the use of the curve obtained by the method of direct comparison. As will be presented in a subsequent paper, the experimental values for the brightness of a black body at different temperatures are slightly more concordant with the computed values if the visibility-curve obtained here is used than with the results computed on the basis of the flicker-photometer visibility-curve, the latter giving relatively too much weight to the red end of the spectrum. And consequently the values of the mechanical equivalent of light computed from the brightnesses of the black body at different temperatures will agree among themselves if the visibility-curve obtained by the direct-comparison method is assumed, whereas this will not otherwise be true. These data will also be presented in the subsequent paper on the subject.

The considerations and data presented in this paper argue for the adherence to the older method of direct-comparison photometry in all ordinary practical work. As a means of avoiding the necessary difficulties in heterochromatic photometry in practice the authors refer to the proposal made years ago[1] that suitable color-screens be calibrated at the Bureau of Standards and distributed for use in photometric laboratories, so that comparisons involving large differences in color may be avoided in all except standardizing laboratories. It is true, however, that the results obtained with the flicker photometer are more consistent even over the comparatively small color intervals encountered in practical photometry, and the conditions of use have been thoroughly standardized, so that this instrument may find valuable application,

[1] *Electrical World*, 54, 195, 1909.

particularly in a standardizing laboratory. But in the opinion of the authors the flicker scale should not supplant that of direct comparison, and consequently the visibility-curve obtained by the direct-comparison method should be used in computing luminosity-curves and relative candle-powers. Especially should the visibility-curve obtained by the method of direct comparison be used in the computation of the mechanical equivalent of light, in which its difference from that obtained by the flicker method is shown in the most pronounced way.

SUMMARY

1. In ordinary photometric comparisons involving difference of color the older method of direct comparison is adequate if a sufficient number of observers is employed to secure a fairly average eye.

2. The relative candle-power values found for sources of different color by the use of the flicker photometer are appreciably different from those obtained by the method of direct comparison. The flicker photometer apparently assigns too much weight to the red end of the spectrum.

3. The average curve of relative visibility for twenty-nine observers obtained by direct comparison, using a step-by-step method, is presented. Special attention is called to the method employed for determining the distribution of energy at the ocular slit.

4. The curve of relative visibility obtained by the method of direct comparison is found to be definitely different from that obtained by other investigators using the flicker method. The latter is somewhat broader and shows relatively too large values in the red end of the spectrum. Evidence will be presented in a subsequent paper to show the consistency between the newly determined visibility-curve and the findings of ordinary photometry in the case of the brightness of a black body at different temperatures.

5. Recommendation is made of the adherence to the older photometric method of direct comparison in practical photometry, and of the use of calibrated color-filters to simplify comparisons otherwise involving large difference in color.

APPENDIX I

In connection with the determination of the effective wave-length of transmission of so-called monochromatic red-glass screens used with optical pyrometers two of the present authors[1] some time ago described a determination of the relative visibility of radiation in the less refrangible end of the spectrum, using an adapted form of a spectral pyrometer. Owing to the large field-brightness obtainable with the pyrometer it was possible to carry the measurements farther into the red than had been possible before. Moreover, since the application of the data was to be made under conditions quite similar to those obtaining in their determination, it seemed unnecessary to enter into any elaborate discussion of these conditions or to question their applicability.

Subsequently L. W. Hartman,[2] working in this laboratory, employed the same method to extend the relative visibility data on the side of short wave-lengths, and incidentally again to furnish values which might be used in computing the effective wave-length of blue pyrometer glasses.

It therefore seemed advisable to one of the authors, in connection with the present investigation as presented in the body of this paper, to carry out a series of determinations of relative visibility with the pyrometer method through the central region of the visible spectrum, thus connecting the previously published data for the red and blue ends of the spectrum. A brief report of the results obtained is presented in this Appendix.

The method employed is identical with that already described in the earlier papers on the subject, except that an unsaturated green-glass screen in front of the eyepiece was used throughout in order to reduce the large differences in color and make more consistent settings possible. The transmission-curve for this screen, plotted in Fig. 4, shows a considerable transmission of light throughout the entire spectral region studied.

In this supplementary investigation, as in the principal one, the method of direct comparison was used, and the determination of the distribution of energy at the eyepiece was carried out in the

[1] *Astrophysical Journal*, **42**, 285, 1915. [2] *Ibid.*, **47**, 83, 1918.

same way. But in some important aspects the conditions in the two experiments were quite different. Thus the brightness in

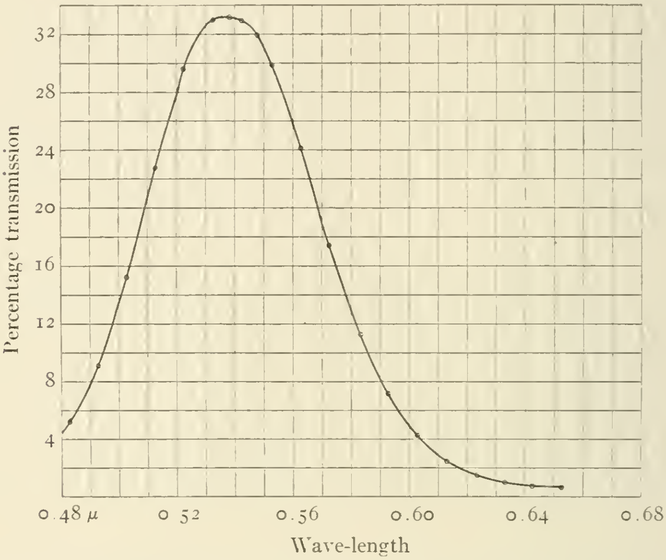

FIG. 4.—Selective transmission of green glass employed in eyepiece of spectral pyrometer.

TABLE III

RELATIVE AVERAGE-VISIBILITY DATA OBTAINED BY THE USE OF A SPECTRAL PYROMETER

(As compared with that obtained in the principal investigation in which the conditions, such as method employed, size of field, etc., were different)

WAVE-LENGTH	RELATIVE AVERAGE-VISIBILITY DATA		WAVE-LENGTH	RELATIVE AVERAGE-VISIBILITY DATA	
	* Spectral Pyrometer	† Spectro-photometer		Spectral Pyrometer	Spectro-photometer
μ			μ		
0.50........	0.329	0.328	0.59........	0.782	0.735
.51........	.552	.515	.60........	.639	.600
.52........	.736	.698	.61........	.489	.464
.53........	.886	.847	.62........	.347	.341
.54........	.958	.968	.63........	.229	.238
.55........	.993	.996	.64........	.161	.154
.56........	1.000	.995	.65........	....	.095
.57........	.999	.944	.66........	....	.052
.58........	.914	.855	....	....	....

the different spectral regions was always the same in the pyrometer experiments (0.024 candles per cm^2), and the measurements were made throughout against a constant greenish comparison field. Moreover, the size of the field was much smaller, though

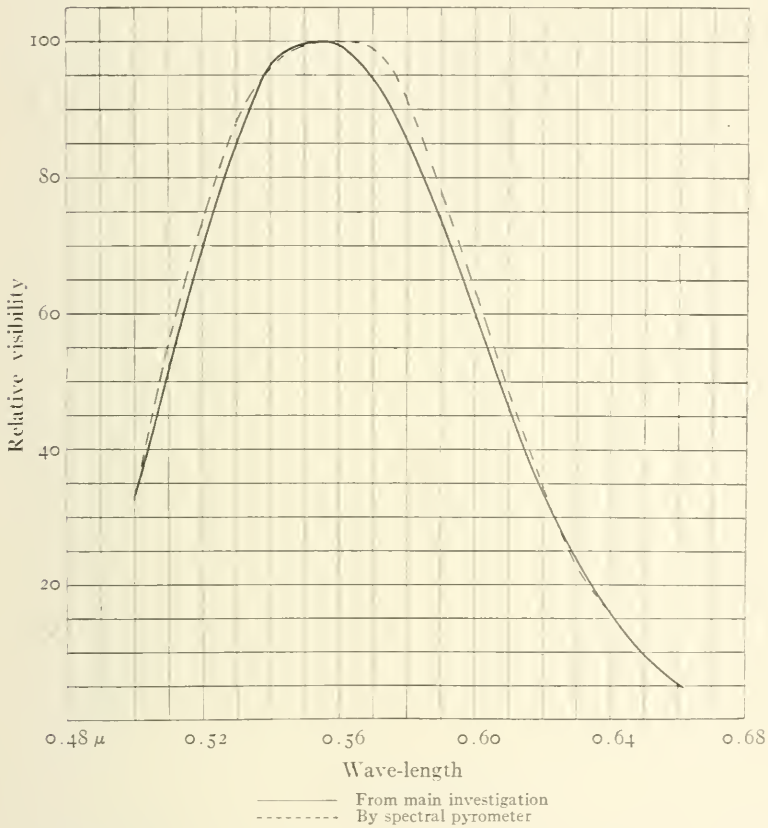

Fig. 5.—Relative average visibility data obtained by the use of a spectral pyrometer as compared with those obtained in the principal investigation in which the conditions, such as method employed, size of field, etc., were different.

the comparison field (pyrometer filament) was not so small as that employed in the earlier investigations of the extreme red and blue regions. The collimator slit was 0.3 mm. and the ocular slit was 0.12 mm. Corrections for slit-width were computed and found to be negligibly small.

Ten observers were employed, taken at random from among the twenty-nine used in the principal investigation, and each observer (with one or two exceptions) made at least two independent sets of measurements. The results, in comparison with those presented in the main body of the paper, are shown in Table III and Fig. 5. It is seen that the curve is somewhat flatter near the center and drops off more rapidly toward the two ends of the spectrum. The previously found visibility data in the red and blue ends of the spectrum, made by the same method, fit nicely on to the ends of this curve. The previously obtained data in the blue fit equally well on to the curve presented in the main body of this paper, but the data for the red end cannot be made to fit with any accuracy.

This curve agrees more closely with that found in the principal investigation than does any one of the curves obtained with the flicker method, and relative candle-powers of a black body at different temperatures computed on its assumption are reasonably satisfactory. Moreover, it conduces to approximately the same value of the mechanical equivalent of light as that calculated from the other curve. But owing to the inherent difficulties encountered in applying the pyrometer method in the middle region of the spectrum, to the difference between the conditions of the experiment and those of ordinary practice, and finally to the more limited number of observers, the authors do not attach as much weight to the results as to those presented in the body of the paper, particularly for application to cases of ordinary practical photometry.

APPENDIX II

The investigation described in the body of the present paper yielded a curve of relative visibility obtained by the method of direct comparison and extending from 0.50μ to 0.66μ. A knowledge of the curve over this interval is sufficient to compute relative candle-powers of a black body over a moderate range of temperature, such as from 1700° K to 2500° K, with errors amounting to only a few tenths of 1 per cent on account of the omission of the ends of the curve in the red and blue. If larger ranges of temperature are employed, the errors arising from this omission may be

appreciable, and at any temperature the application of the visibility data in computing the mechanical equivalent of light will conduce to erroneous values if the luminosities of the two ends of the spectrum are neglected. Finally there are problems, such as the determination of the effective wave-length of transmission of red- and blue-glass screens for use in optical pyrometry, in which a knowledge of the visibility data for the ends of the spectrum is necessary.

TABLE IV

RELATIVE-VISIBILITY DATA FOR ENTIRE SPECTRUM

(As recommended by authors)

Wave-Length	Relative Visibility	Wave-Length	Relative Visibility	Wave-Length	Relative Visibility
μ		μ		μ	
0.40	0.00009	0.52	0.698	0.64	0.154
.41	$.0006_2$	.53	.847	.65	.094
.42	$.004_1$	.54	.968	.66	.051
.43	$.011_5$	.55	.996	.67	.026
.44	.022	.56	.995	68	$.012_5$
.45	.036	.57	.944	.69	$.006_2$
.46	.055	.58	.855	.70	$.003_1$
.47	.087	.59	.735	.71	.0015
.48	.138	.60	.600	.72	$.0007_4$
.49	.216	.61	.464	.73	$.0003_6$
.50	.328	.62	341	.74	$.0001_8$
.51	.515	.63	.238	.75	.00009
				.76	.00005

* Average of 10 out of the 29 observers.
† Average of 29 observers.

It therefore appeared to the authors desirable to submit data on relative visibility covering practically the entire visible spectrum. The most probable values in the opinion of the authors are contained in Table IV.

These values from 0.50μ to 0.64μ are precisely the same as those given in the body of the paper. For the red end are chosen the published data of Hyde and Forsythe[1] brought into agreement with the central region of the curve at 0.64μ. This necessitated a slight change of the values at 0.65μ and at 0.66μ as found in the present investigation. For the blue end the published data of

[1] *Loc. cit.*

Hartman[1] are chosen as the best. The reasons for choosing these data for the red and blue ends are as follows:

1. They were obtained by a direct-comparison rather than by a flicker method, even though the size of the field of view was much smaller than that employed in the investigation of the middle of the spectrum described in the present paper.

2. They were obtained under favorable conditions as to brightness, and are probably more free from errors due to scattered light and slit-width corrections than other published data.

3. They are more definitely applicable in computing the transmitted luminosity of optical pyrometer screens, for which they will probably find their largest application, since they were obtained under the conditions which are found in optical pyrometry.

4. They will serve as well as any other values in computing integral luminosities or the mechanical equivalent of light, since large differences in the accepted visibility data in these extreme regions of the visible spectrum produce negligibly small errors.

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NELA PARK, CLEVELAND, OHIO
May 1918

[1] *Loc. cit.*

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

SIXTH PAPER: ON THE DETERMINATION OF THE DISTANCES OF GLOBULAR CLUSTERS

By HARLOW SHAPLEY

I. INTRODUCTION AND SYNOPSIS

The derivation of parallaxes for globular clusters will contribute to several problems of general astrophysical interest. Through the introduction of a linear scale we may expect to learn much of the actual dimensions of these stellar groups, and of the relation to distance from their centers, of the luminosity, mass, spectrum, and star density, and, eventually, of the internal motions. Since we may also be able to estimate at least a lower limit for the total mass involved in each group, the opportunity is promising for a contribution to the rather meager supply of observed facts for studies in stellar dynamics. Further, a knowledge of the distances of these widely distributed systems, and of the highly luminous variable stars for which parallaxes will be obtained in the course of the same research, will add to our conception of the dimensions of the visible stellar universe. These problems are of enough importance to justify a considerable effort in the study of the distances of globular clusters and their distribution in space. As direct measurement of cluster parallaxes is out of the question, the procedure must be by other methods, such as those involving proper motions and luminosity correlations.

The range in the absolute brightness of stars in many globular clusters is now known to exceed ten magnitudes, and we have as yet no reason to believe that the dispersion of luminosity in any of the typical globular clusters is not strictly comparable with the known dispersion in the general galactic system. Our ordinary investigations of cluster magnitudes, therefore, involve many stars of relatively great luminosity. Among these very brightest stars

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 151.

reddish Cepheid variables of long period frequently appear, while among the stars two or three magnitudes fainter is found the typical cluster-type variable—a Cepheid with a first-type spectrum and a period less than a day.

For some years we have known that the longer-period Cepheids in the galactic system are also giant stars, and a casual examination of their motions indicates a fairly small dispersion in actual luminosity. The possibility is at once suggested, therefore, of utilizing the motions of galactic Cepheids to obtain a mean value of the absolute magnitude of such stars, which, when compared with observed magnitudes of analogous objects in globular clusters, permits an estimate of the distances of the clusters themselves. One obvious advantage of operating with a group of giant stars is that in many of the most distant clusters only the stars of highest luminosity are within reach of our greatest telescopic power. It is also clearly of prime importance in problems of this nature, which involve faint stars and great distances, to have reliable systems of magnitudes established and to be able to ignore with safety the general scattering of light in space.

The methods and results of the investigation of cluster parallax will be discussed in Papers VI to XII, inclusive, of the series of studies of magnitudes in clusters. The present discussion begins with the derivation of a mean absolute magnitude for the isolated Cepheids whose motions are well determined, and then considers the remarkable relation between the period of variation and the total light-emission of such stars. Examples of this interesting correlation, which may perhaps be considered a fundamental law in Cepheid variation, are found among the variables in no less than six different stellar organizations as well as among the galactic Cepheids. Combined with the very homogeneous and accordant data derived from motions, it permits the deduction of absolute magnitudes, and hence of absolute distances, with a surprisingly small computed probable error. In fact, unless some intrinsic weakness in the procedure or some overlooked alternative is found, we may believe that the distances of the Cepheid variables, and of the extremely remote globular clusters in which Cepheid variables have been studied, can be determined with a percentage accuracy

rarely excelled and usually unequaled by direct measures of the nearest stars.

Now it happens that in many globular clusters no variable stars are as yet known; in others the Cepheids similar in period to those for which motions are known in the galactic system are of rare occurrence compared with the typical cluster-type Cepheid with period less than a day. We find, however, that the median magnitude of the typical cluster-type variable shows such an extremely small dispersion in a given cluster that a constant value can be assumed; and when we adopt the reasonable and important hypothesis that the equality of the total light-emission is universal for such variables, the corresponding absolute median magnitude can be accurately derived from the analysis of the motions, magnitudes, and periods of the longer-period Cepheids. The determination of the parallaxes of certain clusters then becomes a by-product of the study of their typical variable stars.

A further step correlates this median magnitude with the magnitude of the brightest stars in a cluster. A means is thus afforded of computing distances of all the clusters (whether or not they contain known variables) for which apparent photographic magnitudes of the brightest stars are measured. Finally, we find from the results derived by the processes outlined above that the parallax of a cluster is very definitely related to its angular diameter, and this yields a method of obtaining from already available photographs the distances of all globular clusters so far discovered.

In the second article (the seventh paper of the general series) the individual distances of globular clusters are derived; their highly significant distribution in space is also discussed, and some computations made of the linear dimensions, of the concentration of stars, and, provisionally, of the total masses in cluster systems.

The parallaxes and co-ordinates in space of all Cepheid variables, for which magnitudes and periods are known, appear in the eighth paper. The chief uncertainty in the results is due to the lack of accuracy and homogeneity in published values of magnitudes. For the isolated cluster-type variables accurate values of the period are not essential in deriving distances, as the dispersion in absolute magnitude is small for periods less than a day.

The next paper contains three notes on certain theoretical and observational aspects of Cepheid variation. An interpretation is offered for some of the results obtained in studying the distances of clusters and variables, the relation of color to period is examined observationally for cluster-type variables, and a composite color-curve is derived for more than a hundred variable stars.

An inquiry into some problems of the frequency of stellar luminosities appears in the tenth paper. The magnitudes of several thousand cluster stars have been measured for this discussion. Some light is thrown on the meaning of the median magnitude of variables, and the propriety of its use for parallax work, rather than the use of maximum or minimum, is established.

The eleventh paper treats briefly of the distances and distribution in space of various objects and classes of objects. The data derived from clusters and variables is supplemented by the results of other investigators for different types and groups of stars.

The last article, summarizing the facts and indications of the preceding papers, contains a preliminary attempt to discern the general plan of the sidereal system. The arrangement proposed has its principal justification in that it appears to be a simple interpretation of the new data and at the same time does not seem to be inconsistent with previous observational results bearing on the structure of the universe.

In the accumulation of the observational material for these studies much credit is due Miss Davis for the measurement of large numbers of stars in many difficult cluster fields. She has also assisted throughout in the reduction and discussion of the observations. Mrs. Shapley has collaborated in the treatment of most of the problems and, in particular, is responsible for large parts of the tenth and twelfth papers. Valuable assistance in the preparation of the papers for the press has been given by Miss Connor. Mr. Pease has freely permitted the use of his long-exposure cluster photographs. Data relative to parallaxes have been kindly furnished by Mr. van Maanen and relative to spectroscopic results by Mr. Adams. Special acknowledgment is due Mr. Seares for many valuable suggestions and criticisms and for painstaking editorial supervision of the whole series of cluster papers.

II. THE MEAN ABSOLUTE MAGNITUDE OF CEPHEID VARIABLES

Hertzsprung[1] and Russell[2] have computed the mean absolute magnitude of Cepheid variables from the proper motions given in the *Preliminary General Catalogue* of Boss, and the former has published his work in some detail. With a few corrections to the observational data the computations are now repeated and somewhat extended. The material is not extensive, and its sufficiency for this problem might well be questioned if it were not for the fortunate circumstances that the data are complete for each star and in most respects homogeneous; that no evidence of preferential motion is found; and that the peculiar motions of such stars, without exception, are small compared with their parallactic drifts.[3] Hertzsprung tabulates 13 stars, but two of them are not typical Cepheid variables and have been excluded;[4] no others have sufficiently accurate proper motions to be added to the list.[5]

Table I contains observational data relative to the group of 11 Cepheids. With the exception of Polaris, all are near the

[1] *Astronomische Nachrichten*, **196**, 201, 1913; see also *Zeitschrift für wissenschaftliche Photographie*, 5, 94, 107, 1907.

[2] *Science*, N.S., **37**, 651, 1913.

[3] See n. 1, p. 103.

[4] Both the period and light-curve of κ Pavonis are subject to considerable perturbation according to Gould (*Uranometria Argentina*, p. 244, 1879), and Roberts also notes the star as an exception to ordinary Cepheid variation. The light-variation of *l* Carinae is peculiar; "an irregular and ill-defined secondary maximum has frequently been observed" (Roberts, *Astronomical Journal*, **21**, 89, 1901). Albrecht has just reported a variation in the spectrum of *l* Carinae from F8 to G5 (*Popular Astronomy*, **25**, 519, 1917), and it may be the star is not so abnormal as was believed when the computations for this paper were made.

The typical Cepheid characteristics of the eleven stars used are attested not only by their light-curves, but also in every case by spectroscopic study. We have no velocity-curves for *l* Carinae and κ Pavonis. The inclusion of both in the discussion would change the final result by less than its probable error. Including κ Pavonis would greatly decrease the certainty of the computed absolute magnitudes, though not altering them seriously, while the inclusion of *l* Carinae alone would make no appreciable difference in either the result or its probable error.

[5] The values given by Perrine in *Astrophysical Journal*, 41, 308, 1915, for Y Ophiuchi and SZ Tauri are too uncertain for the present work. The proper motion of RR Lyrae, period 13.5 hours, is rightly excluded; see the eighth paper of this series, "The Luminosities and Distances of 139 Cepheid Variables," *Mt. Wilson Contr.*, No. 153, 1917.

TABLE I

OBSERVATIONAL MATERIAL FOR ELEVEN CEPHEIDS

Boss No.	Name	R.A. 1900	Decl. 1900	Galactic Longitude	Galactic Latitude	Median Magnitude	Spectrum	Period	τ	v	λ	km/sec.
325..	α Ursae Minoris	$1^h 22^m.6$	$+88° 46'$	90°	+27°	2.12	F8	$3^d.9681$	+0″.016	+0″.041	60°	−15
637..	SU Cassiopeiae	2 43.0	+68 28	101	+ 9	5.9	A8–F5	1.9495	+ 8	+ 14	75	− 7
1629..	RT Aurigae	6 22.1	+30 34	151	+11	5.3	A7–G1	3.72806	− 4	+ 24	118	+22
1815..	ζ Geminorum	6 58.2	+20 43	164	+13	4.0	G	10.15382	− 2	+ 9	126	+ 7
4493..	X Sagittarii	17 41.3	−27 48	329	− 1	4.7	F1–G5	7.01188	− 3	+ 23	59	−14
4564..	W Sagittarii	17 58.6	−29 35	330	− 5	4.7	A8–G2	7.5946	+ 9	+ 12	61	−29
4632..	Y Sagittarii	18 15.5	−18 54	340	− 4	5.8	F4–G4	5.7734	+ 6	+ 12	50	+ 4
5071..	η Aquilae	19 47.4	+ 0 45	8	−14	4.05	A8–G5	7.176382	+ 1	+ 12	39	−14
5098..	S Sagittae	19 51.5	+16 22	23	− 7	5.8	F4–G3	8.381613	− 1	+ 4	30	−12
5370..	T Vulpeculae	20 47.2	+27 52	40	−11	5.8	A9–G1	4.435521	− 11	+ 10	36	− 1
5807..	δ Cephei.......	22 25.4	+57 54	73	+ 1	4.14	F0–G2	5.366386	+0.003	+0.012	52	−17

galactic plane; their distribution in galactic longitude is satisfactory. The median visual magnitudes differ in some instances from those used by Hertzsprung; the spectral types, taken mainly from *Contribution* No. 124,[1] are known to be variable for all but two of the stars. The proper motions have been reduced to the parallactic system of co-ordinates, υ being counted positive in the direction of the antapex. Without further reduction the parallactic motion is conspicuously evident, the unweighted algebraic means being

$$\left.\begin{aligned}\tau&=+0''002\\ \upsilon&=+0''016\end{aligned}\right\} \qquad (1)$$

All values of υ are positive. The distances from the apex of the solar motion, λ, are very favorable for the weight of the solution.

Curves of velocity-variation have been determined for all of these stars, mainly at the Lick Observatory. Seven of the values of V, the observed radial velocity of the center of mass, are taken from Duncan's compilation in *Lick Observatory Bulletin*, No. 151. The value for α Ursae Minoris, computed by Miss Hobe, has been given by Campbell,[2] and those for X Sagittarii[3] and δ Cephei[4] were computed by Moore. The value for SU Cassiopeiae is obtained from an unpublished discussion by Adams and Shapley of the variations in light, velocity, and spectral type.[5]

Some of the steps in the reduction appear in Table II. To eliminate possible effects of the dispersion in distance the proper motions were reduced to the common apparent magnitude $+5$, which is very near the mean apparent median magnitude of the variables, $+4.8$. The weights of the individual values of the reduced parallactic motion, $\upsilon_5 / \sin\lambda$, were determined in the manner suggested and used by Hertzsprung.[6] They depend on the distribution of velocities for this type of star, on the probable errors

[1] *Astrophysical Journal*, **44**, 273, 1916.

[2] *Lick Observatory Bulletin*, **6**, 19, 1910.

[3] *Ibid.*, **5**, 111, 1909.

[4] *Ibid.*, **7**, 153, 1913.

[5] *Mt. Wilson Contr.*, No. 145, 1917. *Note added to proof, April, 1918:* The source of the visual magnitude 6.23 is *Harvard Annals*, **50**, and not the *Preliminary General Catalogue*, as erroneously printed (*Astrophysical Journal*, **47**, 50, 1918).

[6] *Astronomische Nachrichten*, **196**, 203, 1913.

given by Boss for the individual proper motions, and on the distances from the solar apex. The weighted mean value and its probable error are

$$\frac{v_5}{\sin\lambda}=+0''.0161\pm0''.0016. \qquad (2)$$

It is of interest to compare this value with (1). The corresponding value derived by Hertzsprung, if we reduce his result to the same apparent magnitude, is

$$\frac{v_5}{\sin\lambda}=+0''.0146\pm0''.0022$$

from which he derived for the mean parallax of a Cepheid of the fifth apparent magnitude

$$p_5=0''.0035\pm0''.00054,$$

and for the mean absolute magnitude, corresponding to the mean period of 6.6 days,

$$M=-2.3\pm0.35.$$

The larger probable error of his value of $v_5/\sin\lambda$ is attributable mostly to the inclusion of the highly discordant κ Pavonis.

TABLE II

SOLUTION FOR MEAN PARALLAX

Boss No.	Observed		$\frac{v_5}{\sin\lambda}$	Relative Weight	Parallactic v_5	Residual v_5	V_0	π ($M=-2.35$)
	τ_5	v_5						
325..	+0″.004	+0″.011	+0″.013	99	+0″.014	−0.003	− 4km	0″.0128
637..	+0.012	+0.021	+0.022	48	+0.016	+0.005	− 2	22
1629..	−0.005	+0.028	+0.032	61	+0.014	+0.014	+12	30
1815..	−0.001	+0.006	+0.007	83	+0.013	−0.007	− 6	54
4493..	−0.003	+0.020	+0.023	80	+0.014	+0.006	− 4	39
4564..	+0.008	+0.011	+0.013	69	+0.014	−0.003	−18	39
4632..	+0.009	+0.017	+0.022	12	+0.012	+0.005	+18	24
5071..	+0.001	+0.008	+0.013	48	+0.010	−0.002	+ 2	52
5098..	−0.001	+0.006	+0.012	16	+0.008	−0.002	+ 6	24
5370..	−0.016	+0.014	+0.024	11	+0.009	+0.005	+16	24
5807..	+0.002	+0.008	+0.010	79	+0.013	−0.005	− 4	0.0050

With the value of the sun's velocity adopted by Hertzsprung, 20 km/sec., equation (2) gives $p_5=0''.00386$ and $M=-2.06$; but later

determinations from radial motions point to a higher velocity of the sun. We have the following recent values, based on a greater proportion of giant stars than were included in the early determinations and therefore probably more reliable because of smaller peculiar velocities:

		km/sec.	Mean Error
Charlier,[1]	156 B-type stars (mag.<5.0), $V_\odot$	=21.26	
Gyllenberg,[2]	284 B-type stars	=22.06	±0.91
Gyllenberg.	291 A-type stars	=19.77	±1.45
Strömberg,[3]	1400 F, G, K, M, giant stars	=21.48	±1.02

Adopting $V_\odot = 21.5$ km/sec.,

$$\frac{v}{p \sin \lambda} = 4.535$$

$$p_5 = 0\rlap{.}''00354 \pm 0\rlap{.}''00035 \tag{3}$$

and the absolute magnitude, corresponding to the mean period of 5.96 days, is

$$M = -2.26 \pm 0.22.$$

Computing for each star that part of the v-component due to parallactic motion, $0\rlap{.}''0161 \sin \lambda$, and subtracting these values in the sixth column of Table II from the observed values of v_5 in the third column, we obtain in the seventh column values for the peculiar velocity parallel to the direction of the sun's motion. The values of τ_5 and of "residual v_5" are in satisfactory agreement.[4] Their mean values, without regard to sign, are

$$\tau_5 = 0\rlap{.}''0056 \pm 0\rlap{.}''0010.$$

$$\text{Residual } v_5 = 0.0052 \pm 0.0007.$$

[1] *Meddelande från Lunds Astronomiska Observatorium*, Serie II, No. 14, 1916.

[2] *Ibid.*, No. 13, 1915.

[3] *Mt. Wilson Contr.*, No. 144, 1917.

[4] A considerable part of the agreement is a necessary consequence of the magnitude of the observational errors relative to those of the proper motions. The average probable error for the annual motion in one direction is about $\pm 0\rlap{.}''004$. As the peculiar motions are not greatly in excess of the errors of observation, possibly less weight should have been given to (6) in obtaining the final value (7). The difference between (3) and (7), however, is less than half the probable error of either.

Combining all twenty-two values, with half weight for the residual v_5=components because one constant has been derived from them, we obtain the mean value for the peculiar proper motion in one direction

$$\mu_5' = 0''.0055 \pm 0''.00064, \tag{4}$$

a value about one-third that of the derived parallactic motion.

The observed radial velocities corrected for the sun's motion[1] are given in the eighth column of Table II. Their arithmetical mean value is

$$V_0 = 8.35 \pm 1.29 \text{ km/sec.} \tag{5}$$

and we derive from (4) and (5) an independent value of the mean parallax

$$p_5 = \frac{4.74\,\mu_5'}{V_0} = 0''.0031 \pm 0''.0005. \tag{6}$$

Probably the very close agreement of (3) and (6) is partly chance. If in the place of (5) we use a value based upon all known radial velocities of Cepheids, we add to the eleven values in Table II nine others, most of which are only rough estimates,[2] and obtain $V_0 = 9.58$ km/sec., $p_5 = 0''.0027$.

Giving double weight to (3), or, what amounts to the same, combining (3) and (6) with regard to their probable errors, we obtain the final values:

$$\left.\begin{array}{l} p_5 = 0''.0034 \pm 0''.00030 \\ M = -2.35 \pm 0.19 \end{array}\right\} \tag{7}$$

The probable errors of the foregoing mean values include the errors in the observed proper motions and radial velocities and some uncertainties of reduction; but they do not include the errors

[1] The value 21 km/sec. was used for the sun's velocity in this computation, but no difference in the mean value would result from using 21.5 km/sec.

[2] The additional stars are SU Cygni, Y Ophiuchi, SZ Tauri, T Monocerotis, and five southern Cepheids for which very approximate velocities have been estimated recently by Paddock from a few plates of each (*Lick Observatory Bulletins*, **9**, 68, 1917). The value for T Monocerotis is a little more than a guess (*Astrophysical Journal*, **23**, 266, 1906). The velocity-curve of SZ Tauri is by Haynes (*Lick Observatory Bulletins*, **8**, 85, 1914), and for the others Duncan's table furnishes information.

(relatively much less important in the derivation of M) in the apparent magnitudes and the velocity of the sun, nor the uncertainty arising from any systematic motion or drift of the Cepheids as a group.

From the value (7) and the apparent median magnitudes in the seventh column of Table I, the individual parallaxes have been derived and entered in the last column of Table II. The probable error of each value depends only on that of M and the uncertainty of the corresponding apparent magnitude; its average is about $\pm 0''.0007$, or 15 per cent of the tabulated parallaxes. Better values of these parallaxes are derived later.[1]

TABLE III

Boss Number	Observer	Rel. π and P.E.	Absolute π	π from Parallactic Motion and Period
325............	Flint *et al.*	$\pm 0''.018$	$+0''.028$	$+0''.016$
637............	van Maanen	$+0.008 \pm 0.003$	$+0.010$	$+0.004$
1815............	Abetti, Miller	$+0.019 \pm 0.009$	$+0.025$	$+0.004$
5071............	Mitchell	$+0.001 \pm 0.009$	$+0.006$	$+0.005$
____(X Cygni)..	Miller	$0.000 \pm 0.008'$	$+0.006$	$+0.001$
5532............	Lee and Joy	-0.016 ± 0.014	-0.011	$+0.018$

Direct measures of the parallaxes of four of the 11 Cepheids here considered are available for comparison with the results of the present investigation. The data are shown in the first four lines of Table III. Flint's parallax (absolute) of Polaris, $+0''.008 \pm 0''.016$, is combined with six of the more recent determinations listed by Kapteyn in *Groningen Publications*, No. 24. The values of the last column are taken from a later paper of this series. The absolute values are systematically larger than those based upon the parallactic motion, the mean difference being $+0''.010$ for the first four stars, which corresponds to a difference of approximately two magnitudes in the absolute brightness. This discrepancy is large and raises a question as to possible sources of error.

We note, however, that the inclusion of X Cygni and the short-period variable β Cephei (Boss 5532)[2] reduces the difference in the

[1] *Mt. Wilson Contr.*, No. 153, the eighth paper of this series.

[2] Their parallaxes are determined later by the method used to calculate the final values for the 11 Cepheids, although they could not be included in the original group.

mean values to $+0''.003$; and, further, according to an unpublished investigation by van Maanen, there is some evidence that the directly measured parallaxes require systematic corrections sufficient to bring the mean value below that derived from the parallactic motion by $0''.001$. These circumstances illustrate the uncertainty of any conclusion based upon a small number of directly measured parallaxes whose values are as minute as those of the Cepheid variables appear to be.

On the other hand, the smallness of the proper motions makes the calculated parallactic motion sensitive to any systematic errors in the observed motions, provided the errors are so large or so distributed that the mean value of p_5 is affected. An attempt to determine directly from the observations a systematic correction to the proper motions is not expedient, however, as the number of stars is small and they are widely scattered over the sky. The reduced individual motions should reveal any conspicuous error.

If we assume a systematic correction to the proper motions of the amount used by Kapteyn (*Contributions*, Nos. 82 and 147) for some of the B-type stars in the southern hemisphere, and suppose that its effect is not obliterated in the mean result, the deduced absolute brightness might be changed by about one-tenth of a magnitude, an amount clearly insufficient to question the general accuracy of the present result.

In the introduction to the *Preliminary General Catalogue* Boss has suggested as provisional systematic corrections to his proper motions:

$$d\mu_\alpha = +0^s.00021 - 0^s.00015 \sin\alpha \tan\delta$$

$$d\mu_\delta = -0''.0023 \cos\alpha$$

These corrections have been applied to the proper motions of the Cepheids and revised values computed for the τ- and υ-components. In Table IV the probable errors of the proper motions, as given by Boss, should be compared with the proposed systematic corrections. (For the first star, Polaris, a systematic correction of this general nature is of course not appropriate.)

In nearly every case the probable error exceeds the systematic correction, and in the average it is about three times as large. The

revised values of τ and υ should be compared with the analogous values in Table I. The average correction to υ is just one-half the probable error of the proper motion in one direction, and the systematic correction to the parallactic motion is also about one-half of its probable error. The corresponding correction to the adopted absolute magnitude would be less than +0.05, an amount safely negligible. We conclude, therefore, that unless the systematic errors are assumed to be of a wholly different order of magnitude from those derived by Kapteyn and Boss the mean parallax of these Cepheids is essentially correct.

TABLE IV

Boss No.	P.E. of μ_α	P.E. of μ_δ	$d\mu_\alpha$	$d\mu_\delta$	Revised τ	Revised υ	$\Delta\upsilon$
325....	±0^{s}00343	±0″0008	(−0^{s}00225)	(−0″0022)	+0″015	+0″041	0″000
637....	88	40	− 4	− 17	+ 7	+ 16	+ 2
1629....	37	35	+ 12	+ 2	− 2	+ 23	− 1
1815....	11	17	+ 16	+ 6	0	+ 7	− 2
4493....	23	30	+ 13	+ 1	− 2	+ 22	− 1
4564....	35	44	+ 12	+ 0	+ 11	+ 12	0
4632....	74	80	+ 16	− 2	+ 8	+ 12	0
5071....	20	29	+ 21	− 10	+ 3	+ 15	+ 3
5098....	31	40	+ 25	− 11	0	+ 8	+ 4
5370....	59	63	+ 27	− 15	− 11	+ 13	+ 3
5807....	±0.00020	±0.0017	+0.00031	−0.0021	0.000	+0.014	+0.002

There remains, however, the possibility of systematic error in the mean distance due to a preferential drift of the Cepheids as a class—a drift which, in order to escape ready detection by means of the peculiar motions, must be either very small or nearly in the direction of the solar motion. Suppose, for example, that the mean absolute magnitude of the eleven Cepheids were zero rather than −2.35. This corresponds approximately to the extreme systematic difference indicated by the directly measured parallaxes. Then the mean parallactic motion would be 0″05, that is, three times the observed value (which is determined with a computed error of only 10 per cent). To harmonize this assumption with the observed parallactic motion we must assume an annual preferential drift of 0″03 in a direction deviating but a few degrees from the solar apex. Such a drift should reveal itself in the radial

velocities, which show no effect of the required order of magnitude. As a matter of fact, the mean component of these velocities (corrected for solar motion) in the direction of the sun's apex is $+3.3$ km/sec. with a probable error of ± 3.5. The corresponding effect on the mean parallactic motion would be of the order of $0''.003$, or one-tenth that required; and the uncertainty, arising from this source, in the mean parallax given in (7) is perhaps of the order of $0''.0006$.

TABLE V

Boss No.	p'_5	Probable Error	M	Period	π
325	$0''.0029$	$\pm 0''.0009$	-2.7	$3^d.97$	$0''.0110$
637	49	13	-1.6	1.95	32
1629	71	11	-0.7	3.73	63
1815	15	10	-4.1	10.15	24
4493	51	10	-1.5	7.01	58
4564	29	11	-2.7	7.59	33
4632	49	26	-1.6	5.77	33
5071	29	13	-2.7	7.18	45
5098	26	22	-2.9	8.38	18
5370	53	27	-1.4	4.44	36
5807	0.0022	± 0.0010	-3.3	5.37	0.0033

A general solution for the preferential motion, based on all the Cepheids for which radial velocities are available,[1] gives the following values for the velocity of the hypothetical drift, its rectangular components, and the position of its apex:

$$V=6.4\pm 3.0 \text{ km/sec.}$$
$$X=+0.2\pm 6.2 \qquad Y=-1.8\pm 2.6 \qquad Z=+6.1\pm 3.0$$
$$A=275^\circ \pm 195^\circ \qquad D=+74^\circ \pm 131^\circ$$

The relative magnitudes of the probable errors show that as far as this material goes there is no appreciable motion of the Cepheids as a class.

Referring again to the data of Table II, we assume, for the purpose of computation, that the v-component for every star in the foregoing tables is wholly parallactic. Then the parallax of each variable, reduced to apparent magnitude $+5$, is $p'_5=0.22\, v_5/\sin\lambda$, and the corresponding absolute magnitudes are as given in Table V. Proceeding as above, where a constant value of M was used, new

[1] See n. 2, p. 98.

values of π are obtained. Their fairly close agreement with the results of the last column of Table II further emphasizes the relative smallness of the neglected peculiar motion.[1]

The chief interest of Table V, however, lies in the comparison of the absolute magnitude with the period of variation. A marked correlation is at once evident,[2] and if we combine the stars in order of period into small groups so as to eliminate some of the effect of the neglected peculiar motion in v and some of the errors of observation, we find

Number of Stars	Absolute Magnitude	Period
1	−4.1	10.15 days
3	−2.8	7.72
3	−2.1	6.05
3	−1.6	4.05
1	−1.6	1.95

The probable error of the mean absolute magnitude would be further reduced by allowing for this correlation between luminosity and period.[3]

III. THE RELATION OF PERIOD TO LUMINOSITY

From the plot[4] of the numbers in the fourth and fifth columns of Table V a new absolute magnitude is obtained for each star, and

[1] This agreement further suggests that the number of stars is sufficient to give a dependable value of the mean parallax. We may, in fact, group the stars in threes, either at random, or in the order of any characteristic (except period), and the separate means will almost invariably give a value differing by less than half a magnitude from the mean value for the eleven stars.

[2] This might be interpreted as a peculiar distribution of the v-components depending on the length of the period, but further work on the relation of period to luminosity completely disposes of this unlikely alternative.

[3] The correction, however, is very small because the principal source of the probable error remains in the dispersion of the residual v_5-components. Since $M=f(P)$, we observe that the reduced parallactic motion, $v_5/\sin\lambda$, is also a function of the period, and that for the extremes of period the residual v_5 values really contain some portion (positive or negative) of the parallactic motion. A new solution, which involved the reduction of the observed proper motions to apparent magnitudes depending on the period rather than to the common apparent magnitude $+5$, naturally gave no appreciable difference in the mean parallax and absolute magnitude, and reduced the probable error of the latter by less than 0.02.

[4] The deviations from the smooth curve correspond in general to the residual v_5-components, providing we grant that M is uniquely defined by P.

these smoothed values are plotted against the logarithm of the period as large circles in Fig. 1. From a final curve, based on

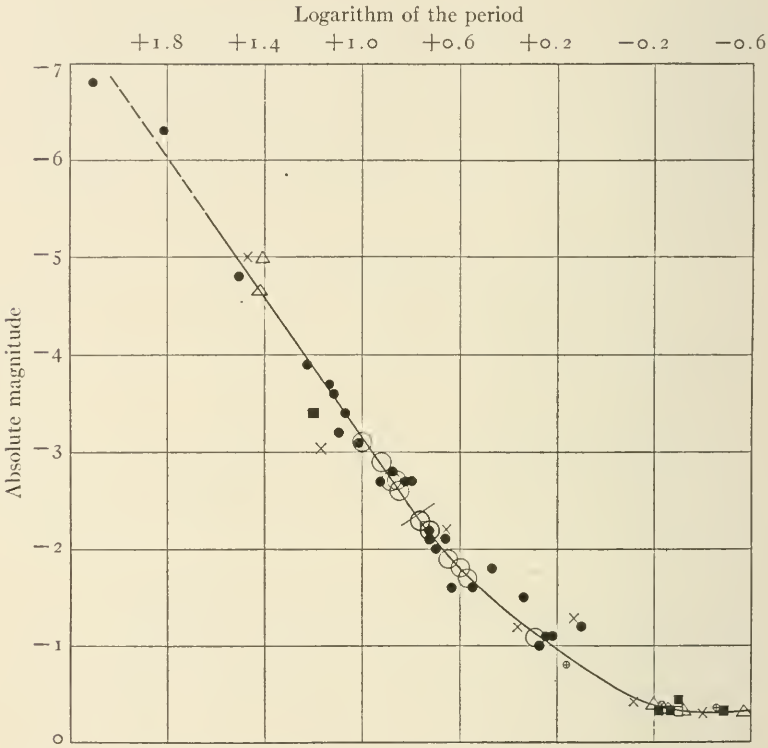

FIG. 1.—Luminosity-period curve of Cepheid variation. The various symbols designate variables from seven different systems. The short bisecting line at absolute magnitude −2.35, log. period 0.775, indicates the mean values for Cepheids of known proper motion. Most of the symbols for periods less than a day represent averages of about ten variables. Of the six largest deviations, four refer to values of particularly low weight. Table XI contains co-ordinates of the adopted curve.

such material as this, we shall presently derive the absolute magnitudes and distances of all Cepheids for which the periods are known.[1]

[1] The eighth paper of this series contains the results for the individual stars (*Mt. Wilson Contr.*, No. 153). Without correcting for the progression of color with period, and assuming a linear relation between period and luminosity, absolute magnitudes have already been computed for two-thirds of the long-period Cepheids by Hertzsprung (*Astronomische Nachrichten*, **196**, 205, 1913), and by Russell for a paper by Russell and Shapley (*Astrophysical Journal*, **40**, 417, 1914). The individual results were not published.

Some years ago Miss Leavitt found a similar relation between the apparent photographic brightness and the length of period for the Cepheid variable stars in the Small Magellanic Cloud.[1] In order to support the result derived above for isolated Cepheids and to obtain a definitive luminosity-period curve, we may reduce her results to the absolute visual system of the present work. From certain globular clusters information that is still more valuable may be obtained through the correlation of the luminosities of the long- and short-period Cepheids. The various sources will be taken up separately.

1. *Small Magellanic Cloud.*—The magnitudes given by Miss Leavitt for stars in the Magellanic clouds are based upon estimates on ordinary photographic plates and are referred to a provisional scale and zero-point. The uncertainty of the zero-point is of no importance for our immediate purpose. As the magnitude scale is probably nearly correct, we shall adopt it as it stands, giving diminished weight to the brightest and faintest stars, and transforming the median brightness of the variables from photographic to visual apparent magnitudes.

The reduction to the visual system will be very small for the short-period variables, but probably as much as two magnitudes for some of the red stars with periods longer than ten days. In the absence of direct color or spectral determinations the change to visual magnitudes must be made on the basis of length of period, using the data already collected for an earlier paper.[2] With a few modifications based on recent spectral classifications,[3] the material is given again in Table VI, the last two columns of which are plotted in Fig. 2. The curve as drawn in the figure has been used for all color corrections; but for periods greater than one day a linear formula

$$\text{Color-index} = -0.55 + 1.5 \log P$$

represents satisfactorily the change of color with period. The probable interpretation of the curve is discussed in a later article.[4]

[1] *Harvard Circular*, No. 173, 1912; *ibid.*, No. 79, 1904; *Harvard Annals*, **60**, 106, 1908.

[2] *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

[3] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, **44**, 273, 1916.

[4] "Three Notes on Cepheid Variation," the ninth paper of this series.

Of the 969 known variables[1] in the Small Magellanic Cloud the periods of 25 have been determined. The designation, logarithm of

TABLE VI

PERIOD AND SPECTRAL TYPE

Limits of Spectrum	Mean Spectrum	Number of Stars	Color-Index	Log. of Mean Period
Bo to A9...	A4	15	+0.15	−0.26
Fo to F9...	F5	27	+0.6	+0.78
Go to G9...	G5	31	+1.0	+1.04
Ko to K9...	K5	9	+1.4	+1.26
M+....	Ma	3	+1.6	+1.52

the period, median[2] photographic magnitude, reduction for color-index by means of the curve in Fig. 2, and the adopted visual magnitude are given for each star in successive columns of Table VII.

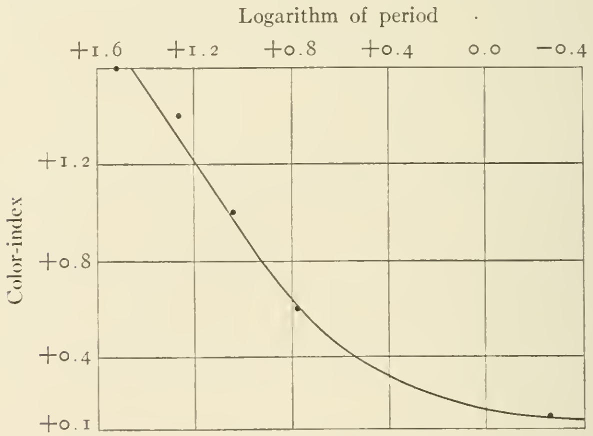

FIG. 2.—Change of median color with period for Cepheid variables

The visual magnitudes of the 25 variables were reduced as follows to the absolute system represented in Fig. 1. The provisional luminosity-period curve, based on the isolated variables as described in an earlier paragraph, covers only a part of the interval for which periods have been determined in the Small

[1] *Harvard Annals*, **60**, No. IV, 1908.

[2] The mean of the values for maximum and minimum.

Magellanic Cloud. To reduce to the absolute scale, values of the absolute magnitude corresponding to the observed periods of the second column have been read from the provisional curve for the 13 variables with $\log P$ between 0.27 and 1.00. The results are in the sixth column of Table VII; in the seventh are the

TABLE VII

CEPHEID VARIABLES IN SMALL MAGELLANIC CLOUD

Harvard Variable	Log. Per.	Median Pg. Mag. (Leavitt)	Color-Index from Curve	Median Vis. Mag.	Absolute Mag. from Curve	Difference	Provisionally Adopted Abs. Mag.
1505....	0.098	15.4	+0.2	15.2	....	....	−1.2
1436....	0.221	15.6	0.3	15.3	....	....	1.1
1446....	0.246	15.6	0.3	15.3	....	....	1.1
1506....	0.273	15.7	0.3	15.4	−1.1	−16.5	1.0
1413....	0.337	15.2	0.3	14.9	1.3	16.2	1.5
1460....	0.464	15.0	0.4	14.6	1.5	16.1	1.8
1422....	0.545	15.3	0.4	14.8	1.7	16.5	1.6
842....	0.632	15.3	0.5	14.8	1.9	16.7	1.6
1425....	0.658	14.8	0.5	14.3	2.0	16.3	2.1
1742....	0.698	14.9	0.5	14.4	2.1	16.5	2.0
1646....	0.726	14.9	0.6	14.3	2.2	16.5	2.1
1649....	0.727	14.8	0.6	14.2	2.2	16.4	2.2
1492....	0.799	14.3	0.6	13.7	2.3	16.0	2.7
1400....	0.823	14.4	0.7	13.7	2.4	16.1	2.7
1355....	0.874	14.4	0.8	13.6	2.7	16.3	2.8
1374....	0.924	14.5	0.8	13.7	−2.8	−16.5	2.7
818....	1.015	14.2	0.9	13.3			3.1
1610....	1.066	14.0	1.0	13.0	Mean diff.	−16.4	3.4
1365....	1.094	14.3	1.1	13.2			3.2
1351....	1.116	13.9	1.1	12.8			3.6
827....	1.129	13.8	1.1	12.7			3.7
822....	1.224	13.8	1.3	12.5			3.9
823....	1.504	13.2	1.6	11.6			4.8
824....	1.818	12.1	2.0	10.1			6.3
821....	2.104	11.6	+2.0	9.6			−6.8

differences between the magnitudes from the curve and the apparent magnitudes. The mean difference, −16.4, is the reduction constant, which, applied to all apparent median magnitudes, gives in the last column the corresponding absolute magnitudes. In making this transformation, the purpose of which is to relate *change* of period with *change* in absolute visual magnitude, we make no assumption regarding the actual luminosity of Cepheid variables in the Magellanic clouds; but as soon as we use the reduction constant as a measure of distance we assume, of course, that variables

of a given period are of comparable luminosity, whether they are in the general galactic system or in separate stellar organizations, such as globular clusters and the Magellanic clouds.

The absolute magnitudes of the last column of Table VII, plotted in Fig. 1 as dots, show that the same relation holds for the Cepheids in the Small Magellanic Cloud and in the galactic system, and that an improved and extended luminosity-period curve may be based upon the combined data. The relation appears so definite that the prediction of the length of period on the basis of magnitude estimates should be possible for most of the 944 other variables in the Small Magellanic Cloud.[1] Two or three hundred of them are fainter than any for which the period has been determined. It is very probable that they are cluster-type variables with periods of the order of twelve hours. With no correction either for color or for divergence of magnitude scale their median magnitudes on the absolute system are about -0.3, agreeing almost exactly with the mean value determined below for cluster-type variables in globular clusters.

It is important to note further that magnitude 16.0 in Miss Leavitt's provisional system marks an abrupt and definite fainter limit to the median brightness of the variables in the Small Magellanic Cloud. The plates, which were made with the 24-inch Bruce telescope with exposures varying from two to five hours, are sufficient to test this matter, for minima are observed as faint as 17.0, but, with one possible exception, no maximum is recorded fainter than 16.0. A similarly definite fainter limit to the interval of magnitude throughout which Cepheids occur has been observed in globular clusters, particularly in ω Centauri, and Messier 3, 5,

[1] Without doubt nearly all variables in both clouds belong to the Cepheid class. In the Small Cloud, however, four variables have an observed range of at least three magnitudes, and are probably long-period variables rather than Cepheids. Possibly a few others have larger ranges of variation than shown by the plates examined by Miss Leavitt and are also long-period variables. It seems to be a definite observational fact that no star that otherwise has typical Cepheid characteristics is known to have a range in excess of two magnitudes. The value, photographically, falls usually between 0.8 and 1.5, regardless of the length of period. Hence the appropriateness of Eddington's search for the physical reason of an upper limit to the amplitude of a pulsation in a gaseous star ("The Pulsation Theory of Cepheid Variables," *Observatory*, 40, 290, 1917).

and 13. The concurrent progression toward a definite limit of luminosity, spectrum, and period suggests that, in the evolutionary sequence of stars, Cepheid variation is abruptly limited at or near the blue end of the giant series because of the changing physical conditions in the interior of the gaseous masses.

2. *ω Centauri.*—Of 132 variables observed by Professor Bailey in the southern cluster ω Centauri,[1] periods were determined for 93, of which 3 are long-period Cepheids. From the observations and notes it has been possible to derive approximate results for two others with periods in excess of a day. In Table VIII the correction to visual magnitude and the reduction to the absolute system follow

TABLE VIII

Cepheid Variables in ω Centauri

Designation	Log. Per.	Median Pg. Mag. (Bailey)	Visual Mag.	Mag. from Curve	Difference	Provisional Abs. Mag.
1..........	1.47	10.45	8.80	−4.70	−13.50	−5.00
29..........	1.17	11.94	10.76	−3.60	−14.43	−3.04
48..........	0.66	12.08	11.60	−2.00	−13.60	−2.20
60..........	0.13	12.74	12.52	−1.01	−13.51	−1.28
61..........	0.36	12.92	12.60	−1.35	−13.95	−1.20
a(37).......	−0.23	13.55	13.40			−0.40
b(19).......	−0.12	13.54	13.37	Mean diff.	−13.80	−0.43
c(34).......	−0.40	13.61	13.49			−0.31

the same plan as that for the Small Magellanic Cloud. The magnitudes of the fifth column were read from the improved luminosity-period curve, but since at this point in the discussion the curve has not yet been extended to periods less than a day, the three groups of cluster-type variables were not used in the reduction. The light-curve of No. 29 is hardly typical of Cepheids, and its magnitude also appears somewhat discordant. In obtaining the reduction constant, −13.80, half weight was given to the approximate magnitudes for Nos. 48 and 61. The numbers in parentheses in the first column refer to the total number of stars in each subgroup. Plotting as crosses in Fig. 1 the values of the second and last columns, the luminosity-period curve is further improved for the ordinary Cepheids and is extended to the cluster-type variables.

[1] *Harvard Annals*, 38, 1902.

3. *Messier 5.*—Professor Bailey has recently determined the periods and light-curves of about 70 variables in Messier 5.[1] Of the three with periods longer than a day, two are certainly Cepheids. The third, No. 50 of Bailey's list, with a period of 106 days, is one component of a close bright double. Its observed magnitude is certainly diminished by the Eberhard effect, possibly to a great extent. Professor Barnard has observed visually all three of these stars.[2] In a recent letter he states that the measures of No. 50 do not give the rapid rise to maximum light that is typical of Cepheid variation. If the star were included without correction for the Eberhard effect, it would deviate more than three magnitudes from the curve.

TABLE IX

CEPHEID VARIABLES IN MESSIER 5

Designation	Log. Per.	Median Pg. Mag. (Bailey)	Visual Mag.	Mag. from Curve*	Difference	Provisional Abs. Mag.
42	1.41	11.72	10.20	−4.55	−14.75	−4.98
50	2.03	13.60	11.6	....	....	−3.6
84	1.42	12.08	10.53	−4.57	−15.10	−4.65
Double Max. (8)	−0.57	14.98	14.84	....	....	−0.34
Group 1	−0.32	14.98	14.85	−0.33	−15.18	−0.33
Group 2	−0.26	14.98	14.83	−0.39	−15.22	−0.35
Group 3	−0.20	14.96	14.80	−0.45	−15.25	−0.38
				Mean diff.	−15.18	

* It should be noted that the luminosity-period curve is slightly corrected after each accretion of data so that the magnitudes in the fifth column are not those derivable from the final curve.

A group of 8 variables with double maxima or peculiarly short periods is found among the cluster-type variables of Messier 5.[3] That they are single stars with an average period of six and one-half hours seems the most probable hypothesis; they are accordingly used to extend the luminosity-period curve and to show that no appreciable decrease in luminosity occurs as the periods become less than twelve hours.

Bailey has collected into three equal groups, in order of length of period, the thirty typical cluster-type variables with most definite light-curves. Each group is given weight 5 in determining the reduction constant. The material is discussed in Table IX

[1] *Harvard Annals*, 78, Part 2, 1917.

[2] *Astronomische Nachrichten*, 147, 243, 1898.

[3] *Harvard Circular*, No. 193, 1916.

and, omitting No. 50, the results are plotted in Fig. 1 as triangles.

4. *Messier 3.*—Bailey's monograph on Messier 3 contains the light-curves of 110 stars, none of which has a period exceeding a day.[1] In his catalogue the only bright star that is certainly variable is one that appears to be irregular. A bright Cepheid, however, was found near the center of the cluster by Barnard, who has determined the period and published a light-curve.[2] The median photo-visual magnitude of this star has been determined from plates made at Mount Wilson and measured by Miss Davis and the writer. Although the light-curve derived by Barnard is more nearly symmetrical than is usual for Cepheid variables, the color and the change of color between maximum and minimum, which is typical of this kind of variable, is clearly indicated by the Mount Wilson measures. A small correction to the final magnitude for Eberhard effect would be appropriate, because of the star's situation in the densest part of the cluster, and would probably eliminate its deviation from the curve in Fig. 1.

One variable of Bailey's list, No. 37, has been specially studied at Harvard[3] and on a series of Mount Wilson plates.[4] Its period is about one-half that of the typical variable of Messier 3, resembling in this respect, as well as in the shape of the light-curve, the eight

[1] *Harvard Annals*, **78**, Part 1, 1913.

[2] *Astronomische Nachrichten*, **172**, 345, 1906. Barnard suspected another bright star of variation, No. 19, in his nomenclature. Referred to his comparison star No. 8 no variation is apparent on the Mount Wilson plates so far examined. The data for photographic magnitudes are as follows:

Plate	Date	19−8	Plate	Date	19−8
2372	1915, April 16	$0^{M}00$	3679	1917, March 28	$+0^{M}06$
2463	June 7	+0.10	3680	28	+0.04
2506	July 6	+0.04			

These stars are among the very brightest in the cluster, and, as might have been inferred from previous investigations, they are red. The color-index of No. 19 is approximately +1.8 magnitudes, and of No. 8, more than two magnitudes, according to measures on five photographic and three photo-visual plates. There is some possibility of Eberhard effect as both stars are near the center.

[3] *Harvard Circular*, No. 193, 1917; *Harvard Annals*, **78**, Part 2, 1913.

[4] *Publications of the Astronomical Society of the Pacific*, **29**, 110, 1917.

stars of Messier 5 that are designated "Double Max." in Table IX. It is entered singly in Table X. The 54 typical cluster-type variables in Messier 3 for which Bailey considered the results most certain are combined, in order of period, into three equal groups, each of which is given weight 10 in deriving the reduction constant. The photo-visual magnitudes are referred to the Mount Wilson system, a series of polar comparison plates being used to obtain the appropriate correction to Bailey's photographic magnitudes. Black squares in the diagram designate the results for Messier 3.

5. *Messier 13.*—Two Cepheid variables in Messier 13 were discovered by Bailey and Barnard and studied by the latter.[1] Of five others found by the writer among the fainter stars[2] four appear

TABLE X

CEPHEID VARIABLES IN MESSIER 3

Designation	Log. Per.	Median Pv. Mag.	Mag. from Curve	Difference	Provisional Abs. Mag.
Barnard 7...	1.20	12.3	−3.7	−16.0	−3.4
Bailey 37....	−0.49	15.40	−0.35	−15.75	−0.33
Group 1.....	−0.30	15.29	−0.34	−15.63	−0.44
Group 2.....	−0.27	15.39	−0.36	−15.75	−0.34
Group 3.....	−0.22	15.39	−0.38	−15.77	−0.34
			Mean diff......	−15.73	

to have periods less than a day. Approximate median photo-visual magnitudes for these four and the earlier two have been obtained from Mount Wilson photographs; but without a complete study of all the light-curves the results have low weight. Carrying through the usual reductions, however, and combining the provisional results into two groups, we obtain the values indicated by open squares in Fig. 1.

6. *Other sources.*—Further numerical results bearing on the luminosity-period relation are not now available, but certain sources may be cited from which quantitative results will come in time and from some of which even now we may infer confirmation of the interdependence of period and luminosity.

[1] Bibliography and discussion in *Mt. Wilson Contr.*, No. 116, p. 78, 1915.

[2] *Ibid.*, pp. 79 ff.

The median photographic magnitudes of the 808 variables discovered by Miss Leavitt in the Large Magellanic Cloud[1] range from 10.5 to 15.8; there is but one fainter exception, which, by showing that fainter stars are easily visible on the plates, proves the rule that Cepheid variation affects only stars brighter than a certain limit. If we should assume the same color correction as used for the Small Magellanic Cloud, the interval of magnitude in which the variables occur is much the same. There can be little doubt that these are Cepheid variables (the ranges of all but two or three are less than two magnitudes[2]), among which the luminosity-period law holds as elsewhere. Good values of the distance and extent of the star cloud will be given eventually through determinations of the magnitudes and periods of these variable stars.

The 50 variable stars in Messier 15 are being studied by Professor Bailey on a series of 75 Harvard and Mount Wilson plates. The periods so far derived, with one exception, are less than a day. The exceptional star has a period of 1.44 days, a provisional value kindly communicated by Professor Bailey; its median magnitude is about 0.43 brighter than the median magnitude of the other variables, according to measures on a few Mount Wilson plates. The results are preliminary and receive no weight in obtaining the luminosity-period curve, but because of the close agreement with results for other clusters, the following data, derived in the usual manner, are plotted in Fig. 1 as small circles containing crosses:

Variable No. 1, Median Mag. (Absolute) $= -0.82$, Log. Per. $= +0.16$
Variable No. 13, Median Mag. (Absolute) $= -0.40$, Log. Per. $= -0.24$
Variable No. 11, Median Mag. (Absolute) $= -0.37$, Log. Per. $= -0.46$

Nos. 13 and 11 are typical of the two subclasses of variables in Messier 15; the median photographic magnitudes, Bailey finds, are in all cases about 15.7.[3]

Variable stars have now been discovered in 26 globular clusters,[4] and from Mount Wilson plates and from reproductions in *Harvard*

[1] *Harvard Annals*, **60**, Part IV, 1908; *Harvard Circular*, No. 82, 1904.

[2] See n. 1, p. 108.

[3] *Popular Astronomy*, **25**, 520, 1917.

[4] To the list in *Harvard Annals*, **38**, p. 2, several have been added through recent discoveries by Miss Davis (*Publications of the Astronomical Society of the Pacific*, **29**, 210. 260, 1917).

Annals, **38**, it is possible to make a preliminary comparison of the relative magnitudes in each system. Thus we find that there are at least 15 clusters, in addition to those examined in some detail in the foregoing discussion, in which some or all of the variables are among the very brightest stars. Clusters which show also a considerable diversity in the magnitude of variables, and which are therefore of importance for the luminosity-period curve, include N.G.C. 104, 362, 1904, 6266, 6397, and 6626.

The final luminosity-period curve, as drawn in Fig. 1 and given numerically in Table XI, is based upon more than 230 stars, and, except for zero-point uncertainty, is probably correct within one or

TABLE XI

CO-ORDINATES OF THE LUMINOSITY-PERIOD CURVE

Logarithm of Period	Absolute Visual Magnitude	Logarithm of Period	Absolute Visual Magnitude
−0.6	−0.34	+0.8	−2.43
−0.5	0.33	0.9	2.79
−0.4	0.3	1.0	3.15
−0.3	0.34	1.1	3.51
−0.2	0.38	1.2	3.87
−0.1	0.50	1.3	4.23
0.0	0.64	1.4	4.59
+0.1	0.81	1.5	4.95
+0.2	0.99	1.6	5.31
+0.3	1.17	1.7	5.67
+0.4	1.37	1.8	6.02
+0.5	1.58	1.9	6.38
+0.6	1.81	2.0	6.74
+0.7	−2.10	+2.1	−7.1

two hundredths of a magnitude. Ten plotted points lie on the curve, 23 are below, and 24 above. The average unweighted deviation is ±0.13 mag.,[1] an amount so nearly of the same order as the errors of the measured magnitudes that for typical[2] Cepheids of given period a rigorously constant median magnitude may be assumed. Almost all of the large deviations from the curve are of

[1] The observational errors in the periods are relatively negligible.

[2] The word "typical" is frequently used to make allowance for such stars as RV Tauri, κ Pavonis, and V Ursae Minoris, which show some general Cepheid characteristics, but because of various recognized irregularities or peculiarities may also be irregular in the relation of period to absolute brightness.

low weight, due to uncertain estimates of magnitude or period, peculiarities in light variation, or possible error in the magnitude scale. Above absolute magnitude -5.5 the curve depends only on the longest period variables in the Small Magellanic Cloud; but the resulting uncertainty is not serious, as few Cepheids have periods longer than forty days.[1]

Future work on the periods and magnitudes of variables in clusters is not likely to alter appreciably the form of the luminosity-period curve; but further investigation of the proper motions of Cepheids may contribute to the certainty of the zero-point, which is now defined both as to amount and accuracy by equation (7). The distances of even the nearest Cepheids are so great that little can be expected from direct parallax measures in the way of quantitative confirmation or improvement of the curve.[2]

IV. THE MEDIAN MAGNITUDE OF CLUSTER-TYPE VARIABLES

The flattening of the luminosity-period curve for magnitudes fainter than -0.5 indicates that for the typical cluster-type variable the median brightness is essentially invariable and is independent of the length of period. As we shall presently relate the magnitude of these variables to the maximum brightness attained in clusters, a further examination of the dispersion of median magnitudes is advisable. From preceding tables we derive the data in Table XII.

Thus the absolute *photographic* magnitude[3] for 183 variables is

$$\text{Median} = -0.23 \tag{8}$$

The deviations from this mean value may be due largely to the errors in the magnitudes of the longer-period Cepheids of each cluster. No marked change of photographic brightness with period appears; the change of visual magnitude with period

[1] Cf. Table II of the eighth paper. Long-period Cepheids are liable to irregularity.

[2] See sec. II, above.

[3] This value, which becomes of much importance in the determination of cluster parallaxes, is independent of the earlier transformations from photographic to photovisual magnitudes with the aid of Table VI.

recorded in Fig. 1 and Table XI is due to the small progression of color with period, assumed in the reductions on the basis of Fig. 2.

Much work bearing on the constancy of the median magnitude of cluster-type variables has been done at Mount Wilson; but a full discussion of the data would be too extensive for the present paper, and the results for only two systems are outlined below.

TABLE XII

Cluster	Number of Variables	Mean Period	Mean Absolute Photographic Magnitude	Deviation
ω Centauri..	34	$0^d.40$	−0.19	$-0^M.04$
	37	0.59	−0.25	+0.02
	19	0.76	−0.26	+0.03
Messier 5...	8	0.27	−0.20	−0.03
	10	0.48	−0.20	−0.03
	10	0.55	−0.20	−0.03
	10	0.63	−0.22	−0.01
Messier 3...	1	0.32	−0.20	−0.03
	18	0.50	−0.31	+0.08
	18	0.54	−0.20	−0.03
	18	0.60	−0.20	−0.03

1. In ω Centauri three subclasses of cluster-type variables are recognized. Omitting those for which the classification is uncertain, we have in Table XIII a summary of the data bearing on the dispersion of median magnitudes. Although the stars of the three groups differ from each other in maximum magnitude and range, as well as in period and form of light-curve, the median values are the same. The distribution of individual deviations agrees closely with the probability curve, as shown in Table XIV.

TABLE XIII

VARIABLE STARS IN ω CENTAURI

Subclass	No. of Variables	Mean Period	Maximum Magnitude	Range of Variation	Median Magnitude	Average Deviation
a...........	33	$0^d.586$	12.99	$1^M.11$	13.55	$\pm 0^M.09$
b...........	15	0.752	13.10	0.87	13.55	±0.10
c...........	28	0.395	13.33	0.56	13.61	±0.09
All.........	76	....	....	....	13.57	±0.10

2. The variables in Messier 3 differ very little from each other in any respect. The comparison stars and some of the variables have been studied by Miss Davis and the writer on a series of 65 photographs; the magnitude scale has been revised and referred to the Mount Wilson system. A sample of the revised data, showing the constancy of the median magnitudes, has been given in *Mt. Wilson Communication* No. 47.[1] For the 54 light-curves

TABLE XIV

Number of Residuals between	Theory	Observation
$0^{M}.00$ and $0^{M}.03$	18	19
0.04 " 0.06	14	12
0.07 " 0.09	13	15
0.10 " 0.12	10	13
0.13 " 0.15	7	7
0.16 " 0.18	5	0
0.19 " 0.24	6	7
≥0.25	3	3

selected by Bailey as most definite the median photographic magnitude is 15.49±0.01, the average deviation from the mean value is ±0.07, and the largest deviation is less than two-tenths of a magnitude. If we include all 110 variables for which periods are known, the mean is 15.50±0.006, with an average deviation ±0.08.

A further examination of the median magnitudes for the 54 selected stars shows the following small systematic variation, which is definitely connected with the range:

Mean range of variation	$1^{M}.04$	$1^{M}.14$	$1^{M}.26$	$1^{M}.33$	$1^{M}.48$
Mean median magnitude	15.58	15.52	15.49	15.47	15.41
Number of variables	10	4	22	9	9

One explanation of this variation is that the duration of exposure was often so long that for some stars the brightness at the top of the sharp-pointed curves was never determined, the measures yielding fainter, more rounded maxima than actually exist. The range

[1] *Proceedings of the National Academy of Sciences*, 3, 480, 1917.

deduced for such stars is always too small, and the systematic error goes into the median magnitude with half its weight. The variation may be due, rather, to errors of observation at maximum light, where usually only a few estimates are available and any error will directly correlate range and median magnitude. The first explanation gives 15.4 as the true median magnitude, the second leaves it at 15.5.[1]

In either case an appropriate and simple correction to the deviations from the mean median magnitude, for the systematic errors in the maxima, leaves the uncertainty in the median magnitude but half as great as given above, and the average deviation for a single star is less than ± 0.05. The distribution of the corrected residuals again accords with the law of error as closely as could be expected for a small number of values:

Number of Residuals between	Theory	Observation
$0^{M}.00$ and $0^{M}.04$	29	27
0.04 " 0.08	16	15
0.08 " 0.12	7	9
>0.12	2	3

The magnitudes in Messier 2, 5, 15, and 22 yield results similar to the foregoing. In each cluster, apparently, the total light variation of short-period Cepheids is confined to a narrow interval of brightness; and in all cases where the observations are sufficient to justify a conclusion the deviations of the median magnitudes from their mean are far within the errors of observation. Hence we are led to place much confidence in the hypothesis that the parallax of a cluster-type variable (or of any cluster containing such stars) may be derived immediately from the measurement of the median magnitude.

[1] The error is probably common to all groups of variables. It does not vitiate the work on cluster parallaxes, for the median value as observed is used to obtain both apparent and absolute magnitudes. A systematic error may be introduced into the determination of the distances of some isolated cluster-type variables, but at most it will be only a few per cent and far within the uncertainty of the various magnitude scales.

V. PARALLAXES OF CLUSTERS FROM THEIR BRIGHTEST STARS

The most important use of accurate median magnitudes is to furnish a starting-point for the study of the absolute luminosity of the brightest stars in a cluster. In Messier 3, for instance, where the magnitudes of all the brighter stars[1] have been accurately measured, there are nearly 300 more luminous than the median magnitude of the variables. A few that are more than two magnitudes brighter appear in the cluster, it may be, by projection only,

TABLE XV

PHOTOGRAPHIC MAGNITUDES OF 30 BRIGHTEST STARS IN MESSIER 3

Star	Color-Index	Pg. Mag.	Deviation	Star	Color-Index	Pg. Mag.	Deviation
206....	+1.01	11.27	Bright	1449....	+1.40	14.18	−0.05
420....	+0.19	13.59	Bright	463....	+1.13	14.25	+0.02
589....	+0.91	13.71	Bright	238....	+1.79	14.27	+0.04
205....	+1.09	13.81	Bright	334....	+1.13	14.27	+0.04
837....	+1.36	13.83	Bright	265....	+1.14	14.29	+0.06
	.			925....	+1.34	14.29	+0.06
1127....	+1.30	13.92	−0.31	1397....	+1.69	14.29	+0.06
1219....	+1.40	14.04	−0.19	398....	+1.13	14.32	+0.09
706....	+1.16	14.06	−0.17	237....	+0.42	14.33	+0.10
1000....	+1.25	14.08	−0.15	309....	+1.19	14.37	+0.14
1128....	−0.60	14.08	−0.15	640....	+1.18	14.40	+0.17
417....	+1.15	14.09	−0.14	1203....	+1.32	14.40	+0.17
740....	+0.70	14.70	−0.13	1208....	+1.23	14.40	+0.17
490....	+1.80	14.13	−0.10	1214....	+1.22	14.40	+0.17
297....	+1.32	14.14	−0.09	177....	+1.00	14.45	+0.22
1392....	+1.22	14.14	−0.09				
				Means..	+1.16	14.23	±0.12

or they may be double or multiple stars. If we limit our study to the region within 9′ from the center and exclude a few, say five, of the very brightest objects, we can feel sure that practically all the remaining bright stars are really typical members of the cluster.

In Table XV the sequence of the 30 brightest objects is shown for Messier 3, the data being taken from an unpublished investigation of the magnitudes and colors of nearly a thousand stars.[2] The

[1] Stars within a concentric circle of nearly 3′ diameter are excluded because of possible systematic errors arising from crowding of images and Eberhard effect.

[2] The numbers are those of von Zeipel's catalogue, *Annales de l'Observatoire de Paris*, 25, 1908.

mean luminosity of the 25 chosen stars is determined with nearly the same accuracy as the median magnitude of the variables. The difference, Med. − Mean Br., is +1.27; hence from equation (8) we find that the brightest stars have an absolute photographic magnitude of −1.5 in the mean, and a maximum just fainter than −2. These bright stars are reddish, however, and the maximum visual absolute brightness exceeds −3 magnitudes in a few cases.

Other clusters which contain short-period Cepheids agree in showing that the maximum photographic luminosity is regularly between 1.5 and 2.0 magnitudes brighter than the median value for the variables. We see, therefore, in this apparent constancy of maximum magnitude, the possibility of an expeditious method of furthering our knowledge of cluster distances. The phenomenon is qualitatively illustrated by an inspection of the photographs reproduced in *Harvard Annals*, 38; and a quantitative confirmation is readily possible through magnitude measures, even when the variables are few in number and their light-curves are unknown. Thus, for Messier 22 a study of three polar-comparison plates gives for the mean magnitude of the 25 brightest stars[1] the value 13.08, with extremes of 12.51 and 13.55, and an average deviation of ±0.19, while the median magnitude of the variable stars,[2] from the measures given in Table XVI, appears to be 14.45. Hence, Med. − Mean Br. = +1.37. For a few clusters, in which the variables have been extensively studied, more accurate values of this difference can be obtained.

All material now available has been considered in discussing the relation of median to brightest magnitude, summarized in Table

[1] Five stars brighter than the "25 brightest" are always excluded in using this method.

[2] Sixteen variable stars are listed by Bailey (*Harvard Annals*, 38, 242, 1902). No. 11 is a very close double and was not measured; Nos. 3 and 14 could not be certainly identified. The variability of Nos. 2, 5, 9, and 16 is not definitely confirmed by the Mount Wilson plates. No. 8 is one of the brightest stars in the cluster and appears to be a long-period Cepheid; similarly, Nos. 5 and 9 are probably bright Cepheids of long period. For the remaining 8 stars a short-period variation of a magnitude or more is fairly well established by these plates. The total interval of brightness is about two magnitudes; the uncertainty of the median magnitude given above is possibly one or two tenths.

XVII.[1] Some of the tabulated material has been adapted from Bailey's published work, but most of it has been derived from Mount Wilson plates, of which the total number entering the discussion for each cluster is noted in the second column. The third column gives the number of cluster-type variables contributing to the mean median magnitude. It should be observed that the final result appears to be independent of the frequency of variable stars.

TABLE XVI

VARIABLES IN MESSIER 22—OBSERVATIONS OF 1917

Number	Plate 3887 Aug. 14.71	Plate 3888 Aug. 14.72	Plate 3892*a* Aug. 14.75	Plate 3892*b* Aug. 14.75	Plate 3940 Sept. 9.70	Plate 3963 Sept. 10.68
1	....	....	....	15.51	14.06	15.37
2	14.96	....	....	15.12	15.15	14.89
3	....	....	....	....	....	16.36?
4	14.80	15.06	....	14.95	15.13	14.38
5	12.72	12.78	12.72	12.75	12.46	12.64
6	15.04	14.79	....	14.37	14.98	14.01
7	15.06	15.10	....	14.78	14.32	15.41
8	12.16	12.17	12.29	12.10	12.76	12.79
9	13.24	13.31	13.21	13.17	13.32	13.15
10	15.14	15.02	....	15.39	15.03	14.38
12	15.01	14.99	....	14.42	14.95	14.93
13	14.20	13.78	13.72	13.58	14.98	13.68
15	14.95	14.99	....	15.36	14.52	14.93
16	14.41	14.54	....	14.52	14.47	14.34

The radius of the concentric circular area in which the bright stars were measured is given in the fourth column of Table XVII. The choice of this quantity is somewhat arbitrary, and small changes in it may affect the mean perceptibly if thereby exceptionally bright stars happen to be included or omitted.[2] The adopted radius

[1] In at least three clusters (N.G.C. 6266, 6626, and 6723), in which magnitudes have not been quantitatively studied, a considerable number of short-period variable stars are from one to two magnitudes fainter than the brightest stars. Qualitatively, therefore, the relation between the median and brightest magnitudes is verified in 10 clusters and the Magellanic clouds and is nowhere controverted. Its quantitative expression, however, is probably less definite than the differences of Table XVII suggest, the close agreement of several values being partially fortuitous. A later study of Messier 15, for instance, indicates an uncertainty of 0.2 mag. in the provisional value given here; but the adopted probable error of the mean difference amply covers this discrepancy.

[2] The mean magnitude for Messier 3 is 14.30 for a radius of 7′ and 14.17 for a radius of 11′.

depends in general upon both the nature of the photographs and the cluster's angular diameter, but mainly upon the galactic latitude. In rich fields the area is necessarily small in order to exclude bright non-cluster stars. We might have adopted the same radius for all clusters and attempted to allow for foreign objects by varying with galactic latitude the number of excluded stars, but the actual angular dimensions differ so greatly that a more flexible procedure seemed advisable. Every effort has been made in this and subsequent work of the same kind to obtain homogeneous results, in each case so choosing the area for measures of bright stars that, with the five brightest rejected, the resulting mean gives a trustworthy value of the maximum luminosity.[1]

TABLE XVII

MEDIAN MAGNITUDES AND THE BRIGHTEST STARS

Cluster	No. of Plates	No. of Variables	Radius	Photographic Magnitude			Weight
				Mean Br.	Median	Diff.	
Messier 3..	65	110	9′	14.23	15.50	1.27	8
5..	3	61	4	13.97	15.26	1.29	4
15..	7	48	6	14.31	15.63	1.32	2
2..	7	7	4	14.61	15.71	1.10	1
22..	6	8	5.5	13.08	14.45	1.37	1
13..	15	4?	6	13.75	15.3:	1.5:	0
ω Centauri.	3	90	....	12.3:	13.9:	1.6:	0
				Mean difference....		1.28	

The mean difference, $+1.28$, combined with (8), gives $M = -1.51$ as the mean absolute photographic magnitude for the bright stars. The probable error of the difference is not likely to be in excess of two-tenths of a magnitude. This estimate makes generous allowance for the real dissimilarities of the clusters (which seem to be small so far as magnitude limits go) and for the uncertainties in excluding peculiar and non-cluster stars. Taking also into consideration the probable error of (7) and the observational errors in the apparent magnitudes, m, we conclude that the

[1] Central condensations and multiple stars were avoided. Counts of stars on the Franklin-Adams charts were frequently of service in estimating the probable frequency of outside stars in a cluster field.

probable error for the difference $M-m$, in the relation $5 \log \pi = M-m-5$, certainly does not exceed 0.4 mag., and, therefore, that the absolute parallax of a globular cluster may be obtained from the apparent magnitudes of its brightest stars with a probable error of less than 20 per cent. For relative parallaxes the probable error does not include the error of (7) or the possibility of systematic error in the magnitude scale; its value is 10 per cent or less for parallaxes derived from homogeneous data.

VI. REMARK ON THE PARALLAXES OF CLUSTERS DERIVED FROM THEIR APPARENT DIAMETERS

The foregoing discussion shows that the mean apparent magnitude of the brightest stars in a globular cluster is a pretty dependable criterion of its distance, thus indicating that all systems are much alike in the maximum luminosity attained by any individual member. In consequence it is a natural assumption that clusters may also be closely comparable in actual size. In fact, the first paper of this series[1] contains a provisional curve correlating decreasing maximum brightness with decreasing angular diameter, and it follows that the apparent size of a globular cluster is also a direct measure of the parallax. As we may obtain the parallaxes of nearly 30 globular clusters by the methods outlined on preceding pages, a curve showing the relation of distance to apparent size can be readily constructed, and using this curve the parallax of any other cluster can be obtained from its diameter. A necessity of such work is homogeneity in the observations, and this is afforded in a highly satisfactory manner by the Franklin-Adams photographic charts, which cover the whole sky and include every known globular cluster. Further discussion of this phase of the work is reserved for the following contribution.

VII. SUMMARY

1. The determination of the distances and distribution in space of globular clusters involves a general treatment of extensive data bearing on the magnitudes, periods, light-curves, proper motions, and radial velocities of Cepheid variables in the Galaxy

[1] *Mt. Wilson Contr.*, No. 115, p. 12, 1915.

and other systems and on the angular diameters of clusters and the number, magnitudes, and colors of their individual stars. Its successful accomplishment will help somewhat to a better understanding not only of the most remote objects known in the stellar universe, but also of the dimensions and dynamics of cluster systems and of the maximum luminosity attainable in stellar evolution. Suggestions relative to the extent and arrangement of the galactic system and to the sun's position therein will be a natural outcome of the work.

2. From parallactic motions the mean absolute magnitude of eleven isolated Cepheid variable stars has been derived with a relatively small computed probable error (sec. II). The luminosities of the individual stars are shown to be uniquely defined by their periods.

3. An extension of these results gives a relation (Fig. 1) connecting the periods of both the ordinary Cepheids and the cluster-type variables with their absolute magnitudes, which permits the derivation of the distances of all such variable stars as soon as their periods and apparent magnitudes are measured; and when we adopt the plausible hypothesis that Cepheids of a given period are comparable wherever found, the relation also yields the parallax of any cluster containing Cepheid variables. Data for more than 200 individual variables from seven different stellar systems contribute to the determination of the luminosity-period relation. Fainter than a definitely fixed luminosity Cepheid variation probably never occurs.

4. Further investigation makes the derivation of cluster parallaxes practically independent of variable stars by substituting the apparent magnitudes of the brightest stars as the criteria of distance (sec. V). Stars brighter than the absolute photographic magnitude -2 are exceedingly rare in clusters.

5. Angular diameters are next employed in extending the work, until finally for all globular clusters in both hemispheres values of the parallax become possible. The distances are derived and considered statistically in the next paper of this series.

Mount Wilson Solar Observatory
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THE ABSORPTION OF NEAR INFRA-RED RADIATION BY WATER-VAPOR

By W. W. SLEATOR

I. INTRODUCTION

The study of the absorption of radiation in the atmosphere was begun by Langley,[1] whose bolographs, or maps of the sun's spectrum, show wide bands attributed by him to water-vapor. Paschen,[2] in work on the emission of radiation by gases, has investigated as well the absorption occurring in steam, and records in particular the effect of changes in temperature upon the wave-lengths of the radiation absorbed. His work has been extended by Eva von Bahr,[3] who has shown that the doublet between wave-lengths 5 and 7 μ is in reality very complex and contains about forty separate absorption bands arranged with some appearance of symmetry on either side of the wave-length 6.26 μ.

The combination principle suggested by Bjerrum,[4] in connection with his application to the case of molecular rotation of the quantum theory, is a fruitful hypothesis in the explanation of near infrared absorption. Bjerrum's ideas have in fact been applied by Eucken[5] to the system of water bands presented in the work of Eva von Bahr, and that system seems to be accounted for in terms of two series of rotation-frequencies corresponding to two principal moments of inertia in the water molecule. More recent work by Rubens and Hettner[6] shows some evidence for the presence in the farther infra-red of a third series of rotation-frequencies. The

[1] *Researches on Solar Heat* (Professional Papers of the Signal Service, No. 15, United States War Department).

[2] *Annalen der Physik*, 51, 4, 1894; 52, 210, 1894; 53, 234, 1894.

[3] *Phil. Mag.*, 28, 71, 1914; *Verhandlungen der deutschen Physikalischen Gesellschaft*, 15, 731, 1913.

[4] *Nernst Festschrift*, p. 90.

[5] *Verhandlungen der deutschen Physikalischen Gesellschaft*, 15, 1159, 1913; *Phil. Mag.*, 28, 71, 1914.

[6] *Chemical Abstracts*, 11, 1357, 1917; *Science Abstracts*, No. 234, p. 223, June 1917.

question of molecular structure is now intimately connected with the investigation of radiation, as is shown by further work of Bjerrum,[1] and important tests of the quantum hypothesis await experimental advances in the same field.[2]

The present paper gives an account of work undertaken to extend our knowledge of atmospheric absorption. It concerns particularly the region of the spectrum between the wave-lengths 1.3 and 3 μ and presents a detailed study of the absorption regions marked Ψ, Ω, and X on Langley's charts.

II. APPARATUS

The work of Eva von Bahr was done with a prism spectrometer and radiomicrometer. In the work recounted here a higher dispersion was secured by the use of gratings, and the uncertainty of wave-lengths determined with the aid of a dispersion-curve was at the same time avoided. The troublesome overlapping of grating spectra has been eliminated by the use of a supplementary prism. The general arrangement is shown in Fig. 1. The apparatus takes the form of two spectrometers, each with fixed arm and single mirror.[3] By a proper setting of the prism a definite and limited portion of the spectrum is presented to the grating spectrometer. The prism serves in fact as an effective screen or filter.

A linear thermopile, incased at T in Fig. 1, and a galvanometer of the Paschen type constituted the detecting system. The ballistic deflections of the galvanometer observed on opening or closing the shutter at O serve to map out the spectrum of the Nernst glower used as the source of energy. The construction of the galvanometer was the first work done on the present problem.

The entire path of the light, shown in Fig. 1 by the dotted lines, is inclosed in a black box of double cardboard. It is of course not possible to make such a box tight, nor to get the air in it absolutely dry. In the section around the glower, however, the air could be saturated. Also energy-curves were plotted from data secured on winter nights, when cold air from out of doors was pumped through

[1] *Verhandlungen der deutschen Physikalischen Gesellschaft*, **16**, 737, 1914.

[2] *Ibid.*, **16**, 614, 1914; **15**, 1159, 1913.

[3] *Annalen der Physik*, **33**, 739, 1910.

sulphuric acid and phosphorus pentoxide into the cases. Sufficient difference appeared between the depths of the bands to enable us to attribute the absorption to water-vapor.

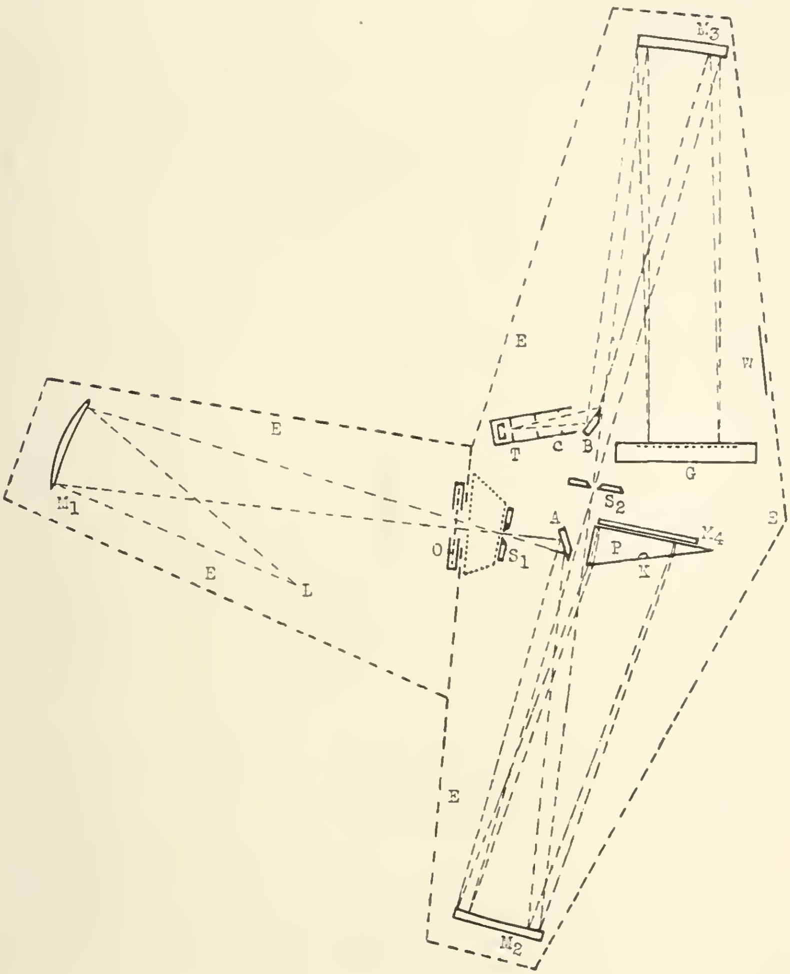

FIG. 1.—The spectrometer, scale 1 to 5

L, Nernst glower; S_1, S_2, slits; M_1, 10 cm mirror, f = 20 cm; *P*, salt prism; M_2, M_3, 10 cm mirrors, f = 50 cm; M_4, *A*, *B*, plane mirrors; *G*, grating; *C*, case for *T*, the thermopile; *W*, window in box *E*; *O*, shutter. The path of the light is $LM_1S_1AM_2PM_4PM_2S_2M_3GM_3BT$. A spectrum appears at S_2. *P* and M_4 rotate together about K, so that any region of the spectrum may be isolated for the grating, and the overlapping of spectra is avoided.

In the region of the spectrum near the wave-length $2.6\ \mu$ some work has been done with steam. The absorption took place in a chamber 15 cm long; which was kept hot by resistance coils, and could be swung into or out of the beam of light. The glass ends of the chamber were compensated for by two pieces cut from the same

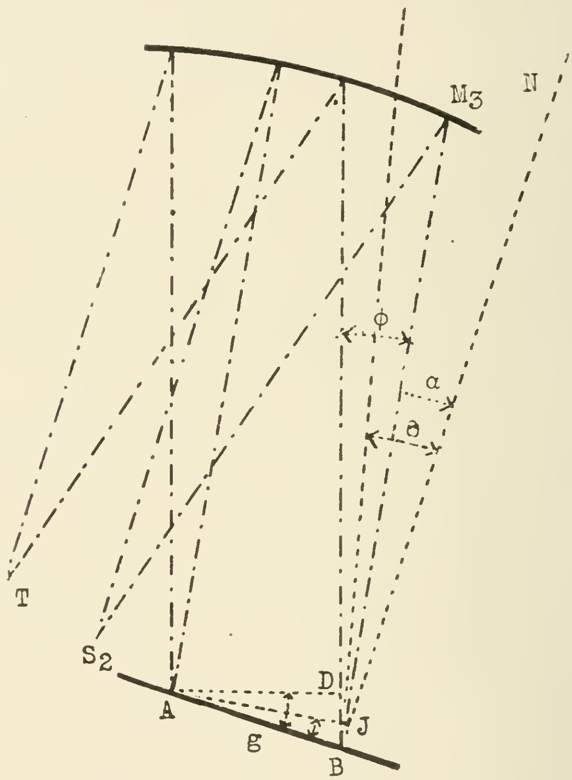

FIG. 2.—The spectrometer constant κ in the equation $\lambda = \kappa \sin \theta$

AB is the grating space, g. Light from S_2 is sent to T. T, S_2, and M_3, the mirror, are fixed, and rotation of the grating leaves ϕ unchanged. The wave-length at T is determined by the relation

$$
\begin{aligned}
n\lambda &= DB + BJ \\
&= g \sin(\alpha + \phi) + g \sin \alpha \\
&= 2g \sin(\alpha + \tfrac{1}{2}\phi) \cos \tfrac{1}{2}\phi \\
&= 2g \cos \tfrac{1}{2}\phi \sin \theta \\
&= \kappa \sin \theta
\end{aligned}
$$

$\theta = \frac{1}{2}(R_1 - R_2)$, R_1 and R_2 being circle readings on the two spectra of one order.

sheet of glass, through which the radiation passed when the steam chamber was drawn aside.

Three gratings have been employed in the course of the work. The smallest has a ruled surface 5 cm wide and about 2400 lines to the inch, and is referred to as the brass grating. The other two have effective surfaces 12.2 cm wide, are ruled on speculum metal, and are referred to as the 7500-line grating and the 15,000-line

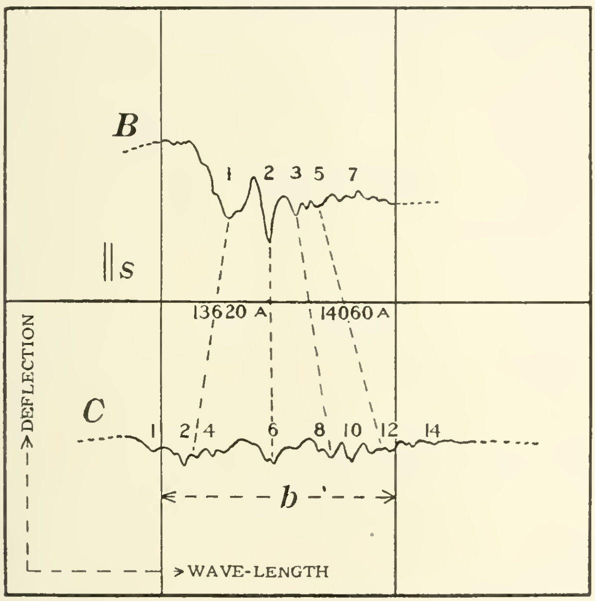

FIG. 3.—Energy-curves, region of 1.3 μ. The slit corresponds to 33 A in *B* and 15 A in *C*.

grating, the figures giving the number of lines per inch. In Figs. 3, 4, 5, and 6 the letters *A*, *B*, and *C* refer respectively to these gratings. They are mounted on the table of a Schmidt and Haensch spectrometer whose circle may be read to ten seconds.

Fig. 2 serves to illustrate the meaning and derivation of the equation

$$\lambda = \kappa \sin \theta,$$

used in computing wave-lengths from data secured with the fixed-arm spectrometer. To obtain the wave-length interval included at

one time in the slit of the thermopile, one considers first the resolving power of the grating. If the source S_2 were very narrow, the width of the central bright band in the diffraction pattern produced by monochromatic light would be given by

$$w = \frac{2\lambda}{b \cos\theta} \cdot 500,$$

where b is the width of the grating in mm, θ the angle by which the grating is turned from the normal, and 500 the focal length of the

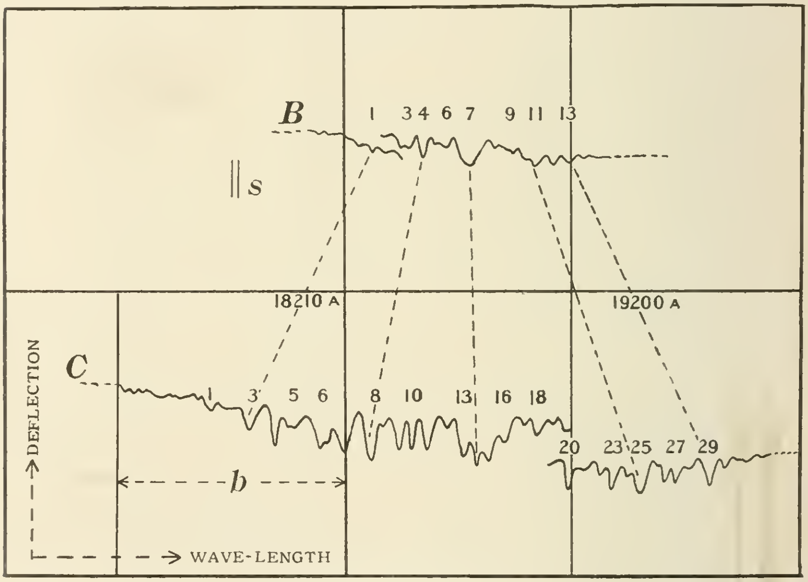

FIG. 4.—Energy-curves, region of $1.8\,\mu$. The slit corresponds to 33 A in B and 14 A in C.

mirror in mm. For the 15,000-line grating w is 0.03 mm at $2.6\,\mu$, and for the brass grating 0.14 mm at $6\,\mu$. The slit S_2 is $\frac{1}{2}$ mm wide, and its image in the plane of the slit of the thermopile is in the first case about 0.52 mm and in the second about 0.57 mm wide. With the slits now in use no great improvement in the purity of the spectrum can be attained by enlarging the grating.

The range of spectrum included in the slit is taken as the interval in wave-lengths between two beams whose centers are $\frac{1}{2}$ mm apart

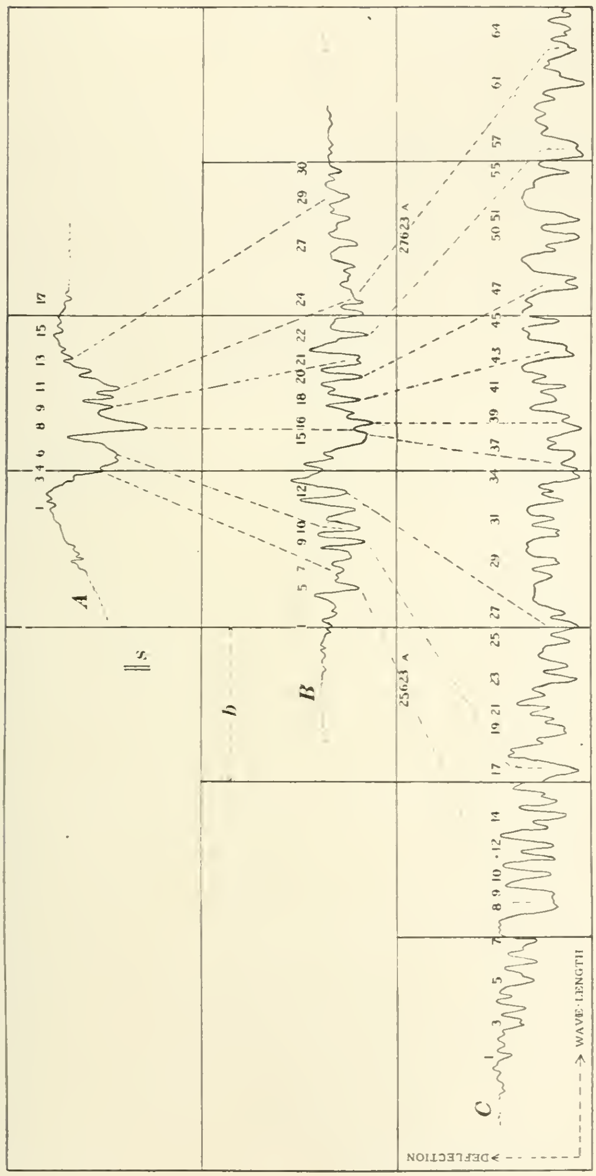

Fig. 5.—Energy-curves, region of $2.6\ \mu$. The slit corresponds to 105 Å in *A*, to 31 Å in *B*, and to 11 Å in *C*. *C* does not give the entire curve of this region as explored with the 15,000-line grating.

in the plane of the thermopile. That slit does not take in the total energy even of a single wave-length, but does receive about three-fourths of the energy in the interval named. This interval corresponds to a variation in ϕ (in Fig. 2) of $\frac{1/2}{500}$ or 0.001 radian. Taking the equation for wave-length as

$$\lambda = 2g \cos \phi/2 \sin (\alpha + \phi/2),$$

and considering that $\phi/2$ is found by measurement to be $1° \ 16'$, we have by differentiation the approximation

$$\begin{aligned} d\lambda &= K \cos \theta d\theta \\ &= K \cos \theta \frac{d\phi}{2}. \end{aligned}$$

This expression gives the values set down with the figures, which are a little too large. In order to throw a single beam from one

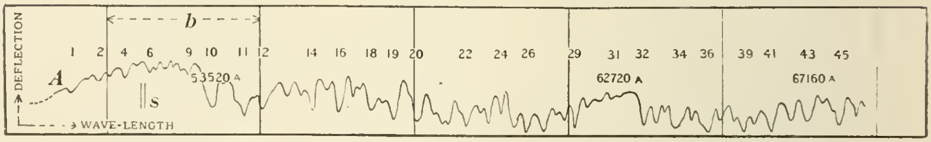

FIG. 6.—Energy-curve, part of the region of 6 μ. The slit corresponds to 100 A

side of the slit of the thermopile to the other, ϕ must be changed by $\frac{1/2}{500}$ or 0.001 radian, as above, and this demands a change in α of 0.0005 radian and a corresponding motion of the grating. 0.0005 radian is $1'.7$, and the spectrum interval may be checked by noting the value of $1'.7$ in units of wave-length along an energy-curve. At $2.6\,\mu$ the 15,000-line grating gives this interval as 10.4 A.

The green line of the spectrum of mercury ($\lambda = 5460.74$ A, Kaye and Laby's tables) was used for visual calibration, the bright line and the slit of the thermopile being viewed objectively. In the case of the brass grating no visible lines could be observed, but the actual grating-space was known, and

$$K = 2g \cos \tfrac{1}{2}\phi$$

was computed from data obtained from measurement.

III. THE CURVES

Figs. 3, 4, 5, and 6 present a summary of the experimental work. Except in Fig. 6 each energy-curve is one of three secured, for two of them the air being saturated and for one dried. The bands are numbered arbitrarily for reference to the tables. The letters *A*, *B*, and *C* refer respectively to the brass, the 7500-line, and the 15,000-line grating. In Fig. 7, as well as in the others, the distance *b* denotes 1° on the circle and *s* shows the relative size of the thermopile slit.

Fig. 7 represents part of the region at 2.6 μ as mapped with the 7500-line grating. The upper curve (lettered *D*) does not show the galvanometer deflections but gives the percentage of incident energy transmitted by steam. The effect of drying the air appears also in this figure, in curve *E*, and it will be noted that the curve for steam accentuates the peculiarities of *F*, the "air-saturated" curve, peculiarities which the curve secured with dried air decidedly obscures. It seems that the temperature of the absorbing vapor has no effect on the position of an individual band. According to Paschen's work, already cited, we conclude that the relative intensities of the bands must change, so that those that are deepest at one temperature are not deepest at another. But the present work cannot confirm nor deny this effect. The effect on the absorption of drying the air, shown in Fig. 7, was found even more definitely with the finer grating, at 1.3 and 1.8, as well as at 2.6 μ. All the deeper bands are less marked when the air is dry. This does not appear so clearly with the shallow bands, but it may be that in the "air-saturated" curves the presence of weak bands is masked by their more prominent neighbors. It is upon such evidence as appears in Fig. 7 that we ascribe the absorption to water-vapor.

With the 7500-line grating the intervals of the spectrum between the absorption bands at 1.3 and 1.8 μ, and between those at 1.8 and 2.6 μ, were carefully explored and were found to present no evidence of atmospheric absorption.

In Figs. 3 and 5 the upper curves are decidedly similar. This similarity suggests that one of these absorption regions may be a harmonic of the other in the sense suggested by Kemble.[1] However,

[1] *Physical Review*, **8**, 689, 1916.

a frequency half that corresponding to band No. 2 in Fig. 3 corresponds nearly to band No. 11 in Fig. 5 rather than to band

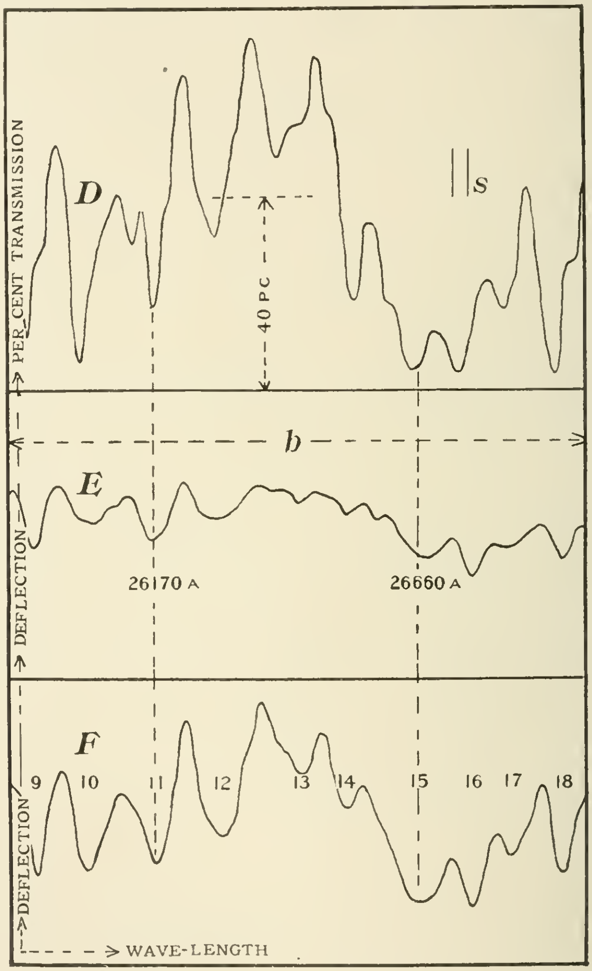

FIG. 7.—Part of the region of 2.6 μ, 7500-line grating. *D* gives the percentage of energy transmitted by steam at 110 to 120° C. and 1 atmosphere pressure. *E* is an energy-curve with the air partly dried, *F* an energy-curve with the air saturated at about 35° C.

No. 8, and the same lack of agreement appears in the lower curves. Evidently the simple idea of harmonics does not apply.

A comparison of the energy-curves in Fig. 5, secured respectively with the 7500- and 15,000-line gratings, shows how the increased dispersion adds to our knowledge of the absorption. Bands No. 8 and No. 9 in the lower curve, for example, appear in the middle one as No. 5, a connection indicated in typical cases by the dotted lines, and appearing more definitely in the tables. These curves must be considered in any numerical computations made with absorption frequencies, for in the curves appear the individualities of the bands, which cannot well be expressed in numbers. In all cases the grating was turned 1′ for each new setting. At 6 and at 2.6 μ six deflections were taken at each minute, and at 1.3 and 1.8 μ four. The computations are based on curves which, for the region at 2.6 μ, are about six feet long.

IV. THE TABLES

On pages 137 to 142 are set forth the wave-lengths and frequencies of the bands, which are denoted by the numbers given with the curves. Corresponding wave-lengths appear opposite each other. The gratings have been calibrated independently, the values given have been calculated from independent curves, and the agreement between opposite numbers is in general satisfactory. In connection with the region of 6 μ is set forth some evidence of the symmetry demanded by Eucken's explanation, but unfortunately the work was not carried far enough to give this evidence much value. Some idea of the accuracy of the determinations of λ and N might be obtained by estimating the effects of various sources of error. But first one may consider the different values secured for the same quantity from the three corresponding curves. The "typical computations of λ and N" show these numbers for certain bands in the region at 2.6 μ—bands taken quite at random. Considering the uncertainty in the value of K we should perhaps affect the wave-length by the probable error of ± 2 A. Errors in the circle of the spectrometer are probably negligible, especially since the calibration involved the same sections of the circle as were employed in the measurements. At 6 μ, where we have only one curve, obtained under less favorable circumstances, we should perhaps write the wave-lengths with an error ± 20 A. A question mark in the tables

TYPICAL DATA

The settings of the grating circle, and corresponding deflections of the galvanometer.

2 A.M., March 22, 1917. Grating temperature, 27°. Period of galvanometer, 6 secs.; E.M.F. applied to glower, 238 volts.

Circle	Deflections in mm	Circle	Deflections in mm
121° 10′......	28–9– 9–8–9–9 = 28.7	120° 58′.....	18– 9– 8–9–9–9 = 18.7
9′......	26–7– 7–6–7–6 = 26.5	57′.....	19–20– 0–0–0–0 = 19.8
8′......	23–2– 1–2–2–2 = 22.0	56′.....	16– 7– 8–8–8–7 = 17.3
7′......	12–5– 6–6–4–5 = 14.7	55′.....	9–11–10–1–0–1 = 10.3
6′......	11–2– 3–2–3–1 = 12.0	54′.....	10– 0– 1–0–1–1 = 10.5
5′......	10–2– 2–1–2–2 = 11.5	53′.....	18–22– 0–1–0–1 = 20.3
4′......	15–8– 7–7–8–7 = 17.0	52′.....	24– 6– 5–5–6–5 = 25.2
3′......	17–6– 6–7–6–7 = 16.5	51′.....	27– 7– 6–7–7–7 = 26.8
2′......	9–8–10–9–9–8 = 8.8	50′.....	27– 8– 6–7–6–8 = 27.0
1′......	5–5– 4–5–5–6 = 5.0	49′.....	25– 7– 6–8–6–7 = 26.5
121° 0′......	6–6– 5–6–6–6 = 5.8	48′.....	23– 6– 4–6–4–6 = 24.8
120° 59′......	14–3– 4–3–4–3 = 13.5	47′.....	22– 3– 1–4–2–3 = 22.5

TYPICAL COMPUTATIONS OF WAVE-LENGTH AND FREQUENCY

REGION OF 2.6 μ

Band No.	Curve No.	λ	Mean λ	N	Mean N
20........	1	26063.4	26066.2	383.69	383.64
	2	26068.8		383.60	
	3	26066.5		383.63	
23........	1	26162.7	26161.6	382.22	382.24
	2	26161.7		382.25	
	3	26160.4		382.26	
27........	1	26321.3	26325.5	379.92	379.86
	2	26330.0		379.79	
	3	26325.3		379.86	
31........	1	26543.7	26546.4	376.74	376.70
	1′	26546.9		376.70	
	2	26549.3		376.66	
	3	26545.7		376.71	
32........	1	26587.4	26589.8	376.12	376.08
	1′	26589.5		376.09	
	2	26590.9		376.07	
	3	26591.4		376.06	
33........	1	26608.7	26609.9	375.82	375.80
	1′	26610.9		375.79	
	2	26610.2		375.80	
	3	26609.7		375.80	

denotes not a doubtful band but a doubtful correspondence. The second column in the tables for the region at 2.6 μ gives the relative depths of the bands as an indication of their importance in other infra-red work. A band numbered 1 seems to represent an absorption of less than 10 per cent, while 5 shows that perhaps 80 per cent of the incident energy is absorbed.

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM

REGION OF 1.38 μ

15,000-Line Grating			7500-Line Grating		
Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
1	13545	738.3			
2	13614	734.5 }	1	13620	734
3	13643	733.0 }			
4	13678	731.1			
5	13710	729.4			
6	13820	723.6	2	13820	723.5
7	13876	720.7			
8	13922	718.3			
9	13954	716.7	3	13950	717
10	14000	714.3	4	14010	714
11	14046	712.0 }	5	14060	711
12	14092	709.7 }			
13	14140	707.2 }	6	14170	706
14	14186	704.9 }			
			7	14240	702

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

REGION OF 1.87 μ

15,000-Line Grating			7500-Line Grating		
Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
1	18110	552.2			
2	18152	550.9			
3	18202	549.4	1	18210	549.1
4	18263	547.6	2	18287	546.8
5	18306	546.3			
6	18363	544.6	3	18398	543.6
7	18414	543.1			
8	18471	541.4	4	18467	541.5
9	18534	539.6	5	18530	539.7
10	18564	538.7			
11	18595	537.8	6	18590	537.9
12	18641	536.4			
13	18676	535.5			
14	18705	534.6	7	18704	534.6
15	18729	533.9			
16	18765	532.9			
17	18806	531.8			
18	18836	530.9	8	18836	530.9
19	18862	530.2			
20	18897	529.2	9	18912	528.8
21	18926	528.4			
22	18968	527.2			
23	18995	526.5	10	18987	526.7
24	19022	525.7	11	19038	525.3
25	19054	524.8			
26	19106	523.4			
27	19133	522.7	12	19126	522.9
28	19157	522.0			
29	19199	520.9	13	19195	521.0
30	19234	519.9	?14	19264	519.1

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

REGION OF 2.6 μ

15,000-Line Grating				7500-Line Grating			Hilger Brass Grating		
Band No.	Relative Intensity	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
1	1	25235	396.28				?1	24850	402.2
2	1	25262	395.85				?2	25040	399.4
3	2	25311	395.09						
4	2	25352	394.45	1	25347	394.52			
5	2	25425	393.31	2	25432	393.21			
6	2	25469	392.63	?3	25481	392.45	?3	25490	392.3
7	2	25520	391.85	4	25517	391.90			
8	3	25608	390.50						
				5	25623	390.27			
9	3	25636	390.08						
10	3	25688	389.29	6	25696	389.17			
11	3	25727	388.70						
				7	25747	388.39			
12	3	25760	388.20						
13	2	25803	387.55				4	25760	388.2
14	4	25830	387.15	8	25837	387.04			
15	3	25864	386.64						
16	2	25880	386.40						
17	5	25940	385.51	9	25941	385.49	5	25920	385.8
18	2	26008	384.50						
19	3	26046	383.94	10	26044	383.97	6	26080	383.4
20	2	26066	383.64						
21	2	26090	383.28						
22	2	26123	382.80						
23	3	26161	382.25	11	26168	382.15			
24	4	26196	381.74						
25	4	26256	380.87						
26	5	26295	380.30	12	26300	380.23	7	26280	380.5
27	4	26325	379.87						
27½	1	26385	379.00						
28	2	26411	378.63						
29	3	26448	378.10	13	26445	378.14			
30	3	26515	377.15						
				14	26528	376.96			
31	2	26547	376.69						
32	3	26590	376.08						
33	2	26610	375.80						
34	5	26645	375.30						
35	5	26661	375.08	15	26663	375.05			
36	5	26696	374.59						
37	3	26720	374.25				8	26730	374.1
38	5	26760	373.69						
				16	26765	373.62			
39	4	26783	373.37						
40	3	26827	372.76						
				17	26837	372.62			
41	3	26853	372.40						
42	2	26880	372.02						

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

REGION OF 2.6 μ—*Continued*

15,000-Line Grating				7500-Line Grating			Hilger Brass Grating		
Band No.	Relative Intensity	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm	Band No.	Wave-Length A	Wave No. per mm
43	4	26930	371.33	18	26936	371.25			
44	1	26974	370.73						
45	3	27006	370.29	19	27002	370.34			
46	1	27044	369.77						
47	4	27084	369.22						
				20	27093	369.10			
48	4	27103	368.96						
49	3	27168	368.08						
50	5	27198	367.67	21	27193	367.74	9	27180	367.9
51	5	27236	367.15						
52	1	27278	366.60						
53	1	27294	366.38						
54	1	27314	366.11						
55	3	27340	365.76						
56	5	27386	365.15						
				22	27395	365.03			
57	5	27407	364.87						
58	2	27445	364.37				10	27430	364.6
59	2	27479	363.91						
60	2	27511	363.49						
61	5	27545	363.04	23	27544	363.06			
62	4	27614	362.14						
63	3	27625	361.99	24	27623	362.02	11	27620	362.0
64	3	37655	361.60						
65	3	27668	361.43						
66	3	27700	361.01						
67	3	27712	360.85						
68	2	27765	360.17						
69	2	27777	360.01	25	27784	359.92			
70	2	27803	359.67						
71	2	27819	359.47						
72	3	27867	358.85	26	27866	358.86			
73	1	27926	358.09						
74	1	27947	357.82						
75	1	27972	357.50						
76	4	28029	356.77	27	28031	356.75	12	28030	356.8
77	2	28077	356.16						
78	1	28103	355.83						
79	2	28140	355.37						
80	3	28197	354.65	28	28198	354.64			
81	2	28267	353.77						
82	2	28336	352.91						
				29	28348	352.76	13	28350	352.7
83	2	28373	352.45						
84	2	28485	351.06						
85	2	28538	350.41	30	28540	350.38	14	28540	350.4
86	1	28590	349.77						
87	1	28658	348.94						
				31	28710	348.31			
							15	28990	344.9
							16	29210	342.4
							17	29750	336.1

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

PART OF THE REGION OF 6 μ

Hilger brass grating

Band No.	Wave-Length A	Wave No. per mm	Distance from Center, 159.6	Mean Distance of One Pair
1	50220	199.1	39.5	
2	50850	196.7	37.1	
3	51110	195.7	36.1	
4	51490	194.2	34.6	
5	51720	193.4	33.8	
6	52040	192.2	32.6	
7	52420	190.8	31.2	
7′	52600	190.1	30.5	
8	52800	189.4	29.8	
9	52960	188.8	29.2	
9′	53090	188.4	28.8	
10	53520	186.8	27.2	
11	54240	184.4	24.8	
11′	54470	183.6	24.0	
12	54660	182.9	23.3	
13	55250	181.0	21.4	
13′	55570	179.9	20.3	
14	55810	179.2	19.6	
15	56170	178.0	18.4	
16	56430	177.2	17.6	
17	56800	176.1	16.5	
17′	56920	175.7	16.1	
18	57170	174.9	15.3	
18′	57470	174.0	14.4	
19	57680	173.4	13.8	
20	58240	171.7	12.1	12.2
20′	58640	170.5	10.9	10.8
21	58830	170.0	10.4	
21′	58950	169.6	10.0	9.9
22	59380	168.4	8.8	8.8
22′	59720	167.5	7.9	7.9
23	59880	167.0	7.4	7.4
24	60160	166.2	6.6	6.4
25	60480	165.3	5.7	5.6
26	60720	164.7	5.1	
27	61140	163.6	4.0	3.9
27′	61440	162.8	3.2	3.2
28	61620	162.3	2.7	2.7
29	61880	161.6	2.0	2.0
30	62160	160.9		
30′	62420	160.2		
31	62720	159.4		
31′	62890	159.0		
32	63440	157.6	2.0	
33	63740	156.9	2.7	
33′	63930	156.4	3.2	
34	64180	155.8	3.8	
35	64530	155.0	4.6	
36	64950	154.0	5.6	
37	65200	153.4	6.2	
38	65510	152.6	7.0	

WAVE-LENGTHS AND WAVE-NUMBERS OF THE ABSORPTION BANDS OF WATER-VAPOR IN THE NEAR INFRA-RED SPECTRUM—*Continued*

PART OF THE REGION OF 6 μ—*Concluded*

Hilger brass grating

Band No.	Wave-Length A	Wave No. per mm	Distance from Center, 159.6	Mean Distance of One Pair
39.........	65730	152.1	7.4	
40.........	65940	151.7	7.9	
41.........	66340	150.7	8.9	
42.........	66810	149.7	9.9	
43.........	67160	148.9	10.7	
44.........	67520	148.1	11.5	
45.........	67920	147.2	12.4	
46.........	68280	146.5	13.1	

V. QUESTIONS OF INTERPRETATION

In addition to her work with water-vapor mentioned above, Eva von Bahr[1] has studied the absorption of HCl in the neighborhood of 3.5 μ. The combination principle, with the quantum distribution of rotation-frequencies, has been applied by Bjerrum[2] to set forth reasons for the presence of these absorption bands of HCl and to calculate directly the size of energy quanta. However, the chaotic arrangement of water bands in the region of 2.6 μ, as they appear in Fig. 5, does not appear to satisfy the demands of these hypotheses, though the region has in a general way a center and a rough appearance of symmetry. Since this work was completed Mr. E. H. Imes has mapped the absorption of HCl at 3.5 μ with the apparatus here described, using an interval of about 30 A in the slit of the thermopile. The wave-lengths are precisely determined by the grating, and a parabolic shift of the vibration-frequency with increasing rotation-frequency is very definitely displayed. Work on HCl showing this effect has been published by Kemble and Brinsmade,[3] though that done here was completed before their work appeared. The curve of Fig. 5 does show a crowding together of bands in the nearer end, which our experience with HCl would lead us at least to consider possible. But the uncertainty, due to

[1] *Verhandlungen der deutschen Physikalischen Gesellschaft*, **15**, 1150, 1913.

[2] *Ibid.*, **16**, 614, 1914.

[3] *Proceedings of the National Academy of Sciences*, **3**, 420, 1917.

the general complexity of the curve, as to which band should be associated with any other to make a pair, and about the location of the vibration center for no rotation, has so far deferred our explanation. The third rotation series mentioned in connection with the work of Rubens and Hettner may perhaps be appealed to as a probable cause of the complexity both at 2.6 and 6 μ.

The appearance of the curves, particularly of Fig. 5, suggests that useful knowledge may be still further advanced by an increase in dispersion. In the case of the 2.6 μ region, however, neither a finer nor a larger grating would serve this purpose; but a source of greater energy, or a more sensitive detector, would enable us to use narrower slits. We hope to profit by the use of a tungsten ribbon lamp in place of the glower. We may be able to build another more sensitive galvanometer.

It should be pointed out that the selective atmospheric absorption near 2.6 and 6 μ may account for the unexpected weakness or even the non-appearance of bright lines predicted by the laws of series in the spectra of elements.

It is hoped to employ the apparatus in the further study of absorption, and it may be that the behavior of carbon dioxide will help to explain that of water-vapor. The work here described was undertaken at the suggestion of Professor H. M. Randall, to whom the author makes grateful acknowledgment of his indebtedness.

University of Michigan, Physics Laboratory
September 25, 1917

Minor Contributions and Notes

A HELIUM STAR WITH LARGE PARALLAX, RADIAL VELOCITY (AND PROPER MOTION?)

Among the stars included in my program for parallax with the meridian circle and afterward selected for photographic determination of parallax was the star Boss *P.G.C.* 1517 = *A.G.C.* 7234 = 72 G Columbae, $\alpha = 6^h 0^m 37^s$, $\delta = -32° \ 10' \ 12''$ (1900), Harvard Mag. 5.6. This star, of type B, was observed at Professor Kapteyn's suggestion, who suspected a parallax of about $+0''.1$, unusually large for this type.

The results obtained from the photographic determination are:

$$\pi = +0''.069 \pm 0.006$$
$$\mu_\alpha = +0''.235 = 0^s.0185$$

The parallax agrees very well with Professor Kapteyn's supposition. The proper motion in α, however, has come out much larger than that given in the Boss *P.G.C.*, which was $-0^s.0001$.

Mr. R. E. Wilson, of the D. O. Mills Observatory, was good enough to determine the star's radial velocity, which he found from three plates to be $+102$ km (that is, $+83$ km corrected for solar motion with $\alpha = 270°$, $\delta = +32°$, $V_o = +19$ km). He remarked that the 6 lines measured were fairly broad, but that the radial velocity will not be more than 5 km in error. The radial velocity is a large one for a B-type star, so that the star must have a considerable space motion, though the transverse proper motion is still very uncertain.

The broad lines may indicate that it is a double star showing the lines of both components.

J. Voûte

Franschhack, South Africa
May 20, 1918

THE

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ON THE EXCESS OF OUTWARD MOTION OF THE STARS OF CLASS B

By C. D. PERRINE

The residual velocities of the B stars, after eliminating the solar motion, show a strong positive tendency, so much so that it gave rise to the belief that there was some sort of a constant error in the positions of the lines of their spectra, or a pressure-effect in the atmospheres of these stars which caused their velocities to be observed too large on the average by +4 or +5 km.

The preference of the B stars for the Galaxy and their distance, together with the hypothesis that their great brightness and peculiar spectral condition result from the action of cosmical matter in those regions, suggest an explanation for this excess of outward motion. It is that a process of selection has been at work, that the B stars are those which have penetrated the region richest in such cosmical matter, and that the observer being on the inside of such a ring of matter these stars, or at least the nearer ones, would have a preference for outward motion. Such a theory requires that only the stars which were moving outward would have passed into this ring of matter and become B-type stars, if we assume that the ring of matter is very extensive. If not of so great extent, then some stars could have entered from the outside. Such stars

would in general be approaching the observer and would be fainter. If such is the true condition, then the brighter and nearer stars of class B should show almost entirely outward motion, and the fainter and more distant of these stars might contain a considerable proportion of approaching stars. This condition would be modified somewhat if the matter were not circular but spiral, for example. In a previous investigation[1] attention was drawn to the fact that there was a progressive change in the residual velocities of stars of class B, the brighter ones showing an excess of outward motion, whereas the fainter stars showed an excess of inward motion. These results were from velocities in which the constant error term K had been removed. As later evidence has been found[2] tending to show that this term K is really an excess of outward motion the foregoing progression indicates not inward motion for the fainter stars but a *smaller* excess of outward motion for these stars. A re-examination was made of these B stars in which they were classified with respect to size of proper motion also. The results are given in Table I.

On account of the well-known differences of the later B stars only classes B to B5 inclusive were used. Three facts are noticeable in these results:

a) Only one star brighter than magnitude 3.0 is approaching, and that is barely over the line (-1 km).

b) The proportion of approaching stars increases toward the fainter magnitudes and is greater for the stars of smaller μ.

c) The residual velocities in the stars of smaller μ decrease toward the fainter stars. This decrease is due to the increasing proportion of stars of approach among the fainter stars and not to a general decrease of outward motion, as is seen from the receding stars which are given just underneath the results for both classes in Table I.

Adams' list of the radial velocities of 500 stars[3] contains 27 stars of classes B to B5 inclusive. With two exceptions they are all fainter than magnitude 5.0, having a mean of 5.4. Of these 10, or 37 per cent, are approaching. The residual velocity of the 27

[1] *Astrophysical Journal*, 41, 319, 1915. [2] *Ibid.*, 44, 244, 1916.

[3] *Mt. Wilson Contr.*, No. 105; *Astrophysical Journal*, 42, 172, 1915.

TABLE I

RESIDUAL RADIAL VELOCITIES OF STARS OF CLASSES B–B5*

	μ=0 to 0″.029						μ=0″.030 and Over					
	Mean Mag.	Residual Velocity V	μ	No. of Stars	Approaching Stars		Mean Mag.	Residual Velocity V	μ	No. of Stars	Approaching Stars	
					No.	Percentage					No.	Percentage
		km						km				
2.2 and brighter....	1.9	+6.4	0″.007	6	0	0	1.6	+5.5	0″.060	9	1	....
2.3 to 2.9.........	2.7	+11.5	.014	5	0	0	2.8	+7.6	.045	7	0	0
3.0 to 3.9.........	3.6	+7.4	.016	22	1	5	3.5	+5.2	.046	16	3	19
		+8.3		21				+7.4		13		
4.0 to 4.9.........	4.6	+3.9	.013	59	17	29	4.5	+6.0	.043	31	7	23
		+8.7		42				+8.9		24		
5.0 and fainter.....	5.2	+1.7	.017	16	7	44	5.3	+5.9	.040	8	2	25
		+9.3		9				+10.0		6		

* These residual velocities resulted from the use of a solar velocity of 19.5 km toward $\alpha=270°$, $\delta=+30°$.

is +2.2 km and of the 17 receding stars +9.3 km. These agree well with the results in Table I.

The results in Table I are shown graphically in Fig. 1, where the curves *A* and *C* relate to magnitude, *B* and *D* to μ.

The foregoing results led to an examination of the galactic stars of other spectral classes with a view to discovering if this preference of approaching motions for the fainter and more distant stars is a general phenomenon or is confined to the B stars. Like

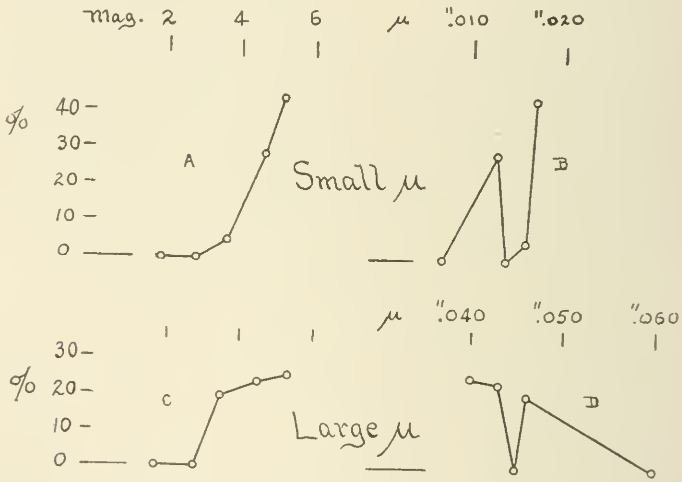


Fig. 1

the preceding this examination was first limited to Campbell's L.O. catalogues because they represent all parts of the sky. Only the *numbers* of stars are considered in this case, the residual velocities being left for later investigation, largely on account of the few stars available in some of the spectral classes. The results are given in Table II. They show that there is essentially no preference of the galactic stars of spectral classes A, F, G, K, and M for positive velocities, the distributions being as uniform between positive and negative velocities as can be expected from the nature of the data. The number of the brighter stars in class M is too small for the predominance of positive velocities to have any significance in the face of the uniformity shown by the larger number of

the fainter stars of the same class. The intermediate position of the B8 and B9 stars is noticeable but of small weight on account of their small number. I conclude, therefore, that among the 852 galactic stars examined only the B-type stars show a numerical preference for receding velocities, all of the other types showing no preference either for receding or approaching motions.

TABLE II

According to Direction of Motion

Campbell's L.O. Catalogues. Galactic

Spectral Class	3.9 and Brighter				4.0 and Fainter				Total No. of Stars
	Receding		Approaching		Receding		Approaching		
	No.	Percentage	No.	Percentage	No.	Percentage	No.	Percentage	
B–B5......	60	92	5	8	81	71	33	29	179
B8–B9.....	5	83	1	17	16	59	11	41	33
A.........	10	36	18	64	41	49	42	51	111
F.........	13	52	12	48	43	47	48	53	116
G.........	5	33	10	67	36	49	38	51	89
K.........	42	49	43	51	103	52	94	48	282
M........	(9	82)	(2	18)	17	55	14	45	42

Among the stars of class A of magnitude 3.9 and brighter there is a preponderance of negative velocities, but the number of stars is too small to justify further consideration at the present time.

An examination of the stars in the non-galactic regions appears to show essentially the same distributions with respect to the numbers of stars receding and approaching as in the galactic regions for the classes later than B. In consequence of this indication the 500 stars of Adams' Mount Wilson catalogue were classified with respect to the numbers of stars having velocities of recession and approach, with the results given in Table III. The early B-type stars show a small preference for receding velocities, agreeing in general with the results from Campbell's catalogues. The A-type stars show a small preference for receding velocities but hardly enough to justify remark. The later B and the classes F, G, K, and M may safely be concluded to show no real preference.

The results of Tables II and III for the approaching stars are shown graphically in Fig. 2. Curves *A* and *B* refer to the stars of magnitude 3.9 and brighter and 4.0 and fainter, respectively, of Campbell's L.O. catalogues. Curve *C* refers to the stars of Adams' Mount Wilson catalogue. The curves for the receding velocities

TABLE III

According to Direction of Motion
Adams' Mount Wilson Catalogue

Spectral Types	Receding		Approaching		Total No. of Stars
	No.	Percentage	No.	Percentage	
B–B5	17	63	10	37	27
B6–B9	34	53	30	47	64
A	80	58	58	42	138
F	17	45	21	55	38
G	47	49	49	51	96
K	39	45	48	55	87
M	23	51	22	49	45

are simply reversals of these curves. An examination of the velocities of approach shows a larger value for the more distant stars of class B–B5 than for the nearer ones. The results are given in Table IV.

TABLE IV

	Small μ				Large μ			
	Mag.	No. of Stars	$\bar{V}$	$\bar{\mu}$	Mag.	No. of Stars	$\bar{V}$	$\bar{\mu}$
Campbell's L.O. catalogues			km				km	
$3^{M}0$ to $3^{M}9$	3.8	1	−11.2	0.″010	3.4	3	−4.2	0.″051
4.0 to 4.9	4.5	17	− 7.8	.015	4.4	7	−4.1	.047
5.0 and fainter	5.2	7	− 8.1	.017	5.6	2	−6.4	.040
Adams' Mount Wilson catalogue	5.6	6	−11.3	.020	4.5	4	−7.7	.042
Mean	......	31	− 8.7	....	......	16	−5.3	....

This would be in accordance with the theory that the approaching stars of early class B have entered the ring of cosmical matter from the outside and that their velocities have been retarded. The

amount of data is too small, however, to place any great confidence in the result, notwithstanding its apparent consistency, or to discuss its bearing further at the present time.

The results in Table I show a progressive increase in the *proportion* of stars with negative velocities, as the stars become fainter. Whether this is real or whether it is really an effect of distance only

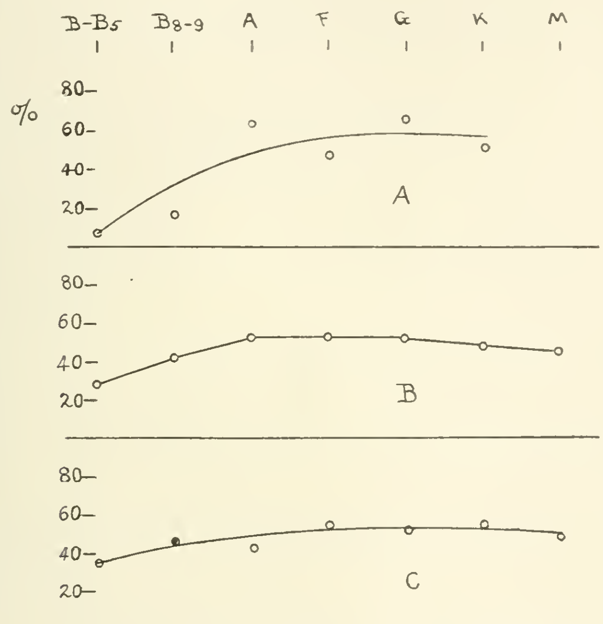


FIG. 2

cannot be determined at present. The proper motions of the stars of class B are all so small that there must be great uncertainty in them and it may be that magnitude is, in this case, a better gauge of distance than the size of proper motion. Direct determinations by parallactic displacement for these stars seem hopelessly beyond our present means. In Table I the positive velocities considered alone show a small increase with decreasing magnitude. Table IV indicates a similar effect in the negative velocities considered separately. The amount of material, however, seems to be too small to justify classification with respect to ellipsoidal regions

which were shown in a recent paper[1] to appear to have caused some of the magnitude-velocity effect.

The residual velocities used in this paper may be considered to be derived using a solar motion of 20 km toward $\alpha = 270°$, $\delta = +30°$. The directions and velocities actually used differ slightly in some cases from the above but are generally within $\frac{1}{2}$ km and 1° of these.

BEARING OF THIS INVESTIGATION ON K TERM

The results of this investigation have a bearing upon the interpretation to be put upon the term K which has been found in the solutions for solar motion from radial velocities. This term may be defined as the excess of velocity with respect to algebraic sign after the elimination of solar motion. Campbell found large positive values for this term for the spectral classes B, K, and M and no appreciable values for the stars of classes A, F, and G, from his solutions from about 1200 stars in all parts of the sky.[2] Using essentially the same apex of solar motion, Adams from his 500 stars found a much reduced value for B, negative (generally small) values for A, F, and K, and negligibly small positive values for classes G and M.[3]

The writer found[4] that a few regions (about one-third of the whole) gave consistently larger positive velocities in the three spectral classes B, K, and M, and that if these were omitted there remained little or no positive residuals for the remaining two-thirds of the stars.

The results obtained in the first part of the present paper indicate a fundamental difference in the distributions of the velocities of recession and approach between the stars of class B and the later spectral classes. Interpreted directly, this further indicates in my opinion a lack of sufficient constancy for the term K to be ascribed to any physical cause and to strengthen the conclusion that these excess velocities are chiefly motion.

[1] *Astrophysical Journal*, **46**, 266, 1917.

[2] *Lick Observatory Bulletin*, **6**, 104, 127, 1911.

[3] *Mt. Wilson Contr.*, No. 105, p. 20.

[4] *Astrophysical Journal*, **44**, 244, 1916.

CONCLUSIONS

I. The stars of class B, particularly those early in the class, after the elimination of solar motion, show a marked preponderance of outward motions.

II. The motions of the B stars of magnitude 2.9 and brighter which were investigated are essentially all *outward*. In the stars of magnitude 3.0 and fainter the proportion of stars with velocities of approach increase for the fainter magnitudes and with the smaller proper motions.

III. The average velocities of recession appear to be essentially the same for all distances, as indicated by size of proper motion, but to increase slightly with decrease of brightness.

IV. In the spectral classes A, F, G, K, and M no preference is shown for either velocities of recession or approach in either galactic or non-galactic regions.

V. The observed distributions of the velocities of approach and recession in the B stars can be explained upon the hypothesis that the peculiar spectral condition of the B-type stars is due chiefly to a ring of distant galactic cosmical matter, and that the nearer and brighter of these stars have entered this ring in general from the inside, whereas a considerable proportion of the fainter and more distant ones have entered from the outside.

Observatorio Nacional Argentino, Córdoba
January 29, 1918

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

SEVENTH PAPER: THE DISTANCES, DISTRIBUTION IN SPACE, AND DIMENSIONS OF 69 GLOBULAR CLUSTERS

By HARLOW SHAPLEY

I. PARALLAXES FROM VARIABLE STARS, APPARENT MAGNITUDES, AND ANGULAR DIAMETERS

Applying the methods discussed in the preceding *Contribution*,[2] the parallaxes of a few clusters are obtained directly from the periods and magnitudes of Cepheid variables; the parallaxes of a considerably larger number are derived from the mean magnitudes of the brightest cluster stars, and the survey is then made complete through measures of diameters of the photographic images of all globular systems. In Table I are listed the clusters for which the variable stars have received a sufficiently detailed discussion to permit a determination of the parallax directly from the luminosity-period curve of Cepheid variation. The apparent diameters and the adopted parallaxes are taken from tables appearing on following pages; the method of weighting the results is also subsequently described.

The parallaxes for Messier 3, 5, 15, and 22, depending almost entirely on the median magnitudes of numerous variable stars, are the most accurate. The computed probable error of the absolute magnitude is ± 0.2; that of the apparent magnitude is estimated to vary from ± 0.05 for Messier 3 to possibly ± 0.2 for Messier 22. The corresponding limits of the probable error of the parallaxes are $\pm 0\overset{''}{.}000007$ and $\pm 0\overset{''}{.}000015$, that is, 10 and 13 per cent, respectively.

For most other clusters, of course, the errors are somewhat greater, particularly for those where the parallax depends solely upon either the magnitudes of the brightest stars or the measured

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 152.

[2] *Mt. Wilson Contr.*, No. 151.

diameter on photographic charts. For the former the estimated average probable error is 20 per cent, for the latter 25 per cent.[1] When two or three sources are available, as in Tables I and V, the errors are less; but in Table I the different parallaxes for each cluster are not all completely independent, as they were used in part to determine the reduction constants and curves. The residuals in the last column of Table I are expressed in millionths of a second of arc, and their minuteness indicates the validity of the methods involving diameters and the magnitudes of the bright stars.

TABLE I

COMPARISON OF CLUSTER PARALLAXES FROM VARIABLES, MAGNITUDES, AND DIAMETERS

Designation		Apparent Diameter	Parallax (Unit Is 0″000001)				Residuals
N.G.C.	Messier		Adopted	From Variables	From Magnitudes	From Diameters	
5272	3	7′0	72	72	71	72	0, − 1, 0
5904	5	8.6	80	80	80	81	0, 0, +1
6205	13	10.6	90	82:	89	91	−8:, − 1, +1
6656	22	16.0	118	116	121	116	−2, + 3, −2
7078	15	5.0	68	67	69	59	−1, + 1, −9
7089	2	7.0	64	65	60	72	+1, − 4, +8
5139		30	153	150	170:	155	−3, +17:, +2
Small Magellanic Cloud		....	52	52	....	....	....

The parallax of the Small Magellanic Cloud, which is given at the end of Table I, is relatively uncertain, for the value from variables can be checked by neither diameter measures nor maximum luminosities, and the zero-point error in the provisional magnitude scale used by Miss Leavitt is unknown.[2]

[1] These estimates appear to be safely conservative. After the smoothing operation, described on a later page, much smaller average errors are obtained for the final parallaxes in Tables V and VIII, thus, including the 10 per cent probable error in the parallaxes due to uncertainty of the zero-point of the absolute scale, the average probable error for all clusters is estimated to be less than 15 per cent. Cf. Table VII.

[2] See sec. III of the preceding *Contribution*. Previous values for the parallax of the Small Magellanic Cloud are: Hertzsprung, $\pi = 0\overset{''}{.}0001$, *Astronomische Nachrichten*, **196**, 204, 1913; Kapteyn, $\pi = 0\overset{''}{.}00004$, *Mt. Wilson Contr.*, No. 82, 71, 1914; Shapley, $\pi = 0\overset{''}{.}00006$, *Mt. Wilson Contr.*, No. 116, 82, 1915. The new value in Table I is probably an improvement over the others because it allows for diversity in color of the variable stars and is based upon more definite knowledge of their absolute magnitudes.

In the preceding paper we found from the study of the bright stars and variables in several clusters that, after excluding the five brightest, the mean absolute photographic magnitude of the 25 most luminous objects is

$$M_{25} = -1.51 \pm 0.28.$$

Adopting this value, we have derived from measures of apparent magnitude the distances of practically all clusters north of declination $-30°$, the southern limit reached with the 60-inch reflector.

Of 300 cluster photographs taken during the last three years as a part of the program, about 175 have been measured for the magnitudes of either the bright stars or the variables; but the material is too extensive to describe in detail. Nearly all of the photographs were made with full aperture and on Seed 27 plates of various emulsions. The exposures vary in length from 10 seconds to 2 hours, but are mostly between 1 and 12 minutes in duration. For the clusters south of $-20°$ the altitude was frequently so low that plates with first-class images could not be secured. Mr. Hoge has assisted with all the observational work.

A summary of the work on each cluster is given in Table II. The designation in the first column is followed in the second with numbers indicating the total number of plates used in all phases of the work and those used in the derivation of the mean magnitude. For the latter purpose the plates, with few exceptions, involve direct polar comparisons on two or more nights; the images of between 50 and 100 cluster stars and of between 20 and 50 Polar Standards were measured at least twice on each plate; and the measures were corrected as usual for scale, distance from center, and differential atmospheric extinction. As the measures, reductions, and discussion cannot well be given for the individual clusters, the method of work is illustrated merely with a summary of the final magnitudes for the bright stars in Messier 2 (Table III) and Messier 53 (Table IV).

The radius of the concentric region in which all bright stars were measured, given in the third column of Table II, does not closely indicate the apparent size of the cluster either actually

TABLE II

SUMMARY OF OBSERVATIONS

N.G.C.	No. Plates	Photographic Magnitude 25 Stars				Weight	Angular Diameter			
		Radius	Mean	Av. Dev.	Extremes		Melotte	Davis	Shapley	Mean
288	2, 2	4′	14.81	±0.16	14.38–15.04	*d*	12′	4′.0	4′.4	4′.2
1904	3, 2	2	15.29	0.13	15.01–15.72	*d*	4.5	2.4	2.2	2.3
4147	4, 2	1.5	16.58	0.18	16.23–16.93	*d*	1	0.8	0.8	0.8
5024	2, 2	5	15.07	0.09	14.94–15.26	*b*	5	5.6	5.4	5.5
5139	3, 2	....	(12.3:)	....	....	*d*	45	32	28	30
5272	65, 4	9	14.23	0.13	13.92–14.45	*a*	18	5.8	8.3	7.0
5904	3, 3	4	13.97	0.15	13.74–14.27	*a*	15	9.1	8.2	8.6
6093	2, 2	3	14.88	0.08	14.72–15.09	*d*	5	4.0	3.0	3.5
6121	3, 2	6	13.84	0.35	13.18–14.40	*c*	20	10.0	11.0	10.5
6205	15, 4	6	13.75	0.11	13.45–13.92	*b*	12	10.2	11.0	10.6
6218	2, 2	6	13.97	0.22	13.56–14.31	*b*	9	8.8	8.8	8.8
6229	3, 2	2.5	16.18	0.13	15.90–16.37	*d*	....	1.0	1.2	1.1
6254	2, 2	5	14.06	0.24	13.35–14.38	*c*	10	10.9	10.0	10.4
6333	2, 2	4	15.61	0.16	15.08–15.88	*b*	3	3.0	3.5	3.2
6341	3, 3	7	13.86	0.13	13.60–14.16	*c*	8	8.4	7.6	8.0
6356	2, 2	1.5	17.16	0.14	16.86–17.44	*d*	1.5	2.0	1.8	1.9
6402	2, 2	5	15.44	0.24	14.85–15.86	*b*	6	3.1	3.8	3.4
6626	3, 3	3	14.87	0.16	14.49–15.11	*b*	5	5.2	4.2	4.7
6638	2, 2	2	16.22	0.19	15.90–16.60	*b*	1.5	1.6	1.8	1.7
6642*	3, 2	1	16.07	0.26	15.51–16.46	*c*	1	1.2	1.2	1.2
6656	6, 3	5.5	13.08	0.19	12.58–13.55	*a*	20	14.5	17.5	16.0
6712	5, 2	1.5	16.10	0.19	15.65–16.36	*d*	2.5	2.2	2.0	2.1
6779	6, 2	2	15.31	0.20	14.98–15.70	*c*	1.5	2.2	2.6	2.4
6864	2, 2	1	17.06	0.13	16.76–17.35	*c*	2	1.6	1.5	1.6
6934	5, 3	3	15.78	0.19	15.33–16.11	*c*	1.5	1.4	1.5	1.4
6981	7, 2	3	15.92	0.17	15.53–16.20	*b*	2	2.2	2.4	2.3
7078	7, 3	6	14.31	0.11	14.13–14.55	*a*	6	4.8	5.2	5.0
7089	7, 3	4	14.61	0.09	14.25–14.76	*b*	8	7.3	6.8	7.0
7099	4, 3	2.5	14.63	±0.27	13.77–15.04	*c*	6	4.6	4.7	4.6

* See n. 1, p. 160.

or relatively. The basis of its choice has been described in the fifth section of the preceding paper.

TABLE III

Magnitudes of Bright Stars in Messier 2 (N.G.C. 7089)

Star*	Photographic Magnitudes				Residuals†	Deviation
	3883P	3902P	3904P	Means		
1	14.09	14.16	14.04	Bright	....	....
2	14.68	14.59	14.53	14.60	+ 8, − 1, − 7	0
3	14.65	14.64	14.70	14.66	− 1, − 2, + 4	+ 6
4	13.54	13.57	13.71	Bright	....	....
5	14.87	14.69	14.62	14.73	+14, − 4, −11	+13
6	14.77	14.69	14.65	14.70	+ 7, − 1, − 5	+10
7	14.73	14.69	14.65	14.69	+ 4, 0, − 4	+ 9
10	14.60	14.49	14.51	14.53	+ 7, − 4, − 2	− 7
11	14.80	14.72	14.70	14.74	+ 6, − 2, − 4	+14
13	14.68	14.69	14.57	14.65	+ 3, + 4, − 8	+ 5
14	14.65	14.57	14.61	14.61	+ 4, − 4, 0	+ 1
16	14.68	14.66	14.57	14.64	+ 4, + 2, − 7	+ 4
17	13.61	14.19	13.92	Bright	....	....
18	14.34	14.56	14.31	14.40	− 6, +16, − 9	−20
19	14.29	Contact‡	14.17	14.23	+ 6, ..., − 6	−37
20	13.54	13.78	13.70	Bright	....	....
21	14.68	14.69	14.57	14.65	+ 3, + 4, − 8	+ 5
23	14.77	14.69	14.57	14.68	+ 9, + 1, −11	+ 8
25	14.57	14.54	14.57	14.56	+ 1, − 2, + 1	− 4
26	14.65	14.72	14.57	14.65	0, + 7, − 8	+ 5
27	13.60	14.02	13.99	Bright	....	....
28	14.33	14.40	14.23	14.32	+ 1, + 8, − 9	−28
29	14.73	14.85	14.70	14.76	− 3, + 9, − 6	+16
30	14.57	14.72	14.46	14.58	− 1, +14, −12	− 2
31	14.73	14.72	14.57	14.67	+ 6, + 5, −10	+ 7
33	14.52	14.69	14.40	14.54	− 2, +15, −14	− 6
36	14.65	14.69	14.46	14.60	+ 5, + 9, −14	0
39	14.73	14.66	14.72	14.70	+ 3, − 4, + 2	+10
40	14.73	14.66	14.53	14.64	+ 9, + 2, −11	+ 4
43	14.65	14.59	14.49	14.58	+ 7, + 1, − 9	− 2
Mean				14.60	Mean	±0.09

* Stars fainter than the brightest thirty are omitted.
† Residuals and deviations from mean expressed in hundredths of a magnitude.
‡ Cluster star in contact with a Polar Standard.

For the average cluster the mean value of the photographic magnitude, in the fourth column, depends upon about 500 measures. Its estimated probable error varies from two- to four-tenths of a magnitude, the principal uncertainty coming from possible non-homogeneity of the clusters and from the error in choosing the area to be measured. The average deviation shows the dispersion

of the magnitudes entering the mean, but gives little indication of the certainty of the result; the extremes also show the dispersion, and the upper limit records the highest luminosity of the individual stars.

TABLE IV

MAGNITUDES OF BRIGHT STARS IN MESSIER 53 (N.G.C. 5024)

STAR*	PHOTOGRAPHIC MAGNITUDES			RESIDUALS†	DEVIATION
	2357P	2568P	Means		
2	15.03	14.98	15.00	+ 3, − 2	− 8
4	15.25	15.21	15.23	+ 2, − 2	+15
6	14.99	15.01	15.00	− 1, + 1	− 8
7	14.97	14.93	14.95	+ 2, − 2	−12
9	14.95	15.10	15.02	− 7, + 8	− 6
11	15.17	15.26	15.22	− 5, + 4	+14
12	15.10	15.26	15.18	− 8, + 8	+10
13	15.00	14.93	14.96	+ 4, − 3	−12
15	15.03	14.91	14.97	+ 6, − 6	−11
16	15.03	15.21	15.12	− 9, + 9	+ 4
17	15.03	14.88	Bright	....	....
18	14.50	14.57	Bright	....	....
20	15.27	15.21	15.24	+ 3, − 3	+16
22	15.15	14.93	15.04	+11, −11	− 4
24	14.87	14.57	Bright	....	....
26	15.05	15.01	15.03	+ 2, − 2	− 5
27	15.05	14.83	Bright	....	....
28	12.90	13.18	Bright	....	....
29	15.28	15.24	15.26	+ 2, − 2	+18
30	14.95	15.10	15.02	− 7, + 8	− 6
31	15.03	15.07	15.05	− 2, + 2	− 3
32	15.30	15.14	15.22	+ 8, − 8	+14
33	15.05	14.93	14.99	+ 6, − 6	− 9
34	15.15	15.33	15.24	− 9, + 9	+16
35	14.94	15.03	14.98	− 4, + 5	−10
36	15.05	15.10	15.08	− 3, + 2	+ 0
37	15.05	15.13	15.09	− 4, + 4	+ 1
38	15.05	15.03	15.04	+ 1, − 1	− 4
42	14.94	15.01	14.98	− 4, + 3	−10
45	14.94	15.04	14.99	− 5, + 5	− 9
Mean			15.08	Mean	±0.09

* Stars fainter than the brightest thirty are omitted.

† Residuals and deviations from mean expressed in hundredths of a magnitude.

Adding the difference between the brighter extreme and the mean magnitude to the adopted absolute value of the latter, we get the maximum brightness in each cluster. Thus we find that the highest photographic luminosity never exceeds magnitude −2.5 (unless some of the excluded five are actually cluster stars), and the maximum usually falls slightly below −2. Contrary to expectation, the

mean magnitude is nearer the brighter extreme in nearly one-third of the clusters, suggesting that among the giants the number of stars does not always increase regularly with decreasing luminosity.

In the seventh column of Table II weights are assigned each cluster on the basis of the quality of the plates, their number, the character of the surrounding stellar field, the certainty of the result for the mean magnitude, and other factors. The remaining columns are described later. Except to remark that the plates for N.G.C. 5139 (ω Centauri) were made with the 10-inch refractor and (because of the very low altitude) are of little value except as a check on Bailey's magnitudes, further space will not be taken for the extensive notes compiled relative to the peculiarities of individual clusters, the observations, the measures, and the investigations of errors.

The equatorial and galactic co-ordinates, the parallaxes, and the co-ordinates in space of the 28 globular clusters[1] for which magnitudes have been measured appear in Table V. For a few clusters the values of the parallax in the sixth column are taken from Table I and for the remainder are computed directly from the mean magnitudes of Table II.

If we plot the parallaxes derived from magnitudes against the diameters of the clusters, as recorded by Melotte,[2] a very definite progression of size with parallax is apparent. Melotte's estimates were made directly from the original Franklin-Adams chart plates; but there is no record of what accuracy was sought or what homogeneity may be expected in the results. Accordingly the diameters have been redetermined from the photographic copies of the plates with the special purpose in view of obtaining results as nearly comparable as possible for all parts of the sky. Measures of the diameters of the images were made independently by two observers, using a finely divided scale under low magnification. By estimating the

[1] N.G.C. 6642 is retained in the table as a twenty-ninth entry. Melotte, with some doubt, classifies it as a globular cluster; Bailey thinks that its stars, few in number, are involved in nebulosity. Mount Wilson plates show a few stars closely crowded, but almost certainly not forming a typical globular cluster. The group is in a rich galactic field.

[2] *Memoirs of the Royal Astronomical Society*, 60, Part 5, 1915.

TABLE V

PARALLAXES OF 28 GLOBULAR CLUSTERS FROM MAGNITUDES AND DIAMETERS

Designation		Position in 1900		Galactic		Parallax (Unit Is 0″.000001)			Distance (Unit Is 100 Parsecs)		
N.G.C.	Messier	R.A.	Decl.	Long.	Lat.	From Mag.	From Diam.	Adopted	Radial	Projected on Galactic Plane	From Galactic Plane
288	....	$0^h 47^m.8$	−27° 8′	214°	−88°	55	52	53	189	7	−189
1904	79	5 20.1	−24 37	195	−28	44	36	39	256	226	−120
4147	....	12 5.0	+19 6	227	+78	24	17	19	526	109	+514
5024	53	13 8.0	+18 42	307	+79	48	62	53	189	36	+186
5139	....	13 20.8	−46 47	277	+16	150	155	153	65	62	+ 18
5272	3	13 37.6	+28 53	8	+77	72	72	72	139	31	+135
5904	5	15 13.5	+ 2 27	333	+45	80	81	80	125	88	+ 88
6093	80	16 11.1	−22 44	320	+18	53	48	50	200	190	+ 62
6121	4	16 17.5	−26 17	319	+15	85	90	88	114	110	+ 30
6205	13	16 38.1	+36 39	26	+40	89	91	90	111	85	+ 71
6218	12	16 42.0	− 1 46	344	+25	80	82	81	123	111	+ 52
6229	....	16 44.2	+47 42	41	+39	29	20	23	435	338	+274
6254	10	16 51.9	− 3 57	343	+22	77	89	83	120	111	+ 45
6333	9	17 13.3	−18 25	334	+ 9	38	44	40	250	247	+ 39
6341	92	17 14.1	+43 15	35	+34	84	78	81	123	102	+ 69
6356	....	17 17.8	−17 43	335	+ 9	19	30	26	385	380	+ 60
6402	14	17 32.4	− 3 11	349	+14	41	46	43	233	226	+ 56
6626	28	18 18.4	−24 55	336	− 7	53	57	54	185	184	− 23
6638	....	18 24.8	−25 34	335	− 8	29	28	29	345	342	− 48
6642*	....	18 25.8	−23 32	337	− 7	31	22	26	385	382	− 47
6656	22	18 30.3	−23 59	338	− 9	118	116	118	85	84	− 13
6712	....	18 47.6	− 8 50	353	− 6	30	33	32	312	310	− 32
6779	56	19 12.7	+30 0	30	+ 7	43	37	40	250	248	+ 30
6864	75	20 0.2	−22 12	348	−28	19	26	22	455	401	−213
6934	....	20 29.3	+ 7 4	20	−20	35	24	30	333	313	−114
6981	72	20 48.0	−12 55	3	−34	33	35	34	294	244	−164
7078	15	21 25.2	+11 44	33	−29	68	59	68	147	129	− 71
7089	2	21 28.3	− 1 16	22	−37	60	72	64	156	125	− 94
7099	30	21 34.7	−23 38	356	−48	59	56	58	172	115	−128

*See n. 1 on p. 160.

elongation of similarly situated bright stars corrections have been made for distortion when a cluster is near the edge of the chart;[1] and to counteract the frequently observed actual ellipticity of figure the mean of the diameters in several directions has been determined for all clusters. The last three columns of Table II contain these closely agreeing measures and their mean, thus affording a fairly homogeneous record of relative apparent dimensions. The diameters estimated from the original plates are usually larger, as would be expected, but even they fall short of the actual dimensions for many clusters.

Plotting the adopted diameters of the last column of Table II against the parallaxes derived from magnitudes in Table V, we obtain the curve in Fig. 1—a curve that demands and apparently justifies the hypothesis that all globular clusters are of nearly the same linear dimensions. There is no reason for supposing that the more distant clusters are actually and systematically smaller than nearer ones; hence, as is reasonable a priori, the non-linear form of this curve may be attributed to the characteristic distribution of luminosity in a cluster and its resulting effect on photographic reproduction. Normal points for the data underlying the figure are given in Table VI and are plotted as black circles.[2] The lower part of the curve is somewhat uncertain, depending only on Messier 22 and ω Centauri, but the uncertainty is not important, for only two or three clusters are large enough to make their parallaxes depend on that part of the curve.

Undoubtedly an improvement can be made in at least some of the parallaxes through the process of smoothing with the aid of the

[1] Many of the clusters appear on two or more charts. The differences in quality from plate to plate, which may be greater for the charts, are not nearly so effective in measures of angular diameter as they would be in estimates of magnitudes. Fortunately about 90 per cent of the clusters are on the Johannesburg plates, which attain a fainter limit of magnitude and are more uniform than those made for the northern sky at Mervel Hill (*Memoirs of the Royal Astronomical Society*, **60**, 167, 1914).

[2] Half weight is given to N.G.C. 6642; also to N.G.C. 6712 because of a slight doubt as to its nature and because it is in such a rich region of the galactic clouds that the measured diameter is a little uncertain. Cf. *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Possibly a doubt should also be expressed as to the perfectly normal nature of N.G.C. 4147. Cf. the eleventh paper of this series.

parallax-diameter curve of Fig. 1. Accordingly, parallaxes corresponding to the measured diameters are entered in the seventh column of Table V and are combined with the parallaxes from magnitudes to obtain the smoothed values of the eighth column. The combination is made with due regard for the relative quality of the magnitude work, assigning for convenience weight unity

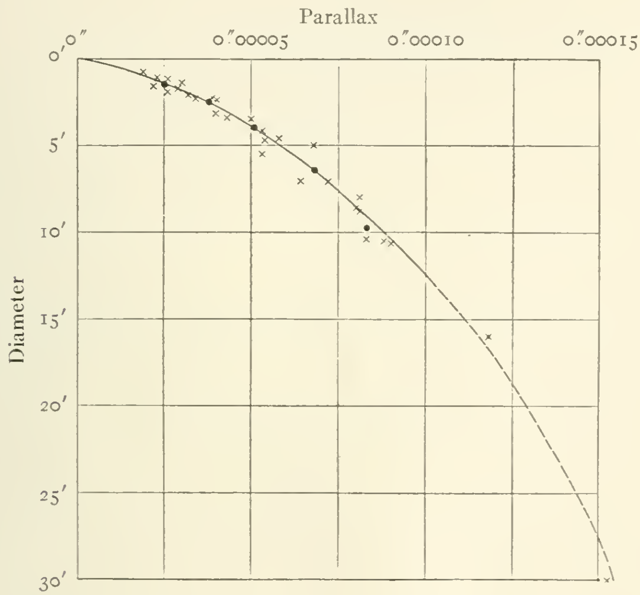


Fig. 1.—The parallax-diameter curve for globular clusters (diameters from Franklin-Adams charts). Dots are normals based upon parallaxes from magnitudes alone; crosses are the finally adopted values for the 29 clusters of Table V.

to all the parallaxes from magnitudes, and, to the corresponding values from angular measurements, the weight zero for group *a*, one-half for group *b*, one for group *c*, and two for group *d*. The adopted parallaxes are plotted in Fig. 1 as crosses.

Before obtaining the parallaxes of other clusters from their diameters alone, we shall note what accuracy may be expected in the results. In Table VII are given the percentage deviations from the parallax-diameter curve, both for the original parallaxes from magnitudes and for the adopted values. Without assigning

weights, the arithmetical[1] mean for the first is 14 per cent, and for the second, 7 per cent. The latter shows the average amount of the discrepancy that will affect the parallaxes of the clusters for which no magnitudes are available. The deviations are partly

TABLE VI

DIAMETERS AND PARALLAXES

Number of Clusters	Mean Parallax	Mean Diameter
7..............	0.″000025	1.′5
5..............	0.000038	2.5
5..............	0.000051	4.0
5..............	0.000068	6.4
5..............	0.000083	9.7
1..............	0.000118	16
1..............	0.000150	30

TABLE VII

DEVIATIONS FROM THE PARALLAX-DIAMETER CURVE

N.G.C.	$\frac{\pi \text{ Mag.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$	$\frac{\pi \text{ Adopt.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$	N.G.C.	$\frac{\pi \text{ Mag.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$	$\frac{\pi \text{ Adopt.} - \pi \text{ Diam.}}{\pi \text{ Adopt.}}$
288....	+0.06	+0.02	6356...	−0.42	−0.15
1904....	+ 20	+ 8	6402...	− 12	− 7
4147....	+ 37	+ 11	6626...	− 7	− 6
5024....	− 26	− 17	6638...	+ 3	+ 3
5139....	− 3	− 1	6656...	+ 2	+ 2
5272....	0	0	6712...	− 9	− 3
5904....	− 1	− 1	6779...	+ 15	+ 8
6093....	+ 10	+ 4	6864...	− 32	− 18
6121....	− 6	− 2	6934...	+ 37	+ 20
6205....	− 2	− 1	6981...	− 6	− 3
6218....	− 2	− 1	7078...	+ 13	+ 13
6229....	+ 39	+ 13	7089...	+ 19	− 12
6254....	− 14	− 7	7099...	+0.05	+0.03
6333....	−0.15	−0.10			

due to real differences in the clusters, but most of the error is within the uncertainty of angular measurement, for the average difference between the estimates of diameter by the two observers is but slightly less than 10 per cent.

Nearly all of the 41 clusters included in Table VIII are south of declination −30°. The few exceptions will be photographed with the 60-inch reflector, when opportunity permits, and the

[1] For brevity the terms "algebraic mean" and "arithmetical mean" are used in the sense of *with* and *without* regard to sign.

TABLE VIII

PARALLAXES OF 41 GLOBULAR CLUSTERS FROM DIAMETERS

N.G.C.	R.A. 1900	Decl. 1900	Galactic		Angular Diameter D, S	Parallax (Unit Is 0".000001)	Distance (Unit Is 100 Parsecs)		
			Long.	Lat.			Radial	Projected on Galactic Plane	From Galactic Plane
104	$0^h 19^m.6$	−72° 38′	272°	−44°	26′.0, 27′.6	148	68	49	− 47
362	0 58.9	−71 23	268	−46	5.4, 6.6	66	152	106	−109
1261	3 9.5	−55 36	238	−51	2.6, 2.5	39	256	161	−199
1851	5 10.8	−40 9	211	−34	4.8, 5.0	58	172	143	− 96
2298	6 45.4	−35 54	213	−15	3.0, 2.7	41	244	236	− 63
2808	9 10.0	−64 27	249	−11	5.2, 4.9	59	170	167	− 32
3201	10 13.5	−45 54	244	+10	6.6, 6.3	68	147	145	+ 26
4372	12 20.1	−72 7	269	− 9	10.8, 9.3	88	114	113	− 18
4590	12 34.2	−26 12	268	+37	4.8, 5.8	62	161	129	+ 97
4833	12 52.7	−70 20	271	− 8	5.6, 4.9	61	164	163	− 23
5286	13 40.1	−50 52	279	+11	3.6, 4.5	51	196	193	+ 37
5634	14 24.4	− 5 32	309	+49	2.2, 2.0	33	303	199	+229
5897	15 11.7	−20 39	311	+30	6.4, 6.2	67	149	129	+ 75
5986	15 39.5	−37 27	305	+13	3.3, 4.1	48	208	203	+ 47
6101	16 14.4	−71 58	284	−15	3.0, 3.7	47	213	206	− 55
6144	16 21.2	−25 49	319	+15	2.8, 2.8	41	244	236	+ 63
6171	16 26.9	−12 50	331	+22	5.8, 5.0	62	161	149	+ 60
6235	16 47.4	−22 1	326	+13	1.0, 1.2	20	500	487	+112
6266	16 54.8	−29 58	320	+ 7	6.4, 5.9	66	152	151	+ 19
6273	16 56.4	−26 7	324	+ 9	5.6, 5.4	63	159	157	+ 25
6284	16 58.4	−24 37	325	+10	1.4, 1.9	27	370	364	+ 64
6287	16 59.1	−22 34	327	+11	1.3, 1.4	23	435	428	+ 83
6293	17 4.0	−26 26	325	+ 8	2.9, 2.2	38	263	261	+ 37
6304	17 8.2	−29 20	323	+ 5	2.0, 1.9	31	322	320	+ 28
6316	17 10.3	−28 1	325	+ 5	1.0, 1.0	19	526	524	+ 46
6352	17 17.8	−48 19	308	− 7	3.4, 3.0	44	227	226	− 28
6362	17 21.5	−66 58	293	−17	7.3, 8.4	77	130	124	− 38

6388...........	17 29.0	−44 40	312	− 7	2.5, 2.2	36	278	276	− 34
6397...........	17 32.5	−53 37	304	−12	18.0, 16.2	120	83	81	− 17
6441...........	17 43.4	−37 1	321	− 5	1.3, 1.2	22	455	453	− 40
6541...........	18 0.8	−43 44	316	−11	6.8, 6.2	68	147	144	− 28
6584...........	18 10.6	−52 15	309	−16	2.4, 2.6	38	263	253	− 73
6624...........	18 17.3	−30 24	330	− 8	2.4, 2.2	35	286	283	− 40
6637...........	18 24.8	−32 25	329	−11	3.3, 3.6	47	213	209	− 41
6652...........	18 29.2	−33 4	328	−12	2.0, 2.0	32	312	305	− 65
6681...........	18 36.7	−32 23	329	−13	4.4, 4.6	55	182	177	− 41
6715...........	18 48.7	−30 36	333	−16	5.7, 5.2	62	161	155	− 44
6723...........	18 52.8	−36 46	327	−18	7.3, 9.2	79	127	121	− 39
6752...........	19 2.0	−60 8	303	−26	16.8, 14.4	114	88	79	− 39
6809...........	19 33.7	−31 10	335	−24	11.7, 13.2	100	100	91	− 41
7006...........	20 56.8	+15 48	32	−20	0.8, 0.7	15	667	626	−228

parallax, as determined from diameters, will receive an independent check from the study of magnitudes.[1] The right ascension, declination, and galactic co-ordinates in Table VIII, as in previous tables, have been taken, whenever possible, from Melotte's catalogue, the results being checked with Bailey's lists and other catalogues. The angular diameters in the fifth column were obtained at the same time as those measured for Table V; their mean value is the basis of the parallaxes in the sixth column. The first cluster of this list is 47 Tucanae.

Tables V and VIII contain all clusters now thought to be definitely globular—a total of 69. The 15 or 20 others[2] frequently

[1] Since writing the above, one highly satisfactory confirmation has been secured. The object N.G.C. 7006 was noted as a small faint cluster by Curtis (*Lick Observatory Bulletins*, **7**, 84, 1912). On the Franklin-Adams plates and charts it appears as a nebulous star and was excluded from Melotte's catalogue of globular clusters. It is not mentioned by Bailey. The apparent diameter as given in Table VIII is only 0.'75—the smallest cluster of the two lists. If our hypotheses are right, therefore, it should be the most distant cluster, and the mean magnitude should be fainter than any hitherto measured. Two polar-comparison photographs were secured in December 1917. A seven-minute exposure on a fast plate shows about two hundred stars brighter than magnitude 18.5, and one star near the center is nearly as bright as magnitude 15; but the mean of the 25 brightest, according to the preliminary measures, is 17.7. The corresponding parallax is a little more than 0."000014, differing by less than a millionth of a second from the value in Table VIII.

[2] Among the clusters thought by some to be globular are the following:

N.G.C.	Angular Diameter	Galactic Long.	Galactic Lat.	Remarks
371	2.'1	268°	−44°	In Small Mag. Cloud. Not typical, Bailey. If globular, it gives for the distance of the cloud $\pi = 0."000033$. Cf. Table I.
2660	1.5	234	− 2	Appears to be globular cluster, Melotte. If so, $R \sin\beta = -1400$ parsecs; $R \cos\beta = 39{,}700$ parsecs.
5466	5.4	10	+72	Apparently an open cluster, Shapley. If globular, $\pi = 0."00006$.
6496	1.5	315	−11	Probably a nebula. Bailey does not mention it.
6535	0.8	354	+10	A group of a few faint stars, Shapley.
6569	1.3	328	− 7	A nebula according to Bailey. If a globular cluster, $\pi = 0."00002$.
6760	1.0	3	− 5	Small cluster of very faint stars, Curtis, Pease. An open cluster, Shapley.
7492	3.5	21	−64	An open cluster, Curtis, Shapley. Very loose globular cluster, Melotte. If globular, $\pi = 0."00005$.

Bibliography: Bailey, *Harvard Annals*, **60**, No. 8, 1908; **76**, No. 4, 1915. Curtis, *Lick Observatory Bulletins*, **7**, 81, 1912; **8**, 43, 1913. Melotte, *Memoirs of the Royal Astronomical Society*, **60**, Part V, 1915. Pease, *Publications of the Astronomical Society of the Pacific*, **26**, 204, 1914. Shapley, *Publications of the Astronomical Society of the Pacific*, **29**, 186, 1917. Shaw, *Helwan Observatory Bulletins*, No. 9, 1912; No. 15, 1915; *Monthly Notices*, **76**, 105, 1915.

admitted to the catalogues are rejected temporarily on the basis of either the Mount Wilson plates or the published opinions of Bailey, Curtis, Shaw, or Melotte. Some questioned groups may be admitted later, and other faint objects now considered nebulous stars or open clusters or faint nebulae may be proved by the large reflectors to be globular clusters. But as far as systems containing stars brighter than the sixteenth photographic magnitude are concerned, the present work may be considered exhaustive.

II. DISTRIBUTION IN SPACE

Some striking features of the arrangement of clusters in space are brought to light by a study of the data in Tables V and VIII. Fig. 2 illustrates the distribution in three dimensions, showing on the plane of the Milky Way the galactic longitudes and projected distances, while the distances from the Galaxy are shown by vectors drawn to scale in the plane of the figure. Distances above (north of) the galactic plane are represented by full heavy lines drawn upward from black circular bases, those below by broken lines downward from open circular bases. To visualize the actual positions in space one needs only to imagine the full-line vectors standing erect on their bases and the broken lines hanging vertically from theirs; the arrow points are then at the positions of the clusters.

The most remote of all the clusters[1] is N.G.C. 7006 with a distance of 67,000 parsecs, the equivalent of 220,000 light-years; the clusters N.G.C. 4147, 6229, 6235, 6287, 6441, and 6864 (M 75) are nearly as far away. Fortunately, of these seven most distant systems six are within reach of the Mount Wilson reflectors. One-fourth of all globular clusters appear to be more distant than 30,000 parsecs (100,000 light-years); and one (N.G.C. 4147) is more than 50,000 parsecs from the plane of the Milky Way. ω Centauri and 47 Tucanae, with distances somewhat less than 7000 parsecs, are the clusters nearest to the sun.

The concentration into a limited interval of galactic longitude is conspicuous; the region from 41° to 195° is completely void of globular clusters. The mean value of all longitudes is 316°, or,

[1] See n. 1, p. 167, of this paper and the later discussion in the eleventh paper.

excluding the five largest and five smallest values, is 318°; but the mean is too much affected by widely divergent values, and the

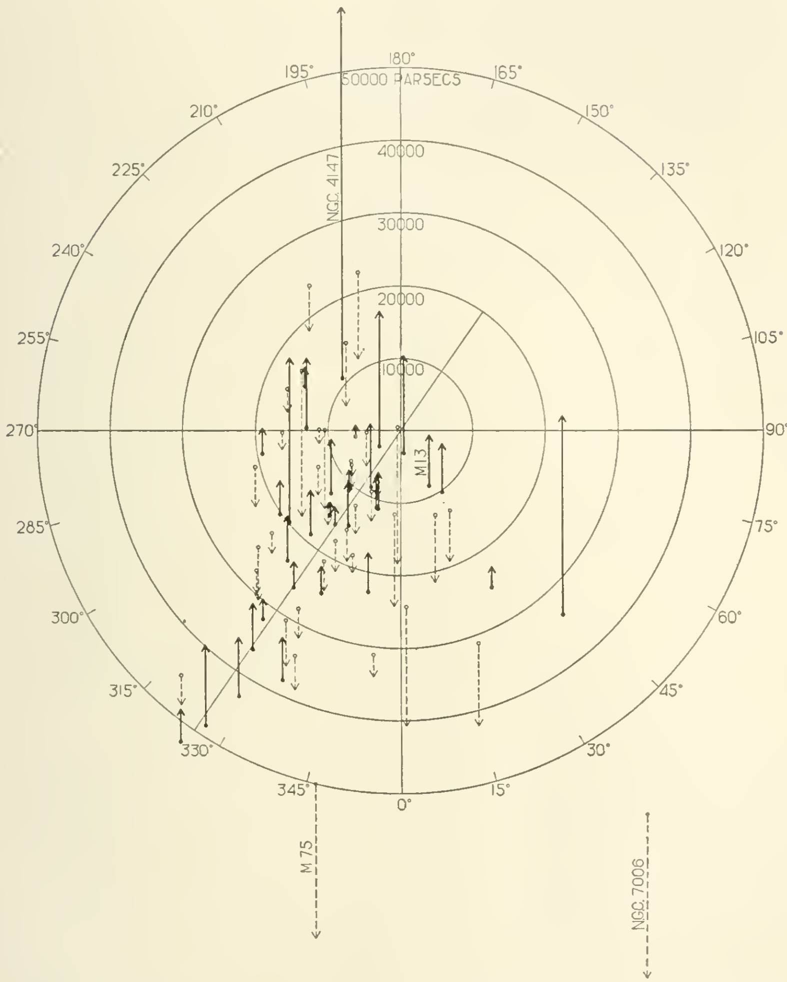


FIG. 2.—Distribution in space of globular clusters. The galactic plane is the plane of the diagram; distances above and below are shown to scale by full-line and broken-line vectors, respectively. Galactic longitudes are indicated in the margin and the scale of distances along the vertical radius. The sun is at the origin of co-ordinates. The diagram illustrates the remarkable distribution in longitude, with a maximum frequency at 325°, and by the absence of very small or zero vectors shows that globular clusters are not found within 1000 parsecs of the plane of the Milky Way. Cf. Fig. 1 of the twelfth paper.

median value,[1] 325°, is preferable in determining the central line of the system of globular clusters. The frequency of longitudes shown by Fig. 3 agrees in placing the center in longitude 325°, the points for the curve depending on the data of Table IX.

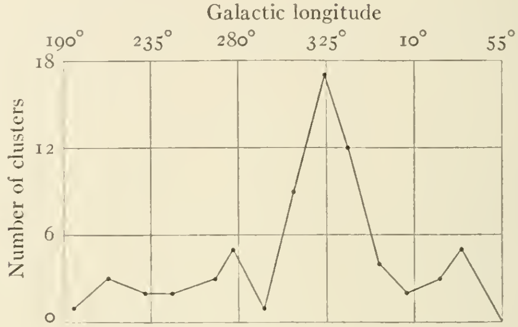


FIG. 3—Distribution of globular clusters in galactic longitude

TABLE IX

DISTRIBUTION OF GLOBULAR CLUSTERS IN GALACTIC LONGITUDE

Interval of Longitude	Number of Clusters	Mean Longitude
195° to 210°	1	195°
210 to 225	3	213
225 to 240	2	232
240 to 255	2	246
255 to 270	3	268
270 to 285	5	277
285 to 300	1	293
300 to 315	9	308
315 to 330	17	324
330 to 345	12	336
345 to 360	4	352
0 to 15	2	6
15 to 30	3	23
30 to 45	5	34

Projecting all positions on to a plane through the sun perpendicular to the Galaxy and including the circle defined by galactic

[1] That is, the longitude of the thirty-fifth cluster when they are taken in order of increasing longitude, beginning with N.G.C. 1904.

longitude 325°, we get the diagram in Fig. 4, which represents the distribution as seen from a great distance in the direction of galactic latitude 0° and galactic longitude 235°. Black dots above the central line represent clusters north of the galactic plane; open circles below represent those south. Thus the ordinates are $R \sin \beta$ and the abscissae $R \cos \beta \cos (\lambda - 325°)$, where R, β, and λ are respectively the distance, galactic latitude, and galactic longitude of a cluster.

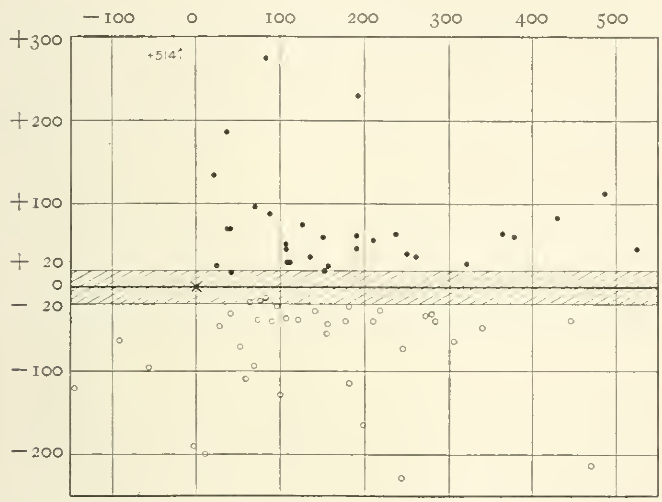


FIG. 4.—Projection of the positions of globular clusters on a plane perpendicular to the Galaxy, illustrating (1) the absence of clusters from the mid-galactic region, (2) their symmetrical arrangement with respect to the Galaxy, (3) the eccentric position of the sun (the cross) with respect to the center of the system of clusters. The ordinates are distances from the galactic plane, $R \sin \beta$; the abscissae are projected distances in the direction of the center, $R \cos \beta \cos (\lambda - 325°)$. The unit of distance is 100 parsecs; the side of a square is accordingly 10,000 parsecs. On this scale the actual diameter of the clusters is about one-fifth the diameter of the circles and dots. The cluster N.G.C. 4147 is outside the boundary of the diagram, as indicated by the arrow.

Fig. 5 differs from the preceding diagram only in having $R \cos \beta$ for abscissae. Hence the sun, as origin, is at the extreme left edge of the figure, and the actual distance of each cluster is represented by the radial distance from the origin.

Figs. 4 and 5 show more clearly than Fig. 2 the remarkable distribution of clusters with respect to the Galaxy. In the first place clusters are impartially distributed above and below, there being 32 north and 37 south of the plane. Although the average value without regard to sign is ± 79 (in units of 100 parsecs), the algebraic mean of all distances from the plane is -1; rejection of

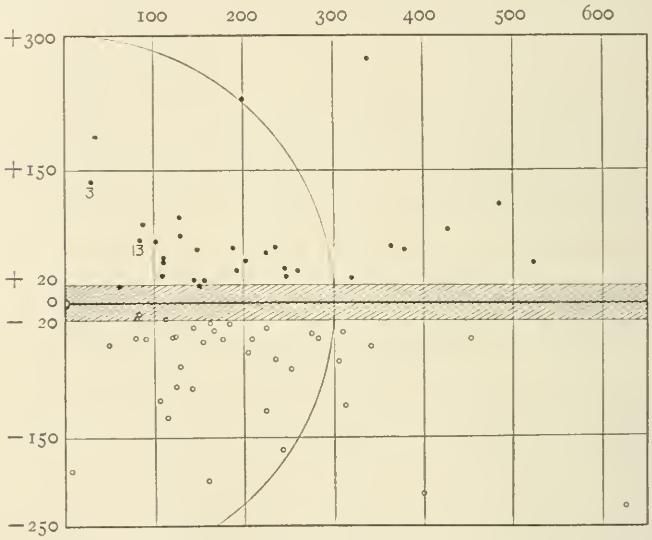


FIG. 5.—Distribution of globular clusters. The ordinates are $R \sin \beta$, as in Fig. 4; the abscissae are distances projected on the galactic plane, $R \cos \beta$. The unit of distance is 100 parsecs. The very small semicircle, with radius corresponding to a parallax of 0".002, illustrates the region around the sun which contains all but a few of the stars in Charlier's B-type cluster. The large semicircle indicates the distance to which the present results are thought to be complete. Messier 3 and Messier 13 are indicated by numbers; the most distant cluster now known, N.G.C. 7006, is near the lower right-hand corner of the diagram. N.G.C. 4147, with co-ordinates 109 and 514, is not shown.

the very distant cluster, N.G.C. 7006, would change this to $+2$. Considering the accidental variation and the size of the distances involved, the algebraic mean is vanishingly small, and we may say confidently that the plane of the Milky Way is also a symmetrical plane in the great system of globular clusters. This relation to the Galaxy holds with good approximation at all distances from the

sun (graphically shown best by Fig. 5), as may be seen from the following tabulation:

	Interval of $R \cos \beta$				
	0–100	100–200	200–300	>300	All
Number of Clusters	11	29	15	14	69
Mean* $R \sin \beta$ Algebraic	+7	+9	−23	−5	−1
Mean* $R \sin \beta$ Arithmetical	±70	±84	±59	±100	±79

*See n. 1, p. 164.

A second phenomenon clearly illustrated by the diagrams is the avoidance of the Milky Way—a result that may be of very exceptional significance. There is no cluster within 1300 parsecs of the plane of the Galaxy, and within 2000 parsecs of that plane there are only five, four of which are among the clusters nearest

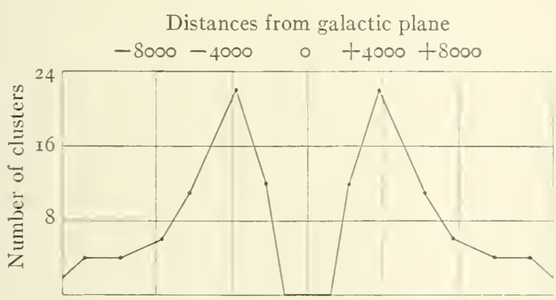


Fig. 6.—Reflected frequency-curve of the distances of globular clusters from the galactic plane, illustrating the equatorial region devoid of globular clusters. The unit of distance is one parsec.

the sun. In Figs. 4 and 5 a shaded region, 13,000 light-years in width, indicates the zone from which globular clusters are practically excluded.

The increasing concentration toward the Galaxy from both sides stops almost abruptly at the boundary of the shaded zone. The frequency of distances from the plane is treated in more detail in Table X, and the undoubted dependence of the clusters on the galactic plane, noted numerically above, is further emphasized by the curve in Fig. 6. The completion of that curve, in a form naturally to be expected for the frequency of objects concentrated toward the

TABLE X

THE FREQUENCY OF DISTANCES FROM THE GALACTIC PLANE ($R \sin \beta$)

Limits of $R \sin \beta$ (Unit Is 100 Parsecs)	$R \cos \beta < 200$					$R \cos \beta > 200$					All Clusters	
	No. of Clusters		Mean $R \sin \beta$			No. of Clusters		Mean $R \sin \beta$				
	North	South	North	South	All	North	South	North	South	All	Number	Mean $R \sin \beta$
< 10	0	0	....	....	....	0	0	....	....	....	0	....
10– 30	4	6	22	20	21	1	1	28	28	28	12	22
30– 50	3	8	37	40	39	5	6	40	39	39	22	39
50– 70	4	0	61	....	61	4	3	61	61	61	11	61
70– 90	3	1	78	71	76	1	1	83	73	78	6	77
90–110	1	3	97	100	99	0	0	....	....	....	4	99
110–130	0	1	....	128	128	1	2	112	117	119	4	118
130–150	1	0	135	....	135	0	0	....	....	....	1	135
150–170	0	0	....	....	....	0	1	....	164	164	1	164
170–190	1	1	186	189	188	0	0	....	....	....	2	188
190–210	0	1	....	199	199	0	0	....	....	....	1	199
210–230	1	0	229	....	229	0	2	....	220	220	3	223
> 230	1	0	514	....	514	1	0	274	....	274	2	394
Totals	19	21	....	....	....	13	16	....	....	....	69	....

Galaxy, would require at least 50 globular clusters within 1500 parsecs of the plane; there is, however, only one, Messier 22, and its distance below the plane corresponds to a parallax of 0."0008. It should be observed, moreover, that not only at great distances, where insufficiency of observations may be urged, do we note this absence of clusters, but also within a distance from the sun of 20,000 parsecs, where the data are quite sufficient (37 clusters) and undoubtedly are complete. Hence we conclude that this great mid-galactic region, which is peculiarly rich in all types of stars, planetary nebulae, and open clusters, is unquestionably a region unoccupied by globular clusters.

To explain this remarkable condition several hypotheses have been considered, such as error in choosing the origin of galactic latitudes,[1] incompleteness of data, general absorption of light in space,[2] clouds or a ring of absorbing matter along the spine of the Milky Way analogous to the dark peripheral rings of spiral nebulae,[3]

[1] The adopted position of the north galactic pole is that given by Gould (*Uranometria Argentina*): $\alpha = 12^h 41^m$, $\delta = +27°21'$. This position differs by less than a degree from those obtained through other reliable and definitive investigations. The most recent, and probably the best, is based on the Harvard Map of the Sky, by Nort, who finds $\alpha = 12^h 44^m$, $\delta = +27°$ (*Recherches Astronomiques de l'Observatoire d'Utrecht*, VII, 84, 112, 1917). An error of $\pm 1°$ in the cluster latitudes might slightly displace, widen, or narrow the zone of avoidance, but nothing short of selectively operative errors of several degrees could seriously obscure it. The frequency-curve of galactic latitudes also shows the zone, but naturally to a less degree. Cf. Fig. 2 of the twelfth paper. The values in Tables V and VIII show no latitude less than 5°. See the remarks relative to N.G.C. 2660 in n. 2, p. 167.

[2] See the notes in a following paper (the eleventh of the series) on the color of stars in the two most distant clusters. The dark obstructing nebulae which are frequently found in and near the Milky Way are undoubtedly capable of obliterating or greatly diminishing the light of any cluster involved in the nebulosity or beyond it. N.G.C. 4372, a large southern cluster (almost certainly globular), which is very faint for its angular diameter, falls alongside a vacant space in the sky. N.G.C. 6144 is near the edge of the ρ Ophiuchi dark nebulosity and appears large for the magnitude of its bright stars. This latter nebulosity may also affect the brightness of Messier 4 (N.G.C. 6121) to some extent. But it is interesting to note that in developing a parallax method that is independent of the magnitudes we have escaped from the errors in parallax that such obstructing material might occasionally have imposed.

[3] Possibly a hypothetical wedge-shaped ring might explain some of the divergence from the galactic plane with increasing distance (Figs. 4 and 5); but insufficiency of material for faint clusters would better account for most of it. The phenomenon, of course, may be real—a widening of the zone of avoidance in the direction of the

and finally the actual absence of globular clusters from the regions rich in stars because of the dynamical impossibility of existence. The first three seem clearly impossible, the fourth improbable or at least unquestionably insufficient, and, therefore, without going at present into the meaning and consequences of such a theory, the last hypothesis is tentatively adopted.

We have found that the center of the elongated and somewhat irregular system of globular clusters lies in the plane of the Milky Way on a line directed toward galactic longitude 325°. The distance along that line may be estimated from an inspection of Fig. 4, and perhaps obtained more accurately from the following consideration of the frequencies of $R \cos \beta \cos (\lambda - 325°)$, in which clusters more than 15,000 parsecs distant from the plane are excluded:

	Interval of Distance in Direction of Center								
	<100	-100 to 0	0 to $+100$	$+100$ to $+200$	$+200$ to $+300$	$+300$ to $+400$	$+400$ to $+500$	>500	All
Number of clusters	1	2	18	20	10	5	3	1	60
Mean of $R \cos \beta \cos (\lambda - 325°)$...	-145	-74	$+59$	$+138$	$+246$	$+342$	$+454$	$+525$	$+158$

From a plot of these numbers we estimate provisionally the distance of the center to be 13,000 parsecs. Incompleteness of data because of faintness will not materially affect the galactic longitude and latitude of this point, but is likely to make the distance too small. The mean value of $R \cos \beta \cos (\lambda - 325°)$ for the 60 clusters, $+158$, is probably nearer the true value, but it is also liable to understate the distance. A definitive value is hardly possible, and, at least until the number of very faint clusters can be considerably increased, we may adopt as the center of the general system of globular clusters a point for which the parallax is between $0''.00006$ and $0''.00004$, with equatorial co-ordinates $\alpha = 17^h 30^m$, $\delta = -30°$, and galactic co-ordinates $\lambda = 325°$, $\beta = 0°$. The position

center of the globular cluster system, combined with a lack of observations for clusters far beyond the center. See n. 2, p. 167; if N.G.C. 2660 is globular, the apparent tendency to widen with distance is somewhat counteracted.

lies in the constellation Sagittarius, a few degrees east of its boundary with Scorpio and Ophiuchus.

III. LINEAR DIMENSIONS OF CLUSTERS

A literal interpretation of the curve in Fig. 1 gives for the dimensions of globular clusters:

	Parallax (Unit Is 0".000001)						
	20	40	60	80	100	120	150
Angular diameter	1'.1	2'.7	5'.2	8'.3	12'.4	17'.2	27'.5
Linear diameter in parsecs.....	16	20	25	30	36	42	53

As remarked before, however, the diameters from the charts do not give a true measure of the clusters, and the hypothesis that size depends on distance from the sun, irrespective of distance from the Galaxy, is quite untenable.

There are, without doubt, real though relatively inconspicuous differences among globular clusters; and some of the differences, such as frequency of certain spectral types, degree of condensation and ellipticity, and possibly total numbers of stars, are being recognized and evaluated through the Mount Wilson studies. Two properties, however, so far seem to show little variation from cluster to cluster—the actual linear diameter and the mean magnitude of the brightest stars. We shall assume, therefore, on the basis of what observational evidence we now have at hand, that all globular clusters are of practically the same dimensions, explaining the apparent decrease in size with increasing distance (tabulated above) as a natural consequence of a central concentration of luminosity and of an intermingling near the edges with non-cluster stars. On that basis an investigation of the dimensions of one cluster will suffice for all.

Long exposures with the 60-inch reflector upon the outer parts of some of the brightest clusters have confirmed the conclusion, obtained from a study of the distribution of variable stars, that the clusters are much greater in extent than would be inferred from ordinary visual or photographic observation. For instance, on the

Franklin-Adams charts the apparent diameter[1] of Messier 3, the cluster chosen for the present illustration of dimensions, is 7′; on the original plates it is 18′, according to Melotte; but the actual diameter is in excess of half a degree.

The distance of Messier 3 is 13,900 parsecs,[2] corresponding to nearly three thousand million times the distance of the sun from the earth. The distance north of the galactic plane is 13,500 parsecs. The accompanying plate is reproduced from a photograph of several hours' exposure made by Mr. Ritchey with the 60-inch reflector. The original negative shows more than twenty thousand stars outside the central burned-out area, the smallest images being fainter than the twentieth magnitude.

The cluster extends beyond the limits of the photograph in all directions. The most distant variable star (undoubtedly a member of the system) is 17′ from the center, corresponding to a projected distance of fourteen million astronomical units. As ordinarily seen and photographed, the cluster covers an area but little larger than one of the squares, but we may be sure that its actual projected area is at least twenty-five times as great; that is, the diameter is about thirty million astronomical units. To cross the cluster, light must travel 470 years.

If we suppose the sun situated at the center of the cluster, all stars with parallaxes greater than 0″1 would be included within the concentric circle. Sirius would be at the distance indicated by the cross, and the projected distance of the bright triplet near the bottom of the picture equals the distance of the Hyades from the sun.

Inclosed in small circles are a few of the variable stars, chosen at random, the close equality of whose magnitudes is to be noted. In some cases they appear as doubles, but the actual separation

[1] The estimates from the charts, listed in Tables V and VIII, refer actually to what appears to be a central core of each system. The scale of the photographs does not permit close differentiation of the outlying members of a cluster from the stars of its surrounding field.

[2] The diagram was made on the basis of a parallax of 0″000074; the final value of Tables I and V indicates that the linear dimensions of the cluster on the plate are too small by 3 per cent, an amount, however, that is far within the probable error.

in the closest of the indicated pairs is more than half the distance separating the sun from α Centauri. In the small square is the image of an exceptional variable, No. 37, for which the period is less than eight hours.

The cluster variables, in the mean, have the absolute magnitude -0.2, photographically nearly six magnitudes brighter than the sun. A star of the brightness and color of the sun would not appear on the photograph, being nearly two magnitudes too faint (21.5). Sirius, located in this cluster, would be of the seventeenth apparent magnitude, corresponding to the star indicated on the photograph by an arrow-point.

The condensation of stars at the center of the cluster may be readily contrasted with that of stars around the sun. Within the circle, which marks a distance from the center corresponding to a parallax of $0''.1$ (approximately two million astronomical units), there are at least 15,000 stars brighter than magnitude 20. (This estimate deducts those stars not within the concentric sphere but appearing by projection within the circular area.) In a sphere of the same radius, with the sun as center, less than twenty stars brighter than the sun are known. But only those which are two magnitudes brighter appear on this photograph of the cluster; there are, accordingly, in the sphere around the sun only four or five stars to compare with the 15,000 in Messier 3.

Finally, we shall make some estimates relative to the probable mass of a globular cluster. We may go astray, to be sure, in assuming similar masses and analogous relations of mass to luminosity for stars in clusters and in the general galactic system. The dynamical conditions in these highly condensed globular systems may conceivably have some important effect upon the amount of matter that goes into a single star as well as upon the speed and nature of subsequent development. Eddington's recent theoretical work on the masses of stars, however, indicates that as long as we deal with typical giants the masses are definitely limited; and, further, the identity of Cepheid phenomena wherever studied—in the galactic system, in the more condensed Magellanic clouds, and in the extremely condensed globular clusters—tends to support the view of the universal comparability of stellar masses.

All that we know of the masses of stars from observation has been summarized recently by Russell.[1] The data come altogether from double stars, but we shall probably not commit serious error in applying the results without alteration to the single stars in the condensed globular clusters, although possibly slightly greater average values for each type would be appropriate for the isolated stars in our general galactic system. For the present approximation we shall simply take a mean value from Russell's data, say an average of four times the sun's mass for every star brighter than the sun. The number of such stars in Messier 3 may be fairly estimated at 40,000, only three-fourths of which have ever been photographed. Hence the entire mass, distributed among the stars which are brighter absolutely than photographic magnitude +5.6, is 160,000 times the solar mass; and something like three-fourths of this amount is within 10 parsecs of the center.

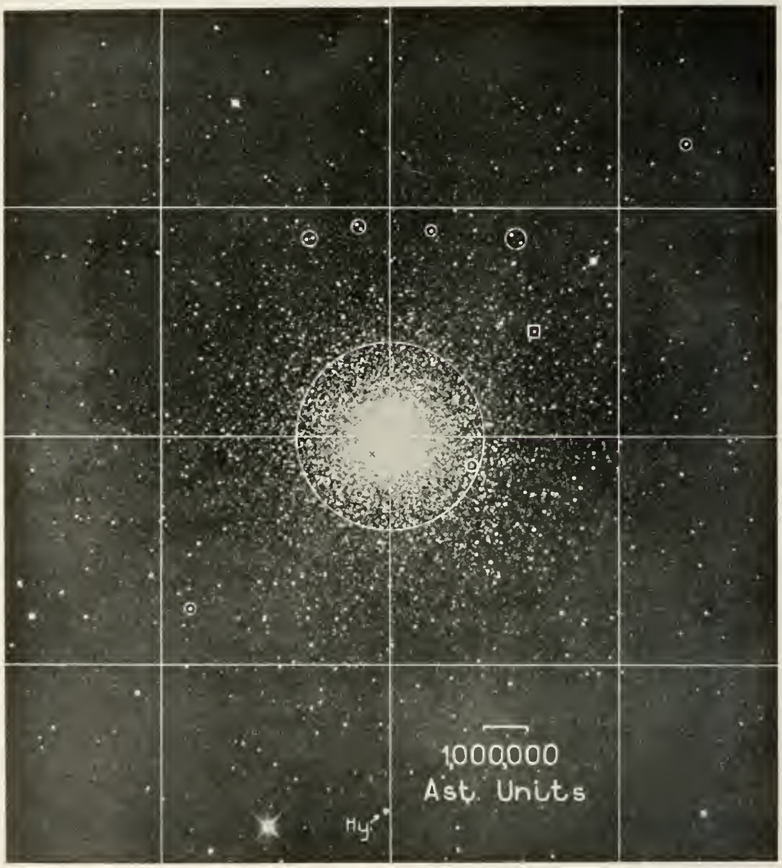
It seems quite futile at present to estimate the total mass of the cluster. Perhaps it is two or three million times the solar mass; probably it is several times the amount estimated for the stars brighter than 5.6—that we must admit on the basis of what little we now know of the relative frequency of dwarfs and giants in the vicinity of the sun; but as a matter of absolute certainty we do not know that there is a single star fainter than the sun, and even our estimate of 160,000 may be 50 per cent too great.

SUMMARY

1. Following the methods outlined in the preceding paper the parallaxes and positions in space are obtained for 69 globular clusters—all that can now be definitely assigned to the globular class. The distances range from 6500 to 67,000 parsecs, the brightest stars in the most distant clusters being fainter than the seventeenth photographic magnitude. The average probable error of a parallax is of the order of 20 per cent, varying considerably with the method used and with the quality of the observational work. Something more than 15,000 measures of magnitudes were made for one phase of the work. Section I contains various

[1] *Popular Astronomy*, **25**, 666, 1917.

PLATE IV



THE GLOBULAR CLUSTER MESSIER 3 (N.G.C. 5272)

The side of a large square is 5,000,000 times the distance of the earth from the sun. The radius of the concentric circle corresponds to a parallax of 0."1 (2,062,650 astronomical units). To cross the circle light must travel for sixty-five years. Small circles contain typical variables; the small square, variable No. 37. If the sun were situated at the center, the Hyades would be at the distance of the triplet at the bottom of the picture; Sirius would be at the distance of the black cross near the center. A star of the luminosity of Sirius is indicated by the arrow-point. Stars of our sun's brightness are nearly two magnitudes too faint to appear on the photograph.

items relative to the method of investigation, comparative accuracy, maximum luminosities, and the frequency of giants.

2. The study of the distribution of clusters in space brings out a number of remarkable features (cf. sec. II), the most significant of which appear to be the absence from the denser stellar regions of globular clusters and the final proof that they are subordinate to the general galactic system. The center of the system of globular clusters is found, with some uncertainty in one co-ordinate. A discussion of the part played in a general theory by the distribution of clusters is postponed to a following paper.

3. The derivation of parallaxes has permitted the discussion of the actual dimensions of clusters and a comparison with familiar distances near the sun. Plate IV sufficiently summarizes the result for a typical system, Messier 3, whose distance from the earth is of the order of 250,000 million million miles. The total mass of a typical globular cluster is estimated to be from a quarter to a half of a million times the solar mass, with much uncertainty as to the upper limit.

Mount Wilson Solar Observatory
December 1917

ALPHA CENTAURI AS A SPECTROSCOPIC BINARY

By JOSEPH LUNT

Alpha Centauri has received a very large amount of attention from double-star observers since Lacaille first measured the position, angle, and distance of the companion in 1752. The stars have made two complete revolutions since Lacaille's first measures, and their orbits have been investigated by many astronomers. In 1893 T. J. J. See,[1] then of the University of Chicago, and Dr. A. W. Roberts,[2] of Lovedale, South Africa, published the elements of the orbit independently. In the words of the latter "the results are almost identical . . . . but as the two sets of elements are independent of each other and were obtained by very different methods, the coincidence can only indicate that the elements found are very near the truth."

In Fig. 1 the orbits of the two stars are shown according to Roberts' elements, the ascending and descending nodes being inserted in their proper positions derived from later spectroscopic observations.

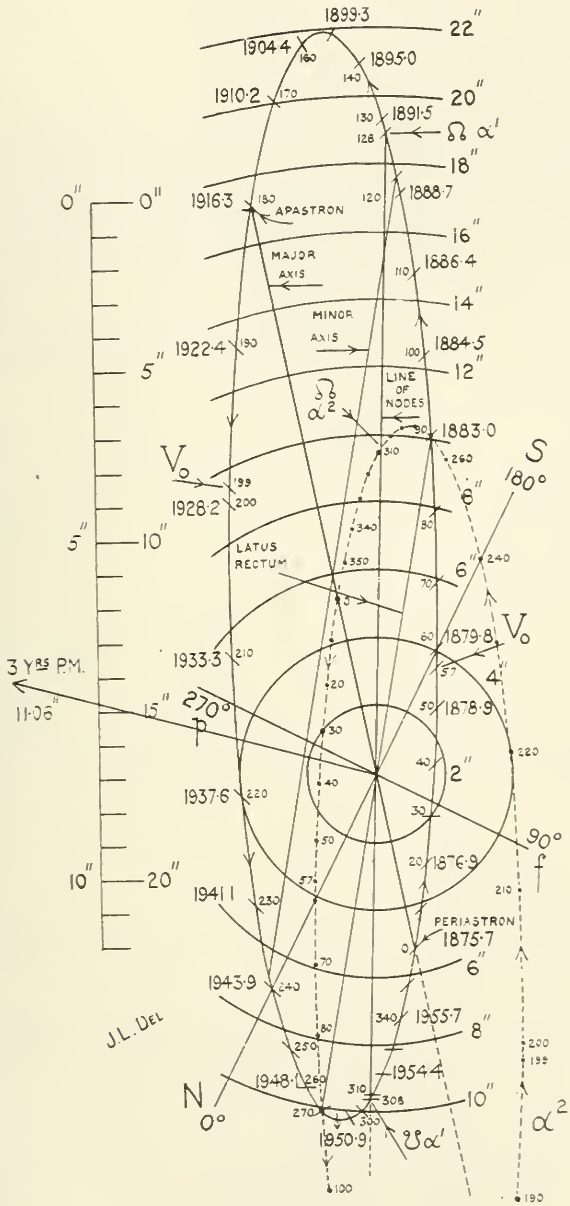
The continuous-line ellipse and the right-hand scale, as well as the concentric circles and arcs, refer to the orbit of the fainter star (α_1) with respect to the brighter star (α_2) regarded as fixed in the center of the concentric circles.

The left-hand scale and the two ellipses taken together refer to the two stars as moving around their common center of mass situated at the center of the concentric circles, the longitudes from periastron at any instant being the same for both stars.

V_0, at longitudes 57° and 199°, indicates the positions in the orbits where the orbital radial velocity becomes zero, and the ascending nodes are the positions in the orbits where the *receding* radial motion becomes a maximum. The right-hand side of the diagram is therefore the near side, and the left the far side, of the

[1] *Monthly Notices*, **54**, 115, 1893.

[2] *Astronomische Nachrichten*, **133**, 106, 1893.



1899·3
22″
1904·4
1895·0
1910·2
20″
1891·5
Ω α′
18″
1916·3
APASTRON
1888·7
16″
MAJOR AXIS
1886·4
14″
MINOR AXIS
1922·4
1884·5
12″
LINE OF NODES
Ω α²
1883·0
V₀
S 180°
1928·2
8″
LATUS RECTUM
6″
1933·3
1879·8
V₀
3 YRS P.M.
11·06″
270°
p
1878·9
1937·6
2″
90°
f
1876·9
1941·1
PERIASTRON
1875·7
1943·9
1955·7
8″
1954·4
J.L. Del
1948·1
N 0°
10″
1950·9
℧ α′
α²
0″ 0″
5″
5″ 10″
15″
10″ 20″

Fig. 1

orbits in space. The small figures represent longitudes from periastron in the orbits.

The figure has been drawn on the assumption of equal masses for the two stars. Roberts[1] gives the masses as $\alpha_2 : \alpha_1 = 51 : 49 \pm 2$ per cent. As the radial velocities are not appreciably affected at present by adopting 51:49 instead of equal masses, the latter value has been used throughout, as the greatest difference will be only 0.1 km in the positions of the maxima and minima of the velocity-curves.

Assuming α_2 and α_1 to be of equal mass, the semiamplitude of the velocity-curve[2] for both stars is

$$\frac{149500000\, a\, \pi \sin i}{365.25 \times 86400\ \pi'' P \sqrt{1-e^2}} = K$$

in nomenclature of spectroscopic binaries. The required data are

	Roberts	See	Doberck*
a	17″.71	17″.705	18″.165
i	79°21′36″	79°44′24″	79°19′
P	81ʸ.185	81ʸ.07	83ʸ.565
e	0.52865	0.52	0.52252

* *Astronomische Nachrichten*, **139**, 273, 1896.

Adopting Gill and Elkin's[3] value of the parallax, 0″.75, the values of K are

Roberts' elements	5.012 km/sec.
See's "	4.993 "
Doberck's "	4.971 "

The following elements based on Roberts' orbit have therefore been adopted

$$P = 29{,}652 \text{ days} = 81^{y}.18$$
$$T = \text{J.D. } 2406150 = 1875.71$$
$$\omega \begin{cases} \alpha_2 = 52^\circ \\ \alpha_1 = 232^\circ \end{cases}$$
$$e = 0.52865$$
$$K = 5.00 \text{ km/sec.}$$

[1] *Astronomische Nachrichten*, **139**, 10, 1896.

[2] See Campbell, *Astrophysical Journal*, **21**, 176, 1905.

[3] *Memoirs of the Royal Astronomical Society*, **48**, 188, 1884.

The orbital radial velocities of the stars will be:

α_2: $V_2 = +1.63$ km/sec. $+\frac{10}{2}\cos(v+52°)$

α_1: $V_1 = -1.63$ km/sec. $+\frac{10}{2}\cos(v+232°)$

where v is the true anomaly and $\pm 1.63 = Ke\cos\omega$. The orbital velocities at intervals throughout the paths have been computed and are given in the fourth column of Table I. Eliminating the orbital velocities from all the available observed radial velocities

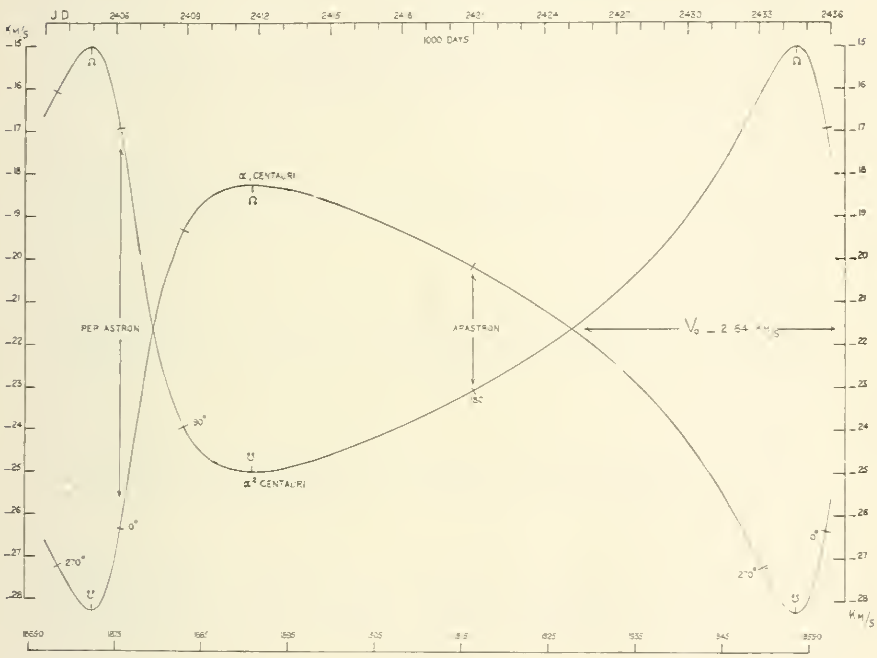


FIG. 2

given in Table II results in the weighted mean of the 28 values for the velocity of the system,

$$V_0 = -21.64 \pm 0.098 \text{ km/sec.}$$

Combining this value with the orbital motion gives the velocity ephemeris for the two stars found in the fifth and sixth columns of Table I and the velocity-curves of Fig. 2, where

$$\text{J.D.} = 2406150 + \frac{29652}{360} m$$

(m = mean anomaly).

TABLE I

α CENTAURI (EPHEMERIS)

TRUE ANOMALY	J.D. 2400000+	DATE A.D.	ORBITAL RAD. VEL. $\frac{\alpha_2}{\alpha_1}$	RADIAL VELOCITY		DIFFERENCE OF RAD. VEL.	REMARKS
				α_2	α_1		
			km	km	km	km	
0°	6150	1875.71	±4.71	−16.93	−26.35	9.42	Periastron α_2 and α_1
10	6360	76.30	3.98	17.66	25.62	7.96	
20	6589	76.91	3.18	18.46	24.82	6.36	
30	6817	77.54	2.33	19.31	23.97	4.66	
40	7060	1878.20	1.46	20.18	23.10	2.92	
50	7330	78.94	±0.59	21.05	22.23	1.18	
57	7536	79.51	0.00	21.64	21.64	**0.00**	Zero orbital radial velocity
60	7630	79.76	∓0.24	21.88	21.40	0.48	
70	7965	1880.68	1.02	22.66	20.62	2.04*	
80	8356	81.75	1.72	23.36	19.92	3.44	
90	8819	83.00	2.31	23.95	19.33	4.62	
100	9373	84.53	2.78	24.42	18.86	5.56	
110	10049	1886.39	3.13	24.77	18.51	6.26	
120	10886	88.68	3.32	24.96	18.32	6.64	
128	11698	90.90	3.37	25.01	18.27	**6.74***	☊ α_1 ☋ α_2
130	11926	91.53	3.37	25.01	18.27	6.74	
140	13210	1895.04	3.26	24.90	18.38	6.52	
150	14779	99.34	3.01	24.65	18.63	6.02	
158	16244	1903.35	2.70	24.34	18.94	5.40	
160	16635	04.42	2.61	24.25	19.03	5.22	
162	17038	05.52	2.52	24.16	19.12	5.04	
164	17450	06.65	2.42	24.06	19.22	4.84	
166	17871	1907.80	2.31	23.95	19.33	4.62	
168	18298	08.97	2.20	23.84	19.44	4.40	
170	18734	10.17	2.09	23.73	19.55	4.18	
172	19173	11.37	1.97	23.61	19.67	3.94	

174	19621	1912.59	1.84	23.48	19.80	3.68	
176	20072	13.83	1.72	23.36	19.92	3.44	
178	20522	15.06	1.58	23.22	20.06	3.16	
180	20976	16.30	1.45	23.09	20.19	2.90	Apastron a_2 and a_1
182	21430	1917.55	1.31	22.95	20.33	2.62	
184	21880	18.78	1.17	22.81	20.47	2.34	
186	22331	20.02	1.02	22.66	20.62	2.04	
188	22779	21.24	0.87	22.51	20.77	1.74	
190	23218	1922.44	0.72	22.36	20.92	1.44	
192	23654	23.64	0.56	22.20	21.08	1.12	
194	24081	24.80	0.40	22.04	21.24	0.80	
196	24502	25.96	0.24	21.88	21.40	0.48	
198	24914	1927.09	∓0.08	21.72	21.56	0.16	
199	25116	27.64	0.00	21.64	21.64	**0.00**	Zero orbital radial velocity
200	25317	28.19	±0.08	21.56	21.72	0.16	
210	27173	33.27	0.93	20.71	22.57	1.86	
220	28742	1937.57	1.80	19.84	23.44	3.60	
230	30026	41.08	2.67	18.97	24.31	5.34	
240	31067	43.94	3.50	18.14	25.14	7.00	
250	31903	46.22	4.28	17.36	25.92	8.56†	
260	32579	1948.07	4.98	16.66	26.62	9.96	
270	33133	49.59	5.57	16.07	27.21	11.14	
280	33596	50.86	6.04	15.60	27.68	12.08	
290	33987	51.93	6.39	15.25	28.03	12.78	
300	34322	1952.84	6.58	15.06	28.22	13.16	
308	34565	53.51	6.63	15.01	28.27	**13.26†**	☊ a_1, ☋ a_2
310	34622	53.67	6.63	15.01	28.27	13.26	
320	34892	54.41	6.52	15.12	28.16	13.04	
330	35135	1955.07	6.27	15.37	27.91	12.54	
340	35363	55.70	5.87	15.77	27.51	11.74	
350	35586	56.30	5.35	16.29	26.99	10.70	
360	35802	1956.90	±4.71	−16.93	−26.35	9.42	Periastron a_2 and a_1

* First maximum difference of orbital radial velocity.

† Second maximum difference of orbital radial velocity.

TABLE II

Year	J.D.	No. of Plates	Measures No.	Measures By	Standard Plate Used	Observed Rad. Vel.	Orbital Rad. Vel.	V_0 Reduced	Rad. Vel. Computed	Difference of Rad. Vel. O−C
					α_2 Centauri (brighter star)					
						km	km	km	km	km
1904.....	2416577*	3	3	W	....	−24.27	−2.62	−21.65	−24.26	−0.01
			4	L	T	24.43	2.60	21.83	24.24	−0.19
1904.....	16662	4	4	S	971	24.43	2.60	21.83	24.24	−0.19
			4	G	Z	23.95	2.60	21.35	24.24	+0.29
1906.....	2417345	7	7	S	971	22.55	2.44	20.11	24.08	+1.53
1906.....	411	4	4	S	971	23.01	2.43	20.58	24.07	+1.06
1906.....	443	14	13	H	971	24.17‡	2.42	21.75	24.06	−0.11
			13	H	T	23.70	2.42	21.28	24.06	+0.36
1907.....	626 †	17	17	H	971	24.18‡	2.37	21.81	24.01	−0.17
			16	H	T	23.90	2.37	21.53	24.01	+0.11
1907.....	793	14	14	H	971	23.97‡	2.33	21.64	23.97	0.00
			14	H	T	23.00	2.33	20.67	23.97	+0.97
1907.....	794	1	1	S	971	22.95	2.33	20.62	23.97	+1.02
1908.....	2418013	10	10	H	971	23.90‡	2.27	21.63	23.91	+0.01
1908.....	052	7	7	S	971	24.99	2.26	22.73	23.90	−1.09
1908.....	163	11	11	S	971	24.83	2.24	22.59	23.88	−0.95
1908.....	171		5	H	2145	25.07	2.23	22.84	23.87	−1.20
1909.....	497 §	3	3	L	⊙	23.97	2.15	21.82	23.79	−0.18
1912.....	2419581	1	1	L	⊙	24.24	1.86	22.38	23.50	−0.74
1912.....	635	4	4	H	2145	24.30	1.84	22.46	23.48	−0.82
1915.....	2420578	3	3	H	2145	−24.95	−1.56	−23.39	−23.20	−1.75
V_0 weighted mean.	....		......	......	....	....	....	−21.66		

	α_1 Centauri (fainter star)									
1904.....	2416578*	3	3	W	....	−19.10	+2.62	−21.72	−19.02	−0.08
1904.....	682 †		3	L	T	17.97	2.60	20.57	19.04	+1.07
1904.....	682 †	3	3	G	Z	17.47	2.60	20.07	19.04	+1.57
1909.....	2418497 §	3	3	L	⊙	18.33	2.15	20.48	19.49	+1.16
1911.....	2419172 §	3	3	L	⊙	20.63	1.97	22.60	19.67	−0.96
1912.....	632 §	3	3	H	2145	21.32	1.84	23.16	19.80	−1.52
1915.....	2420585 §	2	2	H	2145	−20.79	+1.56	−22.35	−20.08	−0.71
V_0 weighted mean .		....	......	......	....	....	....	−21.52		

α_2 and α_1, V_0 weighted mean adopted, -21.64 ± 0.098

W = Wright; L = Lunt; S = Simpson; G = Goatcher; H = Halm; T = Toepfer measuring machine; Z = Zeiss comparator; standard plates: 971, α_2 Centauri; 2145, α Tauri. ⊙ = Daylight.

* *Astrophysical Journal*, **20**, 141, 1904.

† *Annals of the Royal Observatory, Cape of Good Hope*, **10**, Parts I and III.

‡ A correction of −0.882 km/sec. has been applied to the published measures after remeasurement of the shift of the standard plate:
First standardization, 5 plates, JH, −2.012, *Cape Annals*, **10**, 40c.
Second standardization, 20 plates, JAS, -2.894 ± 0.093, *Cape Annals*, **10**, 50c.

§ Table III gives details of these measures.

REAL MOTION IN SPACE

Roberts[1] gives the following values for the annual proper motion:

In R.A.	$-7''.291 \pm 0.032$ (1880)
In Dec.	$+0.750 \pm 0.005$ (1880)

The proper motion is therefore $3''.685$ toward $281°45'$, the component in R.A. being $-3''.608$ and in Dec. $+0''.75$. The correction for the solar motion to proper motions in R.A., expressed in seconds of arc on a great circle, $= -3''.656\ \pi \cos \alpha$, and in Dec. $= +3''.656\ \pi \sin \alpha \sin \delta + 2''.111\ \pi \cos \delta$, where π is the star's parallax and $+3''.656$ and $-2''.111$ are the apparent proper motions, in R.A. and Dec. respectively, of a star with a parallax of $1''$ at 0^h in the equator, due to the solar motion assumed to be 20 km/sec. toward 18^h and $+30°$.

The corrected proper motions are therefore,

In R.A.	$-3''.608$	In Dec.	$+0''.750$
	$+2.165$		$+2.245$
	-1.443		$+2.995$

which combined give a corrected proper motion of $3''.325$ toward $334°16'$ (1880),

$$3.685 \cdot \frac{4.737}{\pi} = 23.27 \text{ km/sec.} = \text{apparent proper motion}$$

$$3.325 \cdot \frac{4.737}{\pi} = 21.00 \quad \text{``} \quad = \text{corrected proper motion}$$

$$2.27 \quad \text{``} \quad = \text{solar component}$$

where 4.737 = the velocity across the line of sight, in kilometers per second, of a star with $\pi = 1''.0$ and proper motion $= 1''.0$. The correction to radial velocities for the solar motion

$$= \eta \frac{\Delta Y}{3460.185} + \zeta \frac{\Delta Z}{3460.185}$$

$$= 0.0050055\eta - 0.00289\zeta .$$

[1] *Astronomische Nachrichten*, **139**, 10, 1896.

where ΔY and ΔZ are the reversed components of the solar motion, i.e., toward 6^h and $-30°$ respectively, expressed in kilometers per second; $\dot{\eta} = -c \sin \alpha \cos \delta$ and $\dot{\zeta} = -e \sin \delta$; c and 3460.185 = the factor for converting astronomical units per twelve hours into kilometers per second.

Adopting the value -21.64 km/sec. for the radial velocity of the system with respect to the sun, and the solar-motion correction -3.41 km/sec., the real motions are

$$\begin{array}{ll} \text{Radial} & -25.05 \\ \text{Transverse} & 21.00 \end{array}\Bigg\}\text{Combined } 32.7 \text{ km/sec.}$$

The system of α Centauri is therefore approaching the sun in a direction inclined $40°$ to the line joining the sun and star, with a velocity of 32.7 km/sec., regarding the sun as stationary. Combining the sun's motion, the velocity becomes 31.8 km, and the angle $47°$.

Alpha Centauri has been under observation spectroscopically only since 1904, and only a small part of the velocity-curve has been determined by observation. In Table II are collected all the observations available. It is self-explanatory.

Table III gives the individual measures not previously published, compared with the computed velocities.

The velocity-curve from See's elements in the region of the spectroscopic observations is practically identical with that from Roberts' elements, and that from Doberck's elements differs only from one- to two-tenths of a kilometer for the same region, Doberck's negative velocities being slightly larger for α_2.

The observations for α_2 Centauri given in Table II may be arranged in the groups indicated by brackets, in which the individual results are accordant.

The accordance within the groups is quite satisfactory, but the groups differ from each other somewhat markedly: they lie within a zone 3.28 km wide, with the velocity-curve almost central. Whether the divergencies from the theoretical velocities are due to instrumental causes or represent real changes has not been determined. The later measures show a distinct tendency to be too low, the mean difference O – C for the plates given in Table III

TABLE III

Plate No.	Date	Sidl. Time	Correction to Sun	Rad. Vel. Observed	Rad. Vel. Computed	Diff. O−C	Measured by
	α_2 Centauri						
			km	km	km	km	
1770	1908 July 24	$17^h 30^m$	−19.45	−24.79	−23.88	−0.91	JAS*
1773	25	15 20	19.50	23.78	23.88	+0.10	JAS
1774	25	15 39	19.52	24.12	23.88	−0.24	JAS
1775	25	16 3	19.54	25.44	23.88	−1.56	JAS
1811	Aug. 15	17 8	21.47	23.49	23.88	+0.39	JAS
				25.03	23.88	−1.15	JH
1812	11	17 48	21.50	25.71	23.88	−1.83	JAS
1815	12	17 0	21.53	25.69	23.88	−1.81	JAS
				25.27	23.88	−1.39	JH
1821	13	17 19	21.59	25.49	23.88	−1.61	JAS
				25.75	23.88	−1.87	JH
1845	21	17 25	21.81	24.56	23.87	−0.69	JAS
				24.56	23.87	−0.69	JH
1861	27	17 3	21.69	24.51	23.87	−0.64	JAS
				24.74	23.87	−0.87	JH
1862	27	17 19	21.70	25.54	23.87	−1.67	JAS
2398	1909 June 25	12 35	12.28	24.10	23.79	−0.31	JL
2417	July 16	16 2	17.84	23.32	23.79	+0.47	JL
2422	17	14 38	17.96	24.48	23.79	−0.69	JL
3674	1912 June 27	13 44	12.98	24.24	23.50	−0.74	JL
3759	Aug. 10	16 9	21.38	24.15	23.48	−0.67	JH
3760	10	16 28	21.39	23.49	23.48	−0.01	JH
3779	30	17 7	21.54	24.60	23.48	−1.12	JH
3780	30	17 36	21.55	24.96	23.48	−1.48	JH
4515	1915 Mar. 8	11 30	+21.14	24.26	23.21	−1.05	JH
4528	25	12 59	+18.10	24.97	23.20	−1.77	JH
4539	31	13 21	+16.65	−25.61	−23.20	−2.41	JH
Mean diff. O−C						−0.97	

			α_1 Centauri				
2399	1909 June 25	13 21	−12.32	−18.23	−19.49	+1.26	JL
2418	July 16	16 39	−17.86	18.41	19.49	+1.08	JL
2423	17	15 14	−17.99	18.35	19.49	+1.14	JL
3123	1911 May 5	14 56	+ 5.53	20.62	19.67	−0.95	JL
3140	15	13 46	+ 1.99	21.93	19.67	−2.26	JL
3151	26	13 34	− 2.02	19.35	19.67	+0.32	JL
3761	1912 Aug. 10	17 9	−21.40	20.46	19.80	−0.66	JH
3762	10	18 2	−21.44	21.56	19.80	−1.76	JH
3781	30	18 14	−21.57	21.94	19.80	−2.14	JH
4529	1915 Mar. 25	13 31	+18.04	20.09	20.08	−0.01	JH
4538	31	12 38	+16.68	−21.48	−20.08	−1.40	JH
Mean diff. O−C						−0.41	

JAS, standard plate, α_2 Centauri 971.

JL, solar (daylight) standards.

JH, standard plate, α Tauri 2145.

* Second Lieutenant J. A. Simpson, R.F.C., killed in action, France, October 23, 1916.

being -0.97 and -0.41 km per second for α_2 and α_1 respectively. The velocity of the system adopted, viz., -21.64 km. gives velocities agreeing with Wright's 1904 observations almost exactly but is half a kilometer higher than his later value -22.18 km.[1]

It will be seen from the figures that the distance between the two stars is slowly decreasing until 1937.6, when they will be 4″ apart; the distance will then increase until 1951, when they will be over 10″ apart, after which time the distance will diminish to less than 2″ in 1959.

The difference of radial velocity is gradually diminishing and will be zero in 1927, when spectroscopic observations will be very desirable, as both stars will have the radial velocity of the system. After that date the difference of radial velocity will rapidly increase until 1953, when the difference will be over 13 km/sec.

Royal Observatory, Cape of Good Hope
May 16, 1918

[1] *Lick Observatory Bulletin*, **6**, 27.

THE NATURE OF A SUPPOSED VARIATION IN THE SOLAR ROTATION IN 1915

By RALPH E. DeLURY

In a brief note[1] evidence was presented which indicated that the conclusion[2] drawn by Mr. H. H. Plaskett "that the sun, during the summer of 1915, underwent a cyclic variation in its rotation rate of 0.15 km . . . . completed in about a month," was not justified, and that this variation in spectroscopic measurements of the velocity of rotation of the sun's equatorial limbs might be due to changes in terrestrial haze. Mr. Plaskett has criticized[3] this explanation without testing my suggestion that remeasurements or an analysis of old measurements should show (on account of the presence of blended spectrum of haze in the observations in question) smaller displacements of weak than of strong spectrum lines, particularly on the days when the lowest values of the rate of rotation were obtained, i.e., on the days of greatest haziness. In what follows new evidence confirming this explanation is given along with an elucidation of the criticisms, which are referred to by their numbers.

1. From a considerable number of observations with respect to haze Mr. Plaskett selected three of my observations, implying that these were all I had made. He places these observations of obviously changed conditions beside his measurements of plates made some hours previously, implying in the words "low" and "high" that the haze explanation has been contradicted, thus:

During the period under consideration, namely, June 21 to August 16, DeLury made three entries in the observation book with reference to haze:

June 21, "Bright"	$V=1.911$ (low)
July 11, "Very hazy"	$V=1.975$ (high)
August 16, "Bright, some water-vapor haze"	$V=1.977$ (high)

[1] *Astrophysical Journal*, 44, 198, 1916.

[2] *Ibid.*, 43, 156, 1916.

[3] *Ibid.*, 45, 144, 1917.

Instead of this he should have adopted the logical procedure of juxtaposing as follows his own observations with reference to haze and the respective values of the velocity of rotation, finding in accordance with the haze explanation that low values of the measurements of solar rotation are associated with haze and higher values with "no haze," thus:

	V in km per sec.
June 24, "B, 3–4, A little haze near sun"	1.846
July 4, "B, 3–4, Haze"	1.893
July 20, "B, 4, No haze"	2.026
July 23, "B, 4–5, Positively no haze"	2.003
August 16, "B. 4, Practically little or no haze"	1.977

That he committed an error in setting his measures of rotation over against my records of changed conditions made later in the day is apparent from a consideration of Table I, where his and my measurements for these three selected days are associated with their respective records of brightness, the observations being in harmony with the haze explanation.

TABLE I

Date	Plaskett			DeLury			
	Time	Brightness	Km per sec.	Time	Brightness	λ	Km per sec.
June 21....	6:45– 8:00	4	1.911	10:18–10:38	Bright	6450	2.007
July 11....	7:00– 7:19	4, "then hazed up"	1.975	9:26–10:58	Very hazy, cirrus clouds	5900	1.785
				11:37–11:49	Brighter, still hazy	5600	1.805
August 16..	8:40–11:49	4, "practically little or no haze"	1.977	12:47–12:54	Bright, some water-vapor haze	5600	1.939

It is significant that on June 21, when he obtained a value of 1.911 km per sec.—one of the low values in the "cyclic variations"—I obtained a few hours afterward 2.007 km per sec., a value about equal to the maximum values in the supposed variation. Probably the record "Bright" indicated a greater brightness than "B, 4" of that date.

On account of the haze which developed after Plaskett's plates were taken on July 11 and which remained throughout my observations these latter (14 plates at λ 5900 and 7 at λ 5600) yield decidedly lower values than his.

On August 16, probably on account of the presence of "some water-vapor haze," my observations (7 plates at λ 5600) yield a value 2 per cent lower than his.

All these results are ascribable to different degrees of haziness in accordance with the records. Further details are given in Table II.

Regarding his record of brightness Mr. Plaskett states:

5 represents a very brilliant day—rare in Ottawa.
4 represents a bright day—normal observing weather.
3 represents a day slightly hazy.

These statements are self-contradictory, presuming that such different degrees of brightness postulate different degrees of haziness. On no day was haze absent. At that time our records of observing conditions were incomplete in that no account was kept of the strength of the spectrum of haze. During the past two years on no day has it been impossible to see the lines in the spectrum from haze just outside the solar limb (at λ 5600, third order, 7 m auto-collimating [5 cm×8 cm] grating spectrograph, slit 0.05 mm, fed by the 45 cm concave mirror, 24.5 m focus). Usually the lines are distinctly visible. Again he states, "Plates taken on days hazier than 3.5 were *not* measured." This is a misreading of the record, as 14 of the 111 plates measured by Plaskett were taken at recorded brightness "3."

Plaskett also asserts: "The record, if it shows anything, shows that in general high values were obtained on hazy days and low values on brighter days, in direct contradiction to DeLury's statement."

That this conclusion is fallacious is seen from the following complete summary of his observations (though it is something to have the admission that there were hazy days). The measurements of the 111 plates may be grouped with regard to brightness "above normal," "normal," and "below normal," using Plaskett's definition of a normal day given above:

	km per sec.
19 plates, B, "5" and "4–5," mean velocity	1.975
50 plates, B, "4," i.e., "normal," mean velocity	1.958
42 plates, B, "3–4" and "3," mean velocity	1.942

It may be noted that 42 of the 111 plates were made on days of brightness below "normal," i.e., on days of haze presumably above normal. Also 43 of the 111 plates were taken at brightnesses recorded as "4–5" and "3–4," terms so vague as to indicate either uncertainty in the estimates of brightness or varying degrees of haziness. When such uncertainty in the records exists, the support which the summary gives to the haze explanation is as satisfactory as can be expected.

The 19 plates of the first group include 15 at "B, 4–5," mean velocity 1.989 km per sec., and 4 at "B, 5," mean velocity 1.919 km per sec. The accuracy of this high record of brightness (the only one of "B, 5") may be questioned, as it was the first made (on July 9) by Plaskett after the three mirrors of the coelostat telescope had been freshly plated (July 7), and it is probable that the much brighter image of the sun produced as a consequence gave the observer an exaggerated impression of the clearness of the sky. If the day were in reality hazy, then the plates should satisfy the criterion for blended spectrum of haze. I have measured the 4 plates and find a difference of 0.054 km per sec. between the values of the velocity of rotation deduced from groups of weak and of strong spectrum lines—a difference which may be interpreted as indicating the presence of a considerable degree of haziness (see Table II). Hence it is reasonable to suppose that record "B, 5" for the brightness during his observations on July 9 is too high relative to the other records and for the cause indicated.

From the foregoing summary of his records and measurements it is quite evident that the following statement which appears in his original paper (p. 147) is not justified: "Observations were never made except on days that were free from haze."

2. Mr. Plaskett states in his note: "DeLury bases his discussion on plates made on two dates, June 24 and July 20, which are described as 'Hazy' and 'Bright,' apparently from memory, as it is not so recorded in the observation book."

On June 24 he recorded "B, 3–4, a little haze near sun," at 7:40 A.M. before his two observations, and "B, 3–4" at 9:10 A.M. after these observations. Memory need not be appealed to for the description "Hazy" for these conditions. On July 20 he recorded

"Bright (4)" three times for intervals 6:23–7:08. 8:03–8:24, and 8:38–8:43 A.M.; also, "No haze." I observed from 9:54–10:11 A.M., allowing these records to stand until a change occurred just after this, when I recorded that it "clouded up." Hence the term "Bright" is justified, "4" representing, according to him, a "bright day, normal observing weather."

In the following two misstatements are made: "On June 24 DeLury's plates were taken through clouds whose presence he notes in the observation book, whereas the writer's plates, exposed earlier, were taken with a clear sky." On this date my measured plates at λ 4230 were taken at 10:18–10:54 A.M., and though I made observations up to noon I did not record "clouded up" until afternoon. Absolutely no mention of clouds was made before noon, so that the statement that my "plates were taken through clouds" is not correct. Also, as shown above, on June 24 he described the conditions as "a little haze near the sun" and "B, 3–4," so that the term "clear sky" by his own definition is incorrect.

In addition to the direct statement of the records as to the relative brightness of the two selected dates, June 24 and July 20, remeasurements of my plates at λ 4230 and measurements by myself of his plates at λ 5900 yield further confirmation of my explanation, for on June 24 the differences between groups of measurements of weak and strong lines are greater than for the same lines on July 20, pointing to a greater degree of haziness on the former date (see Table II).

TABLE II

Date	Time	Brightness	No. of Plates	λ	No. of Lines	Mean Intensity	Velocity in km per sec	No. of Lines	Mean Intensity	Velocity in km per sec	Difference in Velocity
June 24	8:17– 9:10	3–4	2	5900	11	3.7	1.760	3	11	1.855	0.095
	10:18–10:54	....	7	4230	2	2.5	1.714	1	20	1.857	0.143
July 9	8:03– 8:22	5 (mirrors replated)	4	5900	11	3.7	1.888	3	11	1.942	0.054
July 11	10:25–10:58	Very hazy	14	5900	11	3.7	1.767	3	11	1.852	0.085
	11:37–11:49	Brighter, still hazy	7	5600	3	0.3	1.783	3	5.3	1.826	0.043
July 20	6:23– 8:03	4, no haze	4	5900	11	3.7	1.994	3	11	2.022	0.028
	9:54–10:11	....	5	4230	2	2.5	1.982	1	20	2.004	0.022
Aug. 16	12:47–12:54	Bright, water-vapor haze	7	5600	3	0.3	1.939	3	5.3	1.939	0.000

Table II gives in detail the new measurements discussed above. They are all in harmony with the haze explanation. (My 14 plates

at λ 5900, July 11, yielded a velocity of −0.006 km per sec. for the means of 4 atmospheric lines, showing that little error need be feared when the proper precautions in focusing the plate and in illuminating the grating evenly are taken.)

Regarding the possibility of errors of measurement in the results given in my note Mr. Plaskett says: "The difficulties of measurement of broad lines increase the errors and chances of prepossessions, especially with an observer whose last published measures show a probable error for even well-defined lines of 0.06 km, i.e., of the same order as the differences measured."

There are no "chances of prepossessions," for in all of my spectroscopic measurements there is used a movable mask,[1] which, by permitting only one strip of spectrum to be seen at a time, keeps hidden not only the magnitude but also the direction of displacement of the spectrum lines being measured.

In the series referred to the average probable error was 0.052 km per sec. (not 0.06), and in this very series numerous repeated measures of a certain plate have previously shown that about half of this "probable error of a single line" is due to actual differences for different lines on the plate. Also the plates used in the series referred to are of coarser grain and the definition of the lines is poorer than for the plates at λ 4230, referred to in my note.

In Table II are given the means of six measurements of the latter plates (λ 4230, June 24 and July 20), including the single measures reported in my note—three violet right and three violet left. The former single measure differs from these means by 0.002 and 0.013 km per sec. for the Fe lines, and 0.063 and 0.021 km per sec. for the broad Ca line; or, expressed in millimeters, these quantities are 0.0001, 0.0005, 0.0024, and 0.0008. The differences in velocity between the Ca and the Fe lines, namely, 0.143 and 0.022 km per sec., point to an even greater effect of blended spectrum of haze than was supposed from the differences 0.082 and 0.014 km per sec. given in the previous note.

It bears strictly on the whole question to compare these differences, based on measures of a single broad line, with Mr Plaskett's

[1] *Journal of the Royal Astronomical Society of Canada*, 5, 405, 1911.

two-way measurements of *five* solar lines on 111 plates, which when repeated yielded an average difference between the means of 0.055 km per sec., of which differences 18 exceeded 0.100 km per sec. and 6 have a mean of 0.152 km per sec. of the same magnitude as the "cyclic variation of 0.15 km per sec."

The differences derived from the means of the new measurements of the λ 4230 plates of June 24 and July 20 effectively annul the following argument (even if we overlook the fact that it is fallacious to argue as he has done from my measurements[1] at λ 5200 to those of other lines at λ 4230 without previously comparing the extent of the change in character of the two sets of lines in passing from the center of the solar disk to the limbs):

> The measures show evidence, according to the criterion of varying velocity for different line intensity, of about 8 per cent haze. This, on his own hypothesis, should produce about an 8 per cent difference in velocity on the two dates. The difference in mean velocity is actually 12 per cent, leaving a 4 per cent change in velocity to be accounted for.

3. The arguments as to the differences in value of the rotation for low sun and slightly higher sun are based on "meager and inconclusive" data, and the quantities derived are so very small (a few thousandths of a km per sec.) that they are without meaning when such large errors of measurement are present. A very large series of observations of the solar rotation from early morning to late afternoon would give evidence as to the daily development (usually increasing at Ottawa) of haze during that day, and if many days were likewise considered a valuable statement could be made as to the daily development of haze at the locality of the observations. No such measurements could possibly cast doubt on the haze explanation unless at the same time contradictory evidence of the changes in haze were obtained by other reliable means.

It may be said, in conclusion, that the facts presented above make the claim that the sun's rate of rotation varied in 1915 seem very unreasonable. All others who have made spectroscopic investigations of the solar rotation have found variety in their measures, and some have suggested the possibility that the rate

[1] *Astrophysical Journal*, 44, 184, 1916.

varied, e.g., C. A. Young,[1] Halm,[2] Evershed and Royds[3], St. John, Adams, and Ware.[4] However, Halm's low values in 1902–1903 can be readily accounted for by the haziness produced by the volcanoes in the West Indies in 1902, which caused a marked lessening in the values of solar radiation. The low values obtained in 1912–1913 by Evershed and Royds may have been due to the Katmai eruption in June 1912, which also caused a great lessening in the measures of solar radiation. The low values of rotation obtained at Mount Wilson and Ottawa in 1915 also seem to synchronize with low values of radiation, i.e., it may be presumed, with high values of haziness. Apart from haze various instrumental and physical causes produce variations in the measurements of solar rotation. When the effects of all known causes (haze, local convections in the sun, errors of measurement, etc.)[5] are eliminated or accounted for, it probably will be possible to assert that the rate of the sun's rotation does not appreciably vary.

The foregoing may be summarized briefly as follows:

1. The arguments which Mr. H. H. Plaskett advances (in his note criticizing my explanation of his observations of a supposed "cyclic variation" in the sun's rate of rotation in 1915, as due to variations in terrestrial haze) are based on fallacious use, or inaccurate statement, of the records and the measurements.

2. A synopsis of Mr. Plaskett's record of observations and measurements of the solar rotation supports the conclusion that in general high values of the rotation were obtained during brighter conditions and low values during hazier conditions.

3. Measurements of Mr. Plaskett's plates, as well as of mine, satisfy the criterion of blended spectrum of haze in agreement with the record of observational conditions.

4. The *known* terrestrial sources of blended spectrum offer a reasonable explanation of the supposed variation in rotation, so

[1] *The Sun* (1882), p. 100.

[2] *Transactions of the Royal Society of Edinburgh*, **41**, 89, 1904; and *Astronomische Nachrichten*, **173**, 287, 1906.

[3] *Monthly Notices*, **13**, 554, 1913.

[4] *Popular Astronomy*, **23**, 641, 1915.

[5] DeLury, *Astrophysical Journal*, **44**, 178, 1916.

that it is unnecessary to employ the *hypothetical* element of my explanation, which supposes an interplanetary or solar source of blended spectrum.

5. At present there is no sound reason for believing that the rate of the sun's rotation is variable.

I wish here to thank the director, Dr. Klotz, for his approval of the publication of this note.

SOLAR PHYSICS DIVISION
DOMINION OBSERVATORY, OTTAWA
May 1918

ERRATA

Vol. 44, November 1916, "On the Orbits of the Spectroscopic Binaries α Orionis and α Scorpii," by Joseph Lunt:

Page 251, ninth line, *for* what *read* that.

" 257, Table II, last column, third entry, 0.00 *align with* Mean, 2 plates.

Vol. 46, September 1917, "Preliminary Examination of the Planetary Nebulae for Preferential Motion," by C. D. Perrine:

Page 176, tenth line of Table II, *for* B, vS *read* B, S

Vol. 46, November 1917, "Relation to Proper Motion of Preferential Motion and of the Progressions of Spectral Class and Magnitude-Velocity," by C. D. Perrine:

Page 278, eighth line, *for* B. Boss *read* Lewis Boss.

Vol. 48, July 1918, "The 'Astronomical Atom' and the Spectral Series of Hydrogen," by Fernando Sanford:

Page 6, equation (8),

$$for\ R=\frac{3.503\cdot 10^{-6}}{1\lambda}\quad read\ \frac{R}{1\lambda}=3.503\cdot 10^{-6}$$

Vol. 48, September 1918, "The Absorption of Near Infra-Red Radiation by Water-Vapor," by W. W. Sleator:

Page 127, in legend, *for* scale 1 to 5 *read* scale 1 to 9.

THE

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ON THE CONDITIONS IN THE INTERIOR OF A STAR

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I have described a theory of the interior of a gaseous star in two papers in the *Monthly Notices* of the Royal Astronomical Society.[1] After two years' experience it has become possible to make some simplifications in the treatment, and I hope that the following explanation of the theory will be more easily understood. The investigation applies to giant stars of density so low that the laws of a perfect gas are applicable.

At a point in the star distant r from the center, let P be the pressure, T the temperature, ρ the density, and g the acceleration of gravity. The ordinary equation of hydrostatics is

$$\frac{dP}{dr}=-g\rho \tag{1}$$

It is characteristic of the present theory that the pressure of radiation plays an important part in the equilibrium of the star, and we shall include its effects in the equations. The pressure P is accordingly made up of the ordinary gas-pressure p and the radiation-pressure p_R. The latter is equal to $\frac{1}{3}$E, where E is

[1] 77, 16, 596, 1916–1917.

the amount of radiant energy in a cubic centimeter. This may be seen as follows:

Consider the momentum entering and leaving a centimeter cube through one of its faces. We may simplify the problem by resolving the energy into six equal trains of waves proceeding in directions $+x$, $-x$, $+y$, $-y$, $+z$, $-z$, respectively, each containing energy $\frac{1}{6}$E per cubic centimeter. If C is the velocity of light, the energy crossing any face in a second will be that contained in a column of length C; that is to say, energy $\frac{1}{6}$EC will leave the cube and $\frac{1}{6}$EC will enter through that face. But according to Poynting's law the momentum carried by a beam of light is 1/C times the energy. Thus $\frac{1}{6}$E units of outward momentum leave the cube and $\frac{1}{6}$E units of inward momentum enter the cube through this face per second. The net result is a gain of $\frac{1}{3}$E units of inward momentum, which is equivalent to a pressure $\frac{1}{3}$E acting on the face.[1] The same result may be obtained more rigorously by integrating the effects of waves in all directions.

It is well known that the radiant energy in unit volume is proportional to the fourth power of the absolute temperature, so that

$$E = aT^4$$

where a is a universal constant equal to $7.06 \cdot 10^{-15}$, the units being the erg, centimeter, and degree centigrade. Hence

$$p_R = \tfrac{1}{3} a T^4 \tag{2}$$

and

$$P = p + \tfrac{1}{3} a T^4 \tag{3}$$

We obtain another equation by considering the flow of heat through the star. It is fairly obvious that the flow will be proportional to the gradient of the energy-density dE/dr, and inversely proportional to the opacity of the material. The rigorous derivation of the equation has been given by me elsewhere,[2] and also by

[1] Remembering that the energy of light-waves is half kinetic and half potential, the pressure is two-thirds of the kinetic energy in unit volume, which is exactly the same result as for the pressure of gas molecules.

[2] *Monthly Notices*, 77, 18–19, 1916–1917.

Jeans,[1] and it seems unnecessary to repeat it. The following general argument will serve our purpose. The radiation is being forced through matter, as through a sieve, by the pressure acting on it. In a steady state the flow will be proportional to the pressure-gradient dp_R/dr, and inversely proportional to the obstruction of the matter. The obstruction will be proportional to the density ρ and to the specific absorptive power k of the material. Hence if H is the outward flow of energy per square centimeter per second,

$$\mathrm{H}=\text{constant}\times\frac{1}{k\rho}\frac{dp_R}{dr}.$$

On examining the dimensions of this equation, the constant is found to have the dimensions of velocity. Now c is the only velocity with which we are concerned. We thus obtain

$$\mathrm{H}=-\frac{c}{k\rho}\frac{dp_R}{dr} \tag{4}$$

The absence of any numerical multiplier could not be deduced by the argument given here, but it is justified by the more rigorous discussions referred to. The meaning of the mass-coefficient of absorption k is that a layer of material of density ρ and thickness dr will absorb or scatter the fraction $k\rho dr$ of the radiation traversing it. Equation (4) applies only as long as the net flow of heat in any direction is small compared with the gross flow in all directions, and it becomes inaccurate near the photosphere of a star.

From (1) and (4) we can eliminate dr, obtaining

$$dP=\frac{g}{\mathrm{H}}\frac{c}{k}dp_R \tag{5}$$

We shall now make certain approximations, which it will be more easy to justify after deducing the consequences. The fact is that I originally made them expecting to obtain only crude results, but they appear to be verified very closely by observation.

[1] *Monthly Notices*, 78, 28–31, 1917.

I assume that the absorption-coefficient k can be treated as constant throughout the star. I further assume that H is proportional to g throughout the star. In that case we can write

$$\frac{Hk}{gC} = 1 - \beta, \text{ a constant for the star,} \tag{6}$$

and (5) can be integrated at once, giving

$$p_R = (1 - \beta)P \tag{7}$$

so that

$$p = \beta P \tag{8}$$

If L is the total radiation of the star (practically its absolute luminosity), M its mass, R its radius, and G the constant of gravitation, then at the boundary

$$L = 4\pi R^2 H$$

and

$$g = GM/R^2.$$

Thus (6) gives

$$\frac{Lk}{4\pi cGM} = 1 - \beta$$

or

$$L = \frac{4\pi cG}{k} M(1 - \beta) \tag{9}$$

It is true that our equations fail just short of the boundary of the star; but clearly the value of the left side of (6) just within the boundary will be continuous with the value at the boundary, so that no appreciable error is caused by taking the boundary values.

The law of a perfect gas is that p varies as ρT, or

$$p = \frac{\mathfrak{R}}{m} \rho T \tag{10}$$

where m is the molecular weight, or average weight of the ultimate independent particles in terms of the hydrogen atom as unit, and $\mathfrak{R}$ is the universal gas-constant, whose value is $8.29 \cdot 10^7$.

From (2) and (7)

$$P = \frac{1}{3} \frac{a}{1 - \beta} T^4$$

and by (8) and (10)

$$P = \frac{\mathfrak{R}}{\beta m} \rho T.$$

Hence, eliminating T,

$$\mathrm{P}=\kappa\rho^{\frac{4}{3}} \tag{11}$$

where

$$\kappa=\left(\frac{3\mathfrak{R}^4(1-\beta)}{am^4\beta^4}\right)^{\frac{1}{3}} \tag{12}$$

Equation (11) and the hydrostatic equation

$$\frac{d\mathrm{P}}{dr}=-g\rho \tag{13}$$

together determine the condition of equilibrium of the star.

The solution of these equations is a rather difficult problem involving quadratures, but fortunately it has been fully treated by Emden in his book *Gaskugeln* in connection with an entirely different theory of stellar equilibrium. We can use his results.

In finding the solution for a particular star we may suppose that the mass M, radius R, and molecular weight m are given. Since β is unknown, the constant κ is at our disposal, and we must determine it so that in the solution the star actually does terminate at the radius R and contain the assigned mass. Suppose we have found κ for a star of unit mass, unit radius, and unit molecular weight. If we multiply all our lengths by R and all our masses by M, we shall have a star of radius R and mass M. The densities at corresponding points will be multiplied by M/R^3, and the values of g by M/R^2. Putting in these factors in (13), we see that the values of P will be multiplied by $\mathrm{M}^2/\mathrm{R}^4$. But (11) then gives

$$\frac{\mathrm{M}^2}{\mathrm{R}^4}\propto\kappa\left(\frac{\mathrm{M}}{\mathrm{R}^3}\right)^{\frac{4}{3}}$$

so that κ must be multiplied by $\mathrm{M}^{\frac{2}{3}}$ in order that the solution may still fit. Thus by (12)

$$\left(\frac{3\mathfrak{R}^4(1-\beta)}{am^4\beta^4}\right)^{\frac{1}{3}}\propto\mathrm{M}^{\frac{2}{3}}$$

giving the equation

$$1-\beta=\text{constant}\times\mathrm{M}^2m^4\beta^4 \tag{14}$$

to determine β.

The most important result is that R has disappeared, so that the value of β depends on the mass, but *not* on the radius, of the

star. Since it is unlikely that the molecular weight will alter appreciably, β will remain constant throughout the evolution of a star until it becomes too dense to behave as a perfect gas.

Accordingly by (9) kL is constant for stars of the same mass.

The luminosity (or more strictly the total radiation) of a giant star is, for a given mass, inversely proportional to its opacity and is otherwise independent of its density or stage of evolution.

If the opacity is the same for all stars, we have an explanation of the well-known result that the average luminosity of the giant stars is nearly the same for all spectral types. Conversely we deduce that this observational result is due to the opacity of stellar material being independent of the temperature (at high temperatures); and it affords at least a partial verification of our assumption that k can be treated as constant inside any particular star.

The law can also be stated in the form: *for stars of the same mass the effective temperature is proportional to the sixth root of the density.* This is deduced from (6), the emission of radiation per unit area, H, being proportional to the fourth power of the effective temperature, and g being proportional to $\rho^{\frac{2}{3}}$.

To find how the luminosity depends on the mass we have to solve (14) for β. It is thus necessary to know the constant occurring in the formula. This can be obtained only by numerical calculation from Emden's solution.[1] If the hydrogen atom is the unit of m, and the sun's mass the unit of M, I find that (14) becomes

$$1-\beta=0.00260m^4M^2\beta^4 \qquad (15)$$

On theoretical grounds the molecular weight must be taken a little greater than 2. This is because an atom of atomic weight A consists of a nucleus and approximately $\frac{1}{2}$ A electrons; and at the high temperatures within the star the strong ionization will detach most of the electrons, so that they count as independent molecules. The average weight of the ultimate particles must thus be a little

[1] For the sun treated as a perfect gas of molecular weight 18, Emden gives (pp. 101–102, example D), at the center $\rho=74.94$, $P=1.186\cdot 10^{17}$. Hence by (11) $\kappa=3.754\cdot 10^{14}$. Inserting this value in (12), with $a=7.06\cdot 10^{-15}$, $\mathfrak{R}=8.29\cdot 10^{7}$, we obtain $1-\beta=0.0026m^4\beta^4$. In this case $M=1$, so that the constant in (14) must be 0.0026. The other examples given by Emden refer to laws of distribution outside our theory.

greater than 2, whatever may be the material (other than hydrogen). I adopt $m = 2.8$.[1]

Solving (15) for β, we obtain the following results for various values of M.

Mass (Sun = 1)	$1-\beta$	Absolute Magnitude
0.5	0.036	$+2^{M}_{.}6$
1.0	.106	+0.7
1.5	.174	−0.3
3.0	.320	−1.7
4.5	.409	−2.4
6.5	.487	−3.0
13.0	.615	−4.0

The absolute magnitude in the last column is obtained by converting into magnitudes the quantity L, which is proportional to $M(1-\beta)$ by (9). The difference of absolute magnitude is thus the difference of $2.5 \log_{10} M(1-\beta)$. Since we do not know k, the datum point must be found from observation. We may adopt $-0^{M}_{.}3$ as the average absolute magnitude of the giants, and I take this to correspond to a mass 1.5 times that of the sun. By various checks this has been found the most likely value. The magnitude here considered is the bolometric magnitude, corresponding to the total radiation; it agrees with the visual magnitude for types F–G, but the earlier and later types are slightly less luminous in proportion to the total radiation.

As an illustration, the great majority of giant stars of types F and G in Adams and Joy's list[2] are comprised between absolute magnitudes $+2^{M}_{.}0$ and $-0^{M}_{.}5$. We should infer from the foregoing table that their masses are between 0.7 and 1.6 times that of the sun—a remarkably small range.

It is also of interest to notice that $1-\beta$, which represents the fraction of the total pressure due to radiation-pressure, increases

[1] The retention of the last few electrons causes a very slow rise of m. The somewhat odd—but, I believe, satisfactory—value $2\sqrt{2}$ is adopted in order to avoid correcting some lengthy calculations made in a former paper, intended to represent $m = 2$; this change in m eliminates an erroneous factor in that paper (I had taken the radiation-pressure to be $\frac{4}{3}$E instead of $\frac{1}{3}$E).

[2] *Astrophysical Journal*, 46, 334, 1917.

from insignificance to predominance in the range of masses covered by the table. I think that this points to radiation-pressure as the cause determining the aggregation of the matter of the universe into stars usually between these limits of mass rather than into units of a quite different order of magnitude.

Knowing by observation the approximate rate of radiation of energy from a star of given mass, we can determine k, which is now the only unknown in equation (9). The sun, magnitude $5^{\mathrm{M}}_{.}1$, radiates $3.8 \cdot 10^{33}$ ergs per second; and a giant star of mass 1.5, which is 5.4 magnitudes brighter, radiates 145 times as fast. We have

$L = 5.5 \cdot 10^{35}$ ergs per second; $M = 2.91 \cdot 10^{33}$ gm; $c = 3 \cdot 10^{10}$ cm per second; $G = 6.66 \cdot 10^{-8}$; $1 - \beta = 0.174$. Hence $k = 23$ C.G.S. units.

This means that radiation would be reduced to about a third $(1/e)$ of its original intensity in passing through a layer of $1/23$ gm per square centimeter, or a thickness of about 30 cm at atmospheric density. The stars are thus amazingly opaque to radiation, and this accounts for the large radiation-pressure which exists. If the radiation within the star were light of ordinary character it would be difficult to understand so high an opacity; but at the high temperatures in the interior the radiation is of short wave-length and is more akin to X-rays. Laboratory experiments on the absorption of X-rays give results of the same order of magnitude as that here found in the stars.

Our conclusion that at reasonably high temperatures the coefficient of absorption becomes constant, or nearly so, is not a hitherto known physical law; but I think that the evidence of the magnitudes of giant stars makes it fairly certain. Laboratory experiments on X-rays show a strong dependence of the absorption-coefficient on the wave-length (and therefore on the temperature) of the radiation absorbed; but they are concerned with streams of much less intensity than in the stars. In the laboratory it is a question of how often an absorbing atom can intercept one of the occasional bundles of radiation flying past; in the star it is a question rather of how fast it can deal with the bundle and get ready to catch another.

We assumed at an earlier stage that H/g remains constant at different points in a star. From Emden's solution it is possible to test how far this holds, assuming that the energy comes solely from contraction. It is found that from the boundary to the center, H/g increases slowly in the ratio 1 at the boundary to 1.7 at the center. The variation is insignificant from our point of view. We could take account of it and proceed to a second approximation; but it is very uncertain whether the energy radiated by a star is derived from contraction.

Further applications, and in particular a tentative extension to stars of density too great to behave as a perfect gas, will be found in the second of the two papers referred to. I conclude this introductory account by giving a table of the temperature and density according to this theory within a star of mass 1.5 times the sun, mean density 0.002 gm cm^{-3}, and molecular weight 2.8. The unit of r is rather arbitrary, the radius being taken as 6.9 units; in this star the unit happens to be approximately 1,000,000 kilometers. For any other star the densities must be changed in proportion to the mean density and the temperatures in proportion to $M^{\frac{2}{3}}\rho^{\frac{1}{3}}\beta$. The calculation of these numbers depends on *Gaskugeln*, page 80, Table 7.

DENSITY AND TEMPERATURE IN A TYPICAL GIANT STAR

r	ρ	T
		Abs. C.
0	0.1085	4,650,000°
1	.0678	3,980,000
2	.0215	2,710,000
3	.00503	1,670,000
4	.00100	974,000
5	.000149	517,000
6	.0000093	207,000
6.9	.0000000	....

More than half the mass is within the sphere $r=2$, and less than 1/20 is outside $r=4$. A star of this density would be of type about F7.

Cambridge, England
August 15, 1918

THE VARIATION IN LIGHT AND COLOR OF RS BOÖTIS[1]

By FREDERICK H. SEARES and HARLOW SHAPLEY

A variation in the light of RS Boötis[2] was first noted by Mrs. Fleming on a photograph made at Arequipa on May 24, 1906.[3] Other plates in the Harvard collection, some made as early as 1890, indicated a period of about 12 hours and an amplitude of one magnitude. Visual observations by Seares and Haynes at the Laws Observatory in June and July, 1908, gave the provisional elements:[4]

$$\text{Max.} = \text{J.D. } 2418115.626 + 0.37722 \text{ E, G.M.T.} \tag{1}$$

and revealed a variation similar to that of the short-period variables so numerous in certain globular clusters.

Only a few such objects were known among the stars at large. Williams, during 1900–1903, had recognized Y Lyrae, UY Cygni, and RZ Lyrae; and observations at the Laws Observatory in 1906–1908 had identified RV Capricorni, SU Draconis, and SW Andromedae. RR Lyrae, RR Geminorum, and one or two others had previously been discovered; but the total number of isolated cluster-type variables, now about forty-five, did not then exceed a dozen. The discovery of RS Boötis therefore emphasized the interest of this peculiar type of variability, and the visual observations begun at the Laws Observatory were accordingly continued with some persistence until July, 1910. The results of these measures, 542 observations in all, are collected in Table I.[5]

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 159.

[2] B.D. $+32°2489$, 9.3, $14^h27^m21^s$, $+32°23'.4$ (1855). For a modern position cf. *Astronomische Abhandlungen der Hamburger Sternwarte*, **1**, No. 3, 1909.

[3] *Harvard Circular*, No. 124; *Astronomische Nachrichten*, **174**, 105, 1907; **176**, 188, 1907.

[4] *Laws Observatory Bulletins*, **1**, 240, 1908.

[5] These observations were made with the assistance of a grant from the Gould Fund of the National Academy of Sciences.

TABLE I

VISUAL OBSERVATIONS OF RS BOÖTIS

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
2417-					712.661..	59	*b*H	10.22	+16
660.611..	80	*a*H	10.17	− 2	.672..	70	*b*H	10.30	+17
		*b*H	10.06	−13	.685..	83	*b*H	10.36	+15
.685..	154	*a*H	10.68	+20	.700..	98	*b*H	10.47	+20
		*b*H	10.50	+ 2	.728..	126	*b*H	10.61	+22
.752..	221	*a*H	10.94	+32	.748..	146	*b*H	10.65	+19
		*b*H	10.86	+24	715.618..	374	*b*H	10.33	− 1
.786..	255	*a*H	10.94	+28	.635..	14	*b*H	10.36	+30
		*b*H	10.98	+32	.649..	28	*b*H	10.45	+59
667.608..	285	*a*H	10.85	+16	.667..	46	*b*H	10.33	+35
		*b*H	10.80	+11	.683..	62	*b*H	10.38	+30
.651..	328	*a*H	10.94	+24	.695..	74	*b*H	10.53	+38
		*b*H	10.83	+13	.727..	106	*b*H	10.43	+13
.697..	374	*a*H	10.48	+14	.744..	123	*b*H	10.55	+17
		*b*H	10.46	+12	.761..	140	*b*H	10.47	+ 3
.728..	28	*a*H	10.30	+44	.774..	153	*b*H	10.52	+ 4
		*b*H	10.36	+50	.790..	169	*b*H	10.55	+ 2
.764..	64	*a*H	10.25	+16	.812..	191	*b*H	10.54	− 3
		*b*H	10.19	+10	.827..	206	*b*H	10.59	− 1
.869..	169	*a*H	10.53	0	.845..	224	*b*H	10.46	−17
		*b*H	10.62	+ 9	.862..	241	*b*H	10.47	−18
.910..	210	*a*H	10.62	+ 1	.880..	259	*b*H	10.43	−24
		*b*H	10.63	+ 2	716.617..	241	*b*H	10.57	− 8
675.909..	285	*a*H	10.96	+27	.640..	264	*b*H	10.76	+ 9
		*b*H	10.95	+26	.656..	280	*b*H	10.73	+ 4
679.751..	353	*a*H	10.24	−39	.671..	295	*b*H	10.71	+ 1
		*b*H	10.39	−24	742.623..	212	*a*H	10.35	−26
687.590..	268	*b*H	10.81	+13			*b*H	10.25	−36
.826..	127	*b*H	10.70	+30	.654..	243	*b*H	10.50	−15
.849..	150	*b*H	10.62	+15	.681..	270	*b*H	10.54	−14
.868..	169	*b*H	10.63	+10	.701..	290	*b*H	10.59	−11
.890..	191	*b*H	10.60	+ 3	.737..	326	*b*H	10.75	+ 5
689.703..	117	*b*H	10.52	+16	.777..	366	*b*H	10.41	− 6
.735..	149	*b*H	10.53	+ 6	.806..	18	*b*H	9.94	− 6
.765..	179	*b*H	10.63	+ 8	.832..	44	*b*H	10.30	+34
.810..	224	*b*H	10.82	+19	.865..	77	*b*H	10.32	+15
.840..	254	*b*H	10.77	+11	762.632..	224	*a*H	10.62	− 1
.872..	286	*b*H	10.80	+10	.634..	226	*b*H	10.56	− 7
690.585..	245	*b*H	11.05	+40	.660..	252	*a*H	10.62	− 4
.601..	261	*b*H	10.96	+29	.663..	255	*b*H	10.57	− 9
697.580..	70	*b*H	10.28	+15	.682..	274	*a*H	10.78	+10
.605..	95	*b*H	10.25	− 1	.686..	278	*b*H	10.89	+20
.632..	122	*a*H	10.75	+37	.688..	280	*a*H	10.92	+23
		*b*H	10.46	+ 8	.692..	284	*b*H	10.54	−15
.658..	148	*b*H	10.76	+30	.714..	306	*a*H	10.89	+19
.690..	180	*b*H	10.81	+26	.716..	308	*b*H	10.68	− 2
.733..	223	*b*H	10.87	+24	.749..	341	*a*H	10.74	+ 5
.778..	268	*b*H	10.89	+21	.751..	343	*b*H	10.84	+16
.793..	283	*b*H	10.99	+30	.795..	10	*a*H	10.23	+11
.811..	301	*b*H	10.80	+10	.798..	13	*b*H	10.42	+34
.873..	363	*b*H	10.59	+ 7	.838..	53	*a*H	10.02	− 1
712.649..	47	*b*H	10.26	+27	.840..	55	*b*H	10.08	+ 4

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
763.643..	103	*a*H	10.59	+30	037.936..	70	*a*H	10.11	− 2
.645..	105	*b*H	10.34	+ 4			*b*H	10.23	+10
.677..	137	*a*H	10.44	+ 1	057.667..	180	*a*H	10.19	−36
.678..	138	*b*H	10.32	−11			*b*H	10.33	−22
.702..	162	*a*H	10.57	+ 6	.683..	196	*b*H	10.34	−24
.704..	164	*b*H	10.54	+ 3	.701..	214	*b*H	10.34	−27
.707..	167	*a*H	10.62	+10	.725..	238	*b*H	10.30	−35
.709..	169	*b*H	10.46	− 7	115.626..	32	*a*S	9.72	−13
777.629..	128	*b*H	10.40	0	.665..	71	*a*S	10.02	−12
.654..	153	*b*H	10.59	+11	.708..	114	*a*S	10.38	+ 4
.701..	200	*b*H	10.50	− 9	.711..	117	*a*S	10.49	+13
.750..	249	*b*H	10.82	+16	.745..	151	*a*S	10.79	+32
.798..	297	*b*H	10.63	− 7	.786..	192	*a*S	10.85	+28
787.612..	302	*a*H	10.77	+ 7	.790..	196	*a*H	10.72	+14
.614..	304	*b*H	10.41	−29			*b*H	10.61	+ 3
.632..	322	*a*H	10.80	+10	.836..	242	*a*S	11.00	+35
.634..	324	*b*H	10.98	+28	.840..	246	*a*H	10.68	+ 2
.706..	19	*a*H	9.73	−26	.843..	249	*a*S	10.80	+14
.708..	21	*b*H	10.14	+18	.883..	289	*a*S	11.05	+35
788.606..	163	*a*H	10.44	− 7	.887..	293	*a*H	10.79	+ 9
.608..	165	*b*H	10.56	+ 5	117.619..	138	*a*H	10.36	− 7
.624..	181	*a*H	10.61	+ 6			*b*H	10.32	−11
.626..	183	*b*H	10.45	−11	.657..	176	*a*S	10.58	+ 4
.642..	199	*a*H	10.61	+ 2			*b*S	10.61	+ 7
.645..	202	*b*H	10.60	+ 1	.704..	223	*a*S	10.56	− 7
.661..	218	*a*H	10.44	−18			*b*S	10.58	− 5
.664..	221	*b*H	10.78	+16	.741..	260	*a*S	10.47	−20
.685..	242	*a*H	10.73	+ 8			*b*S	10.63	− 4
.687..	244	*b*H	10.65	0	.783..	302	*b*S	10.61	− 9
.709..	266	*a*H	10.55	−13	.787..	306	*b*H	10.67	− 3
.712..	269	*b*H	10.83	+15	.829..	348	*b*S	10.62	− 4
.735..	292	*a*H	10.69	− 1	.832..	351	*b*H	10.70	+ 6
.737..	294	*b*H	10.63	− 7	.865..	7	*b*S	10.21	+ 4
.759..	316	*a*H	10.96	+26	.868..	10	*b*H	10.15	+ 3
.761..	318	*b*H	10.74	+ 4	.872..	14	*b*S	10.00	− 6
994.683..	212	*a*H	10.77	+16	.874..	16	*b*H	9.98	− 5
		*b*H	10.58	− 3	.878..	20	*b*S	9.80	−17
.709..	238	*a*H	10.38	+27	.880..	22	*a*S	9.74	−20
		*b*H	10.46	+19	.883..	25	*a*H	9.74	−16
.776..	305	*a*H	10.47	−23	.886..	28	*a*S	9.85	− 1
		*b*H	10.47	−23			*b*S	9.86	0
.874..	26	*a*H	9.81	− 7	.890..	32	*a*H	9.72	−13
		*b*H	9.87	− 1	.893..	35	*a*S	9.76	−11
.898..	50	*a*H	9.66	−35	.894..	36	*a*H	9.71	−17
		*b*H	9.79	−22	.897..	39	*a*S	9.81	−10
.913..	65	*a*H	9.89	−21	118.604..	369	*a*S	10.31	−12
		*b*H	9.81	−29	.607..	372	*a*S	10.37	0
2418-					.610..	375	*a*S	10.26	− 6
037.598..	109	*b*H	10.42	+10	.611..	376	*a*S	10.17	−13
.656..	167	*b*H	10.41	−11	.614..	1	*a*S	10.08	−19
.706..	217	*b*H	10.65	+ 3	.617..	4	*a*S	10.01	−21
.767..	278	*b*H	10.54	−15	.620..	7	*a*S	9.89	−28
.899..	33	*b*H	10.13	+27	.622..	9	*a*S	9.83	−31

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
118.625..	12	*a*S	9.81	−28	131.785..	344	*b*S	10.80	+13
.627..	14	*a*S	9.84	−22	.788..	347	*b*H	10.62	− 4
.629..	16	*a*S	9.72	−31	.790..	349	*b*S	10.76	+11
.631..	18	*a*S	9.72	−28	.792..	351	*b*H	10.75	+11
.635..	22	*a*S	9.71	−23	.801..	360	*b*S	10.63	+ 6
.638..	25	*a*S	9.77	−13	.803..	362	*b*H	10.52	− 2
.639..	26	*a*S	9.69	−19	.805..	364	*b*S	10.50	− 1
.643..	30	*a*S	9.74	−11	.807..	366	*b*H	10.62	+15
.646..	33	*a*S	9.82	− 4	.810..	369	*b*S	10.40	− 3
.649..	36	*a*S	9.86	− 2	.812..	371	*b*H	10.47	+ 8
.651..	38	*a*S	9.82	− 8	.819..	1	*b*S	10.16	−11
.668..	55	*a*S	9.93	−11	.822..	4	*b*H	10.00	−22
.670..	57	*a*S	9.93	−12	.826..	8	*b*S	10.09	− 6
.685..	72	*a*S	10.15	+ 1	132.633..	60	*a*H	10.07	0
.687..	74	*a*S	10.06	− 9			*b*H	10.02	− 5
.708..	95	*a*S	10.26	0	.667..	94	*a*H	10.27	+ 1
.710..	97	*a*S	10.34	+ 7			*b*H	10.30	+ 4
.731..	118	*a*S	10.23	−13	.718..	145	*a*H	10.45	0
.733..	120	*a*H	10.44	+ 7			*b*H	10.40	− 5
.769..	156	*a*S	10.58	+ 9	.764..	191	*a*H	10.68	+11
.772..	159	*a*H	10.55	+ 5			*b*H	10.59	+ 2
.835..	222	*a*S	10.50	−12	139.615..	250	*a*S	10.39	−27
.838..	225	*a*S	10.74	+11			*b*S	10.54	−12
.840..	227	*a*H	10.66	+ 3	.618..	253	*a*H	10.76	+10
.844..	231	*a*H	10.62	− 2			*b*H	10.74	+ 8
		*b*H	10.64	0	.690..	325	*a*H	10.70	0
120.604..	105	*a*H	10.14	−16			*b*H	10.69	− 1
.606..	107	*a*H	10.20	−11	.714..	349	*a*S	10.53	−12
.610..	111	*a*S	10.17	−16			*b*S	10.62	− 3
.612..	113	*a*S	10.19	−15	.718..	353	*a*H	10.53	−10
.614..	115	*a*H	10.31	− 4			*b*H	10.57	− 6
.616..	117	*a*H	10.26	−10	.723..	358	*a*S	10.43	−16
.619..	120	*a*S	10.25	−12			*b*S	10.53	− 6
.622..	123	*a*S	10.12	−26	.728..	363	*a*H	10.49	− 3
.626..	127	*a*H	10.38	− 2			*b*H	10.48	− 4
		*b*H	10.35	− 5	.732..	367	*a*S	10.38	− 8
.631..	132	*a*S	10.29	−13			*b*S	10.33	−13
		*b*S	10.35	− 7	.736..	371	*a*H	10.31	− 8
122.615..	229	*a*S	10.44	−20			*b*H	10.42	+ 3
		*b*S	10.48	−16	.740..	375	*a*S	10.26	− 6
.671..	285	*a*S	10.54	−15			*b*S	10.43	+11
		*b*S	10.45	−24	.747..	5	*a*H	10.16	− 4
.716..	330	*a*S	10.72	+ 2			*b*H	10.23	+ 3
		*b*S	10.50	−20	.751..	9	*a*S	9.92	−22
131.654..	213	*a*H	10.58	− 3			*b*S	10.24	+10
		*b*H	10.61	0	.756..	14	*a*H	10.02	− 4
.697..	256	*a*S	10.58	− 9			*b*H	10.07	+ 1
		*b*S	10.62	− 5	.760..	18	*a*S	9.86	−14
.700..	259	*b*H	10.65	− 2			*b*S	9.91	− 9
.743..	302	*b*S	10.65	− 5	.765..	23	*a*H	9.89	− 4
.745..	304	*b*H	10.73	+ 3			*b*H	9.97	+ 4
.774..	333	*b*S	10.77	+ 7	.770..	28	*a*S	9.63	−23
.776..	335	*b*H	10.76	+ 6			*b*S	9.84	− 2

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
139.774..	32	aH	9.80	−5	178.624..	19	aH	10.02	+3
		bH	9.91	+6			bH	10.05	+6
.779..	37	aH	9.82	−7	.626..	21	aH	9.95	−1
.782..	40	aH	9.76	−16			bH	10.03	+7
.786..	44	aH	9.89	−7	.630..	25	aH	9.76	−14
		bH	9.97	+1			bH	9.89	−1
.799..	57	aH	9.97	−8	.633..	28	aH	9.89	+3
.801..	59	aH	9.96	−10			bH	9.94	+8
143.631..	115	aS	10.33	−2	.638..	33	aH	9.87	+1
		bS	10.29	−6			bH	9.90	+4
.649..	133	aS	10.17	−25	.642..	37	aH	9.93	+4
		bS	10.27	−15			bH	9.89	0
153.677..	351	aS	10.69	+5	.646..	41	aH	9.85	−8
		bS	10.57	−7			bH	9.97	+4
.683..	357	aH	10.75	+15	.656..	51	aH	9.97	−5
		bH	10.74	+14			bH	10.05	+3
.705..	2	aS	10.45	+20	.667..	62	aH	10.00	−8
		bS	10.24	−1			bH	10.12	+4
.712..	9	bH	10.16	+2	.695..	90	aH	10.21	−3
.713..	10	bH	10.07	−5			bH	10.18	−6
.716..	13	bS	10.06	−2	473.682..	373	aSh	10.20	−16
.717..	14	bS	9.92	−14			bSh	10.17	−19
.721..	18	bH	9.82	−18	475.673..	100	aH	10.08	−20
.725..	22	bH	9.71	−23			bH	10.24	−4
.727..	24	bS	9.79	−12	.711..	138	aH	10.18	−25
.729..	26	bS	9.67	−21			bH	10.32	−11
.733..	30	aH	9.84	−1	.740..	167	aH	10.22	−30
.735..	32	aH	9.86	+1			bH	10.39	−13
169.639..	89	aH	10.10	−14	477.761..	302	aH	10.69	−1
		bH	10.04	−20			bH	10.46	−24
.656..	106	aH	10.23	−7	.785..	326	aH	10.86	+16
		bH	10.18	−12			bH	10.49	−21
170.659..	354	bH	10.69	+6	.842..	6	aH	10.17	−1
.699..	17	bH	10.13	+11			bH	10.34	+16
.719..	37	bH	10.02	+13	.849..	13	aSh	10.24	+16
176.612..	271	aH	10.88	+20			bSh	10.18	+10
		bH	11.00	+32	.854..	18	aH	10.06	+6
.633..	292	aH	10.73	+3			bH	10.17	+17
		bH	10.91	+21	.867..	31	aSh	9.71	−14
.653..	312	aH	10.73	+3	.869..	33	aSh	9.82	−4
		bH	10.80	+10	.872..	36	aH	9.73	−15
.675..	334	aH	10.82	+12			bH	9.67	−21
		bH	11.05	+35	479.704..	359	aH	10.69	+11
.702..	361	aH	10.88	+33			bH	10.74	+16
		bH	10.85	+30	.709..	364	aH	10.57	+6
.716..	375	aH	10.76	+44			bH	10.54	+3
		bH	10.89	+57	.733..	11	aH	10.22	+11
178.612..	7	aH	10.11	−6			bH	10.15	+4
		bH	10.18	+1	.739..	17	aH	9.95	−7
.615..	10	aH	10.20	+8			bH	9.91	−11
		bH	10.10	−2	.760..	38	aH	9.87	−3
.618..	13	aH	10.08	0			bH	9.95	+5
		bH	10.22	+14	.765..	43	aH	9.78	−17

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
479.765..	43	*b*H	9.84	−11	504.658..	33	*a*H	9.86	0
.788..	66	*a*H	10.05	− 6			*b*H	9.85	− 1
		*b*H	9.91	−20	.674..	49	*a*H	9.99	− 1
501.609..	2	*a*H	10.40	+15			*b*H	9.96	− 4
		*b*H	10.29	+ 4	.681..	56	*a*H	10.01	− 4
.614..	7	*a*H	10.28	+11			*b*H	9.93	−12
		*b*H	10.26	+ 9	.698..	73	*a*H	10.20	+ 5
.620..	13	*a*H	10.04	− 4			*b*H	10.12	− 3
		*b*H	10.16	+ 8	.704..	79	*a*H	10.19	+ 1
.627..	20	*a*H	10.09	+12			*b*H	10.13	+ 5
		*b*H	10.02	+ 5	837.627..	190	*a*Sh	10.52	− 5
.635..	28	*a*H	9.92	+ 6			*b*Sh	10.69	+12
		*b*H	9.89	+ 3	.637..	200	*b*Sh	10.60	+ 1
.640..	33	*a*H	9.91	+ 5	.643..	206	*a*Sh	10.60	0
		*b*H	9.83	− 3			*b*Sh	10.76	+16
.646..	39	*a*H	9.91	0	.678..	241	*b*Sh	10.66	+ 1
		*b*H	9.75	−16	.683..	246	*b*Sh	10.61	− 5
.653..	46	*a*H	10.00	+ 2	.691..	254	*b*Sh	10.66	0
		*b*H	9.95	− 3	.721..	284	*b*Sh	10.58	−11
.675..	68	*a*H	10.09	− 3	.736..	299	*b*Sh	10.67	− 3
		*b*H	10.11	− 1	840.642..	186	*b*Sh	10.31	−25
.680..	73	*a*H	10.12	− 3	.716..	260	*a*Sh	10.59	− 8
		*b*H	10.09	− 6			*b*Sh	10.74	+ 7
.707..	100	*a*H	10.14	−14	863.721..	250	*a*Sh	10.53	−13
		*b*H	10.16	−12			*b*Sh	10.58	− 8
.714..	107	*a*H	10.32	+ 1	865.621..	263	*b*Sh	10.76	+ 9
		*b*H	10.21	−10	.624..	266	*b*Sh	10.67	− 1
.737..	130	*a*H	10.56	+15	.678..	320	*b*Sh	10.55	−15
		*b*H	10.38	− 3	.719..	361	*b*Sh	10.27	−28
.742..	135	*a*H	10.49	+ 7	.724..	366	*a*Sh	10.35	−12
		*b*H	10.45	+ 3			*b*Sh	10.24	−23
502.643..	281	*a*H	10.55	−14	.728..	370	*b*Sh	10.27	−14
		*b*H	10.58	−11	.735..	0	*b*Sh	10.18	−11
.649..	287	*a*H	10.60	−10	.741..	6	*b*Sh	10.07	−11
		*b*H	10.60	−10	.744..	9	*b*Sh	10.06	− 8
504.612..	364	*a*H	10.77	+26	.747..	12	*b*Sh	10.02	− 7
		*b*H	10.78	+27	.753..	18	*b*Sh	9.84	−16
.617..	369	*a*H	10.62	+19	.756..	21	*a*Sh	10.23	+27
		*b*H	10.73	+30	.760..	25	*a*Sh	10.06	+16
.623..	375	*a*H	10.45	+13			*b*Sh	9.91	+ 1
		*b*H	10.50	+18	.767..	32	*b*Sh	10.03	+18
.628..	3	*a*H	10.31	+ 7	.770..	35	*a*Sh	10.05	+18
		*b*H	10.28	+ 4	.775..	40	*a*Sh	10.12	+20
.633..	8	*a*H	10.27	+12			*b*Sh	10.14	+22
		*b*H	10.12	− 3	.786..	51	*b*Sh	10.04	+ 2
.638..	13	*a*H	10.19	+11	871.754..	359	*a*Sh	10.54	− 4
		*b*H	10.13	+ 5	.759..	364	*a*Sh	10.32	−19
.643..	18	*a*H	10.01	+ 1			*b*Sh	10.28	−23
		*b*H	9.99	− 1	.766..	371	*a*Sh	10.26	−13
.648..	23	*a*H	9.83	−10	.773..	1	*a*Sh	10.01	−26
		*b*H	10.02	+ 9	.778..	6	*a*Sh	10.06	−12
.653..	28	*a*H	9.89	+ 3	.784..	12	*a*Sh	10.02	− 7
		*b*H	9.87	+ 1	.789..	17	*a*Sh	10.13	+11

TABLE I—*Continued*

J.D.	Phase	Star, Obs.	Vis. Mag.	O−C	J.D.	Phase	Star, Obs.	Vis. Mag.	O−C
876.652..	353	*a*Sh	10.52	−11	879.705..	10	*a*Sh	10.09	− 3
.656..	357	*a*Sh	10.59	− 1	.708..	13	*a*Sh	10.09	+ 1
		*b*Sh	10.46	−14	.718..	23	*a*Sh	9.98	+ 5
879.635..	317	*a*Sh	10.50	−20	.720..	25	*a*Sh	9.92	+ 2
		*b*Sh	10.61	− 9	.732..	37	*a*Sh	10.12	+23
.662..	344	*a*Sh	10.76	+ 9	.736..	41	*a*Sh	10.10	+17
.694..	376	*a*Sh	10.23	− 7	.745..	50	*a*Sh	10.15	+14
.697..	2	*a*Sh	10.22	− 3			*b*Sh	10.01	0

An important question concerning cluster-type variables is that of a change in color paralleling their fluctuations in light. Their many analogies with the Cepheids, which were known to vary in color, had suggested such a possibility, although the cluster variables were not then classed with the Cepheids as they now usually are. The photometric methods developed for use with the 60-inch reflector afforded a ready means of measuring color; and photographic and photo-visual observations of RS Boötis on three successive nights in July, 1914, gave the data in Table XI, and established beyond question a variation in color similar to that previously detected in the case of the longer-period Cepheids.

These two series of measures, made with different instruments by wholly different methods and separated by an interval of several years, are discussed in the present paper for the purpose of revising the light-elements and determining mean light- and color-curves. The publication of the results has been long delayed, partly through occupation with other work, but more particularly by a series of mischances in securing the polar-comparison plates necessary for the determination of the zero-points, so that it was not until recently that the magnitudes could finally be reduced to the international system.[1]

THE VISUAL OBSERVATIONS

Most of the visual observations listed in Table I were made with an equalizing wedge photometer attached to the $7\frac{1}{2}$-inch equatorial of the Laws Observatory; those for J.D. 2417762–88,

[1] Results of a preliminary reduction were given in *Publications of the Astronomical Society of the Pacific*, 26, 202, 1914.

however, were obtained with a Zöllner photometer and a $4\frac{1}{2}$-inch equatorial. An account of the methods of observation and reduction will be found in *Laws Observatory Bulletins*, **1**, 94, 187, 1905, 1907.

The decimals of the Julian Day in the first column of the table are expressed in Greenwich mean time. The unit for the phase in the second column is 0.001 day. The letters in the third column indicate the comparison star (a=B.D. $+32°2481$, mag. 9.2; b=B.D. $+32°2483$, mag. 9.4) and the observer (S = Seares, H = Haynes, Sh = Shapley). The visual magnitude has been referred to the international zero-point by methods described below; the residuals in the last column (unit = 0.01 mag.) are referred to the mean visual light-curve given in Table V.

The measures with the wedge photometer were usually arranged so that a single group of settings gave values for the difference in brightness of the variable and each of the comparison stars a and b, and, at the same time, the magnitude difference of the comparison stars themselves. The first step in the reduction was the reference of the comparisons made with star a to star b, in order that all the observations might be expressed as differences of the form $v-b$. This required a knowledge of the difference $a-b$.

TABLE II

VALUES OF $a-b$ BY HAYNES

J.D.	Mean	Av. Dev.	No. Values	J.D.	Mean	Av. Dev.	No. Values
7660	−0.72	± 6	4	8118	−0.71	....	1
7667	.65	5	7	8120	.61	....	1
7675	.65	....	1	8131	.62	....	1
7679	.51	....	1	8132	.70	± 7	4
7697	.79	....	1	8139	.61	4	10
7742	.78	....	1	8169	.70	2	2
7762	.59	18	8	8176	.55	6	6
7763	.78	2	4	8178	.59	5	13
7787	.34	23	3	8475	.49	2	3
7788	.59	21	8	8477	.72	16	5
7994	.61	± 10	6	8479	.65	4	7
8037	.61	....	1	8501	.69	7	14
8057	.53	....	1	8502	.60	1	2
8115	.71	....	1	8504	−0.66	± 6	14
8117	−0.61	....	1				

The most extensive series of values for $a-b$ is by Haynes. His results for each night are collected in Table II, which gives the

Julian Day, the mean for the night, the average deviation of a single observation from this mean, and the number of values. The accidental error of measures with the Zöllner photometer (J.D. 7762–88) is large; otherwise the agreement is satisfactory, and there is no evidence of variability in the comparison stars. The mean for the series is $a-b=-0.64$ mag. (108 obsns.). Similar, but less extensive, series by Seares and Shapley give -0.64 (22 obsns.) and -0.55 (13 obsns.), respectively. The adopted value is $a-b=-0.64$ mag. (143 obsns.).

After adding $a-b$ to the values of $v-a$, the entire series of results for $v-b$ was plotted with the G.M.T. of observation as abscissa. With the aid of an approximate light-curve drawn on tracing paper, a number of epochs were determined for which the variable, during increasing light, was equal to star b. The results and their estimated relative weights are in the second and third columns of Table III. With two similar epochs derived from the

TABLE III

EPOCHS FOR $v=b$; REVISION OF PERIOD

E	J.D. Epoch $v=b$	Wt.	Red. to Sun	$O-C_1$	Cal. Epoch	$O-C_2$
−988	7742.783	1	−2	−1	7742.786	−5
935	7762.796	1	−1	+14	7762.785	+10
869	7787.690	1	+1	+6	7787.689	+2
−320	7794.853	1	−3	+10	7994.846	+4
0	8115.588	1	−1	0	8115.593	−6
+6	8117.857	5	−1	+5	8117.857	−1
8	8118.604	5	−1	−3	8118.612	−9
43	8131.816	4	0	+4	8131.818	−2
45	8132.574	1	0	+7	8132.573	+1
64	8139.736	6	0	0	8139.742	−6
101	8153.703	5	+1	+6	8153.704	0
146	8170.690	1	+2	+14	8170.684	+8
167	8178.610	4	+3	+11	8178.608	+5
960	8477.841	4	−2	+12	8477.835	+4
965	8479.724	5	−1	+10	8479.721	+2
1023	8501.611	5	0	+12	8501.607	+4
1031	8504.630	6	0	+13	8504.625	+5
2004	8871.762	2	0	0	8871.772	−10
2025	8879.693	2	+1	+8	8879.696	−2
5873	0331.683	5	0	+19	0331.682	+1
+5876	0332.812	6	0	+16	0332.814	−2

Heliocentric Time = Geocentric Time $-0^d.00416 \cos(\odot - 201°)$.
The unit for the reduction to the sun and for $O-C$ is 0.001 day.

Mount Wilson photo-visual observations on J.D. 2420331–2, these are the data used for the revision of the elements of variation.

For the derivation of final elements the following provisional values were used

$$m_{v=b}=\text{J.D. } 2418115.587+0.377333\ \text{E} \qquad (2)$$

The epoch is that of elements (1) corrected by -0.039 day, which is an estimate for the interval separating $v=\max.$ from $v=b$. The period is a value by Pračka[1] based on an interval of 1185 periods.

The representation by elements (2) is shown by the differences $O-C_1$ in Table III. Discussed in the usual manner by least squares, these residuals give the corrections and probable errors,

$$E-E_0=+0.0057\pm0.00085 \text{ days}$$
$$P-P_0=+0.00000206\pm0.00000035 \text{ days},$$

whence

$$\left.\begin{array}{l} E_{v=b}=\text{J.D. } 2418115.593, \text{ G.M.T.} \\ P=0.37733506 \text{ days} \end{array}\right\} \qquad (3)$$

The very small uncertainty in the period, $\pm0\overset{s}{.}03$, is to be attributed to the long interval of seven years, or nearly seven thousand complete cycles of variation, covered by the observations. The residuals $O-C_2$ corresponding to the revised constants are all small. The epoch numbers show that the rather conspicuous grouping of signs can scarcely be accepted as evidence of irregularity in the period. The residuals for Pračka's epochs[2] ($E=-1185$ and -1143), neither of which was used in revising the elements, are $+0.002$ and $+0.084$ days, respectively.

The mean visual light-curve has been derived from the 491 observations made with the wedge photometer, those with the Zöllner instrument being omitted because of large accidental error (see Table II). The values of $v-b$ were arranged in the order of heliocentric phase and divided into 29 groups of 17 each (16 observations in the first and last groups), whose means are in the first two columns of Table IV. The resulting light-curve, reduced to the international zero-point, is in Table V. The normal points and the curve are illustrated in the upper part of Fig. 1.

[1] *Bulletin international de l'Académie des sciences de Bohême*, **15**, 1910.

[2] *Loc. cit.*

Table IV also gives the deviation of the normals from the curve and the probable error of a single observation based upon the

TABLE IV

NORMAL POINTS FOR MEAN VISUAL LIGHT-CURVE

Phase	Mean $v-b$	Dev.	P.E. One Obsn.	Rel. Wt.	Phase	Mean $v-b$	Dev.	P.E. One Obsn.	Rel. Wt.
0.003....	−0.12	+4	± 9	12	0.136...	+0.06	+5	± 8	15
.008....	.20	+3	10	11	.160...	.24	−6	8	15
.012....	.21	−2	7	21	.188...	.25	0	12	7
.015....	.31	+4	11	8	.214...	.30	−1	12	7
.019....	.34	+1	9	12	.232...	.26	+6	11	8
.024....	.46	+5	8	16	.251...	.37	−3	12	7
.028....	.40	−6	14	5	.266...	.39	−3	12	7
.033....	.47	+1	7	23	.286...	.42	−4	14	5
.037....	.46	+3	9	14	.306...	.32	+6	8	14
.044....	.31	−5	12	6	.333...	.43	−5	9	13
.056....	.30	+3	8	16	.353...	.27	+4	10	11
.068....	.22	+2	9	13	.361...	.27	−4	10	9
.083....	.12	+1	8	15	.368...	.11	+1	11	8
.104....	− .07	+5	8	16	0.374...	+0.07	−5	±14	5
0.120....	+0.10	−5	±12	7					

TABLE V

MEAN VISUAL LIGHT-CURVE AND COLOR-CURVE

Phase	Vis. Mag.	C.I.	Phase	Vis. Mag.	C.I.	Phase	Vis. Mag.	C.I.
0.000....	10.29	+0.30	0.055...	10.04	0.00	0.220...	10.62	+0.44
.005....	10.20	.20	.060...	10.07	+ .03	.230...	10.64	.45
.010....	10.12	+ .10	.065...	10.10	.05	.240...	10.65	.45
.015...	10.05	.00	.070...	10.13	.08	.250...	10.66	.46
.020....	9.97	− .09	.075...	10.16	.10	.260...	10.67	.46
.025....	9.90	.12	.080...	10.19	.12	.270...	10.68	.47
.026....	9.88	.13	.090...	10.24	.17	.280...	10.69	.48
.027....	9.87	.14	.100...	10.28	.21	.290...	10.70	.49
.028....	9.86	.14	.110...	10.32	.24	.300...	10.70	.49
.029....	9.85	.15	.120...	10.37	.28	.310...	10.70	.50
.030....	9.85	.15	.130...	10.41	.31	.320...	10.70	.51
.031....	9.85	.14	.140...	10.44	.34	.330...	10.70	.51
.032....	9.85	.13	.150...	10.47	.36	.340...	10.69	.52
.033....	9.86	.13	.160...	10.50	.38	.350...	10.65	.52
.034....	9.86	.12	.170...	10.53	.39	.355...	10.62	.52
.035....	9.87	.12	.180...	10.55	.40	.360...	10.57	.51
.040....	9.92	.09	.190...	10.57	.41	.365...	10.49	.50
.045....	9.97	.06	.200...	10.59	.42	.370...	10.41	.45
.050....	10.01	− .03	.210...	10.61	.43	.375...	10.32	.35
0.055....	10.04	0.00	0.220...	10.62	+0.44	0.380...	10.24	+0.25

Phases are counted from instant when $v=b-0.03$ mag. To refer them to the instant of maximum brightness, subtract 0.031 from the tabular values. To obtain the ordinates of the mean photographic light-curve, add the color-index to the corresponding visual magnitude.

individual deviations (last column, Table I). The error varies considerably from point to point, which accounts for the differences in the weights given in the last column of Table IV.

The reference to the international zero-point is based upon b = 10.32 mag., a value found by comparing the mean curve defined by Table IV with the photo-visual observations after

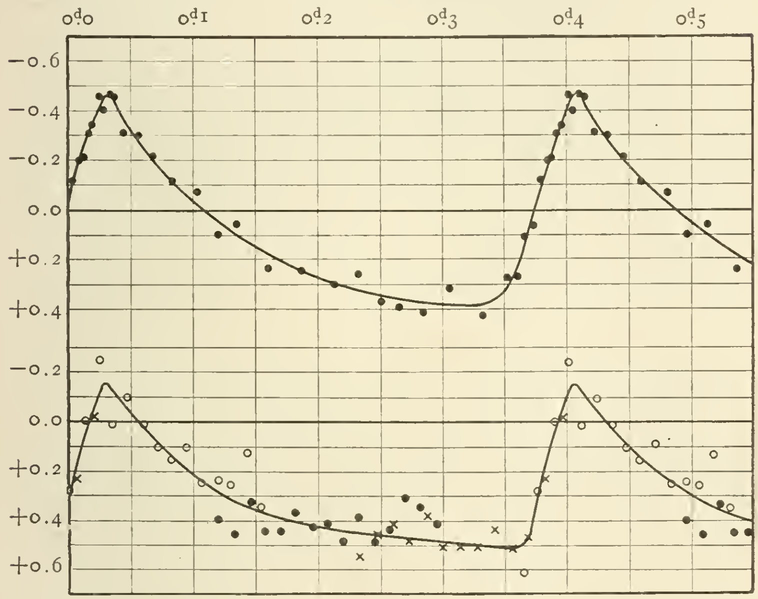


FIG. 1.—Mean visual light-curve of RS Boötis (above) and curve showing the variation of color (below). The ordinates of the lower curve are color-indices. The circles, crosses, and dots represent results obtained on different nights. The zero ordinate of the light-curve corresponds to visual magnitude 10.29.

reduction to the international system by means of polar comparisons. The photo-visual observations themselves (residuals in last column, Table XI) are well represented by the mean visual curve.

The phases had been referred, as accurately as possible, to the instant for which $v=b$. The light-curve shows that, in the mean, $v=b$ for phase −0.002; further, that the maximum visual brightness, 9.85 mag., occurs for phase 0.031. The subtraction of this

quantity will therefore refer the tabulated phases to the epoch of maximum; finally, from (3) we obtain the adopted elements

$$\text{Max.} = \text{J.D. } 2418115.624 + 0^{d}.37733506 \text{ E, G.M.T.} \qquad (4)$$

THE PHOTOGRAPHIC AND PHOTO-VISUAL OBSERVATIONS

The photographic and photo-visual observations collected in Table XI were obtained by the methods commonly used with the 60-inch reflector.[1] The plates were Seed 27's and Cramer Instantaneous Isos, respectively, the latter exposed behind a yellow filter. Relative magnitudes referred to arbitrary zero-points were determined for the comparison stars by means of images registered on the same plate with diaphragms of different aperture. The results were subsequently reduced to the international zero-point by comparisons with the North Polar Standards.[2]

The photographs of the variable were as follows:

1914, July 16. Plates 1613–1641

Photographic (odd-numbered plates): five 1^{m} exposures; apertures 32, 32, 32, 14, 9 inches; 32 and 14 used to establish scale.

Photo-visual (even-numbered plates): three 2^{m} exposures; apertures 9, 14, 32 inches; images with 9 and 14 invisible or too faint for measurement; 32-aperture images reduced with relative magnitudes of comparison stars in Table VII.

1914, July 17. Plates 1648–1677

Photographic (even-numbered plates): same as July 16.

Photo-visual (odd-numbered plates): three 2^{m} exposures; apertures 60. 40, 32 inches; 60 and 40 used to establish scale.

1914, July 18. Plates 1684–1709

Photographic (even-numbered plates): six 1^{m} exposures; apertures 32, 32, 32, 14, 14, 14 inches, with order reversed on alternate plates; reduced with relative magnitudes of comparison stars in Table VII.

Photo-visual (odd-numbered plates): four 2^{m} exposures; apertures 60, 40, 40, 60; used to establish scale.

To illustrate the method used for such data, the reduction for plate No. 1648 is given in Table VI. The first six columns contain

[1] Seares, *Mt. Wilson Contr.*, No. 80; *Astrophysical Journal*, **39**, 307, 1914.

[2] Seares, *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

star number and its distance from the optical axis of the instrument, the aperture, the scale reading (mean of two measurements), the distance correction, and the corrected reading s. The seventh column contains Δs, the difference in s for the apertures used to establish the scale (14 and 32, corresponding to a magnitude difference of 1.01).[1] By a process illustrated in Fig. 2, we find the relation between scale reading and magnitude necessary for the determination of the relative magnitudes.

TABLE VI

REDUCTION OF PHOTOGRAPHIC PLATE NO. 1648

Star	Dist.	Ap.	Mean s	D.C.	Corr. s	Δs 14–32	Mag.	Mean Mag.	Zero-Point	Final Mag.	Resid.
1.....	3.2	32	10.0	−0.1	9.9	1.8	1.35	1.40	8.36	9.71	+0.05
		32	10.0	−0.1	9.9		1.35				
		32	10.0	−0.1	9.9		1.35				
		14	11.8	−0.1	11.7		1.37				
		9	14.2	+0.1	14.3		1.60				
2.....	3.4	32	11.6	0.0	11.6	2.1	2.30	2.31	8.29	10.62	−0.02
		32	11.4	−0.1	11.3		2.15				
		32	11.6	0.0	11.6		2.30				
		14	13.6	0.0	13.6		2.27				
		9	16.8	−0.1	16.7		2.53				
3.....	2.6	32	12.4	0.0	12.4	2.3	2.71	2.83	8.27	11.14	−0.04
		32	12.8	−0.1	12.7		2.86				
		32	12.7	−0.1	12.6		2.82				
		14	14.7	+0.2	14.9		2.82				
		9	18.0	−0.1	17.9		2.96				
5.....	3.6	32	15.0	+0.3	15.3	2.7	3.97	3.85	8.33	12.16	+0.02
		32	14.6	+0.2	14.8		3.79				
		32	14.6	+0.2	14.8		3.79				
		14	17.8	−0.1	17.7		3.86				
		9	tf	......	......		......		Mean		
Var...	4.0	32	12.7	−0.1	12.6	.....	2.82	2.80	8.31	11.11	......
		32	12.4	0.0	12.4		2.71				
		32	12.4	0.0	12.4		2.71				
		14	14.8	+0.3	15.1		2.89				
		9	17.7	−0.1	17.6		2.87				

We first plot the value of Δs for each star against the mean s for the three 32-aperture images—the largest aperture used. This gives the lower curve in Fig. 2, whose significance is as follows: Let s and Δs represent the abscissa and ordinate of any point on

[1] Seares, *Mt. Wilson Contr.*, No. 80, p. 11; *Astrophysical Journal*, **39**, 317, 1914.

the curve, and m the magnitude corresponding to the scale reading s; $s'=s+\Delta s$ will then be the reading for magnitude $m'=m+\Delta m$, where $\Delta m=1.01$, the reduction constant of the 14-aperture as compared with that of 32 inches. All readings refer to the 32-aperture.

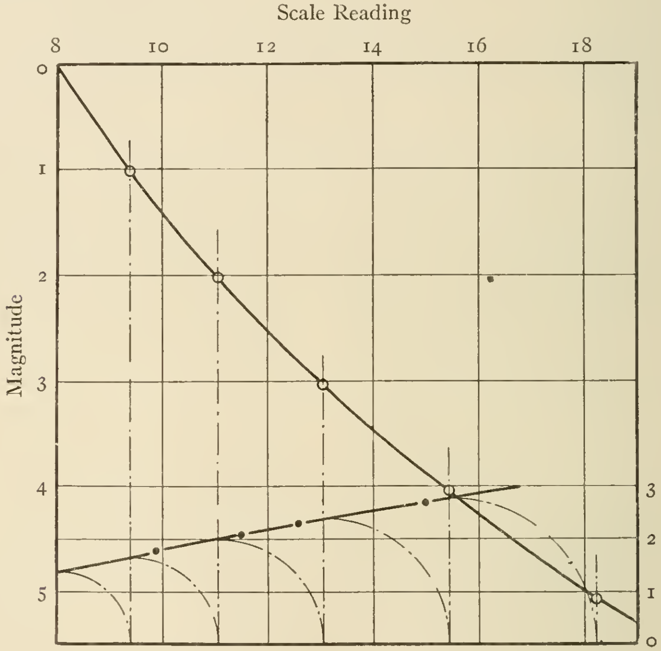


FIG. 2.—The ordinates of the lower curve are values of Δs, indicated in scale intervals on the right. This curve is used to construct the upper curve, which shows the relation between scale reading and magnitudes for Plate 1648.

With the aid of this curve we next construct the scale reading and magnitude curve in the upper part of Fig. 2. Since for the present the zero-point is arbitrary, we assume $m=0$ for $s=8$ and thus begin the accompanying tabulation. From the Δs curve, for abscissa 8 we find the ordinate 1.40, and derive $m'=0.00+1.01=1.01$ as the magnitude for scale reading $s'=8.00+1.40=9.40$. Inserting these numbers in the second line of the table, the process is repeated with m' and s' as a new m and s. Thus, for abscissa 9.40, we have the ordinate 1.65, which gives 11.05 as the scale

reading for $m' = 2.02$. A plot of the corresponding values of s and m thus found gives the required curve. The operations can be even more quickly accomplished with a pair of dividers and a ruler, by a process sufficiently obvious from the figure.[1]

m	s	Δs
0.00	8.00	1.40
1.01	9.40	1.65
2.02	11.05	2.00
3.03	13.05	2.38
4.04	15.43	2.78
5.05	18.21	....

From this curve we now read the magnitudes entered in the eighth column of Table VI, care being taken in the case of exposures with the 14- and 9-apertures to subtract the relative reduction constant—1.01 or 1.97—in order that all the results may refer to the 32-aperture.

Each photographic plate for July 16 and 17 and each photovisual plate for July 17 and 18 gave a series of relative magnitudes similar to that in the ninth column of Table VI. The results for the comparison stars were reduced to a common, though still arbitrary, zero-point and combined into the means which appear in Table VII. The internal agreement is illustrated by Table VIII,

TABLE VII

Comparison Stars—Relative Magnitudes

Star	B.D. No.	α 1855	δ 1855	Photographic Mag.			Photo-visual Mag.		
				July 16	July 17	Mean	July 17	July 18	Mean
1...	+32°2485	14h26m 8s9	+32°16′8	0.30	0.24	0.27	0.47	0.50	0.48
2...	....	26 7.7	16.7	1.06	1.10	1.08	1.23	1.19	1.21
3...	32 2487	26 17.3	22.1	1.58	1.59	1.59	1.87	1.89	1.88
4...	32 2486	26 15.2	25.1	2.75	2.81	2.77	2.11	2.10	2.10
5...	....	26 51.6	27.6	2.64	2.63	2.64	2.97	3.03	3.00

which shows the deviations of the individual plates for July 16 from the weighted means in the seventh and last columns of

[1] A numerical process, equivalent to the graphical methods given here, is described in *Mt. Wilson Contr.*, No. 80, p. 24; *Astrophysical Journal*, **39**, 330, 1914.

Table VII. Except for very faint stars, the average residual ranges from 0.04 to 0.06 mag. The systematic deviations are naturally one-half the differences between the mean results for the separate nights given in Table VII.

TABLE VIII

Comparison Star Residuals for July 17, 1914

Plate	Photographic					Plate	Photo-visual				
	1	2	3	4	5		1	2	3	4	5
1648.....	+7	−3	−4	.....	−1	1649.....	+3	−1	−3	+4	+1
1650.....	−3	+2	+5	−13	−4	1651.....	−2	+2	−7	+5	0
1652.....	+6	+1	−5	.....	−2	1653.....	−10	−1	+8	+3	+28
1654.....	0	−3	+2	−13	+1	1655.....	−19	0	−1	+20	+30
1656.....	+1	+3	−4	−4	+2	1657.....	+2	+2	−4	−1	+5
1658.....	+1	−2	+4	−7	−1	1659.....	+3	+2	−7	+1	+9
1660.....	0	−8	0	+4	+9	1661.....	+12	+2	−5	−9	−11
1662.....	+1	−3	+4	.....	−3	1663.....	0	−6	+5	0	+7
1664.....	+4	−10	0	+10	−3	1665.....	+6	−2	+2	−6	.....
1666.....	+4	0	−9	.....	+5	1667.....	+14	+8	−9	−14	−17
1668.....	+13	−6	−5	.....	0	1669.....	−2	0	+4	−2	+2
1670.....	+7	−2	−5	.....	−1	1671.....	−4	−5	+4	+6	.....
1672.....	+2	−2	+5	.....	−7	1673.....	−1	−9	+15	−5	−21
1674.....	+9	+7	−4	.....	−11	1675.....	+8	−7	+3	−5	.....
1676.....	+3	−5	+7	.....	.....	1677.....	+11	−7	+4	−6	.....
Syst. diff.	+4	−2	−1	−4	−1	Syst. diff.	+1	−1	+1	−1	+3
Av. dev..	±4	±4	±4	±8	±4	Av. dev..	±6	±4	±5	±6	±12

REDUCTION TO INTERNATIONAL ZERO-POINT

The reduction to the international zero-point depends on four pairs of comparisons with the Polar Standards, made on August 25 and 26, 1917. The results for the comparison stars in the second and third columns of Table IX are based directly on the scale and zero-point of the Standards themselves. By comparison with the mean relative magnitudes in Table VII, we find for the latter the zero-point corrections in the fifth and sixth columns of Table IX. Adding the mean correction to the relative magnitudes, we have, finally, a second series of magnitudes, given in the last three columns of the table. These refer to the international zero-point, but depend upon the scales established for the individual plates. We thus have for the comparison stars two series of results, quite independently derived. Table X exhibits the agreement and gives the

adopted magnitudes, which are the means, with equal weight, of the two determinations.

Incidentally, attention may be directed to the color-indices of the comparison stars. The values in Table IX found by comparing photographic and photo-visual magnitudes agree well with each other and with those in the last column of Table X derived by the exposure-ratio method (three plates),[1] which affords a valuable control.

TABLE IX

MAGNITUDES OF COMPARISON STARS

Star	From N.P. Comp.			Zero-Point		From Rel. Mags.		
	Pg.	Pv.	C.I.	Pg.	Pv.	Pg.	Pv.	C.I.
1......	9.73	9.18	0.55	9.46	8.70	9.78	9.24	0.54
2......	10.62	10.01	0.61	9.54	8.80	10.59	9.97	0.62
3......	11.10	10.63	0.47	9.51	8.75	11.10	10.64	0.46
4......	12.23	10.89	1.34	9.46	8.79	12.28	10.86	1.42
5......	12.22	....	....	9.58	....	12.15	11.76	0.39
Means	....	....	....	9.51	8.76	....	....	....

TABLE X

INTERNAL AGREEMENT AND ADOPTED MAGNITUDES

Star	N.P. Comp. *minus* Rel. Mag.			Adopted Mag. and Color			C.I. from Exp. Rat.
	Pg.	Pv.	C.I.	Pg.	Pv.	C.I.	
1..............	−5	−6	+1	9.76	9.21	0.55	0.56
2..............	+3	+4	−1	10.60	9.99	0.61	0.66
3..............	0	−1	+1	11.10	10.64	0.46	0.30
4..............	−5	+3	−8	12.26	10.88	1.38	1.28
5..............	+7	....	....	12.18	11.76	0.42	0.37

FINAL MAGNITUDES—THE COLOR-CURVE

We are now in a position to finish the reductions in Table VI. Subtracting the magnitudes of the comparison stars in the ninth column from their adopted values in Table X, we find the mean

[1] Seares, *Mt. Wilson Communication*, No. 33; *Proceedings of the National Academy of Sciences*, **2**, 521, 1916. For a test of the colors of the North Polar Standards, see *Mt. Wilson Communication*, No. 38; *Proceedings of the National Academy of Sciences*, **3**, 29, 1917.

zero-point correction for the plate, namely, 8.31 mag., and, finally, the magnitudes and residuals in the last two columns of Table VI. The main result, however, is the magnitude of the variable, 11.11, which has been transferred to Table XI, where it appears with similar results from the other plates.

The magnitudes in Table XI—both photographic and photo-visual—have been compared with the mean visual light-curve in Table V. The photo-visual residuals in the last column of Table XI (unit = 0.01 mag.) are appreciably smaller than the visual residuals (see last column, Table I, and probable errors in Table IV); as already stated, they show that the photo-visual observations are very satisfactorily represented by the adopted visual curve.

The comparison of the photographic measures with this curve gives the color-indices of the variable in the eighth column of Table XI. These are plotted in the lower part of Fig. 1 and show at a glance the change in color paralleling the variation in light. The ordinates of the mean color-curve are in Table V alongside the visual light-curve. The deviations of the color-indices from the mean color-curve are appropriately entered in Table XI under the heading "Photographic O − C," for they are numerically equal to the deviations of the observed photographic magnitudes from the mean photographic light-curve obtained by adding the two series of ordinates in Table V. These residuals are even smaller than those in the last column of the table, owing, probably, to the larger number of exposures made on the plates from which photographic magnitudes have been determined.

The maximum and minimum brightness are: photographic, 9.70 and 11.21; visual, 9.85 and 10.70. The amplitudes, 1.51 and 0.85, respectively, are in the ratio of 1.78 to 1.

SYSTEMATIC DEVIATIONS FROM MEAN LIGHT-CURVE

The systematic deviations of the photographic and photo-visual results from the mean curves are negligibly small—in no case exceeding 0.03 mag. per night. The variation on the three nights involved seems therefore to have been quite normal.

The residuals of the much longer series of observations in Table I, on the other hand, present numerous instances of

TABLE XI

Photographic and Photo-visual Observations

Pg. and Pv. Plate Nos.	G.M. Date		Phase		Photographic		C.I.	Photo-visual	
	Pg.	Pv.	Pg.	Pv.	Mag.	O−C		Mag.	O−C
1914, July 16. J.D. 2420330									
1613–1614....	.670	.679	120	129	10.77	−12	+0.40	10.41	0
1615–1616....	.683	.690	133	140	10.88	−14	.46	10.61	−17
1617–1618....	.697	.704	147	154	10.79	+ 2	.33	10.55	− 7
1619–1620....	.708	.716	158	166	10.94	− 7	.45	10.40	+12
1621–1622....	.720	.727	170	177	10.98	− 6	.45	10.47:	+ 7
1623–1624....	.731	.738	181	188	10.92	+ 3	.37	10.64	− 7
1625–1626....	.746	.754	196	204	11.01	− 1	.43	10.70	−10
1627–1628....	.758	.766	208	216	11.03	+ 1	.42	10.59	+ 3
1629–1630....	.770	.778	220	228	11.11	− 5	.49	10.70	− 6
1631–1632....	.782	.792	232	242	11.03	+ 6	.39	10.65	0
1633–1634....	.796	.804	246	254	11.15	− 3	.49	10.71	− 5
1635–1636....	.808	.815	258	265	11.11	+ 2	.44	10.74	− 7
1637–1638....	.820	.828	270	278	10.99	+16	.31	(10.30)	(+39)
1639–1640....	.832	.842	282	292	11.04	+13	.35	invis.	......
1641.........	.846	......	296	......	11.12	+ 7	+0.42	....	......
1914, July 17. J.D. 2420331									
1648–1649....	.670	.676	365	371	11.11	−12	+0.62	10.48	− 9
1650–1651....	.682	.688	0	6	10.57	+ 2	+ .28	10.14	+ 4
1652–1653....	.695	.700	13	18	10.08	+ 7	.00	9.96	+ 4
1654–1655....	.706	.712	24	30	9.67	+13	− .24	9.71	+14
1656–1657....	.716	.723	34	41	9.88	−14	+ .02	9.93	0
1658–1659....	.729	.735	47	53	9.90	+ 4	− .09	9.96	+ 7
1660–1661....	.742	.747	60	65	10.09	+ 1	+ .02	10.01	+ 9
1662–1663....	.753	.758	71	76	10.25	− 3	.11	9.90	+27
1664–1665....	.764	.770	82	88	10.36	− 3	.16	10.25	− 2
1666–1667....	.776	.782	94	100	10.35	+10	.09	10.27	+ 1
1668–1669....	.789	.796	107	114	10.56	− 2	.25	10.37	− 3
1670–1671....	.802	.807	120	125	10.61	+ 4	.24	10.40	− 1
1672–1673....	.812	.818	130	136	10.67	+ 5	.26	10.31	+12
1674–1675....	.824	.830	142	148	10.58	+21	.13	10.54	− 8
1676–1677....	.836	.841	154	159	10.83	+ 2	+0.35	10.58	− 8
1914, July 18. J.D. 2420332									
1684–1685....	.670	.678	233	241	11.19	−10	+0.55	10.71	− 6
1686–1687....	.684	.690	247	253	11.12	0	.46	10.63	+ 3
1688–1689....	.697	.703	260	266	11.09	+ 4	.42	10.70	− 2
1690–1691....	.710	.718	273	281	11.16	− 1	.48	10.62	+ 7
1692–1693....	.725	.731	288	294	11.08	+11	.38	10.65	+ 5
1694–1695....	.737	.744	300	307	11.22	− 3	.52	10.79	− 9
1696–1697....	.751	.757	314	320	11.22	− 2	.52	10.76	− 6
1698–1699....	.766	.772	329	335	11.21	0	.51	10.89	−19
1700–1701....	.778	.784	341	347	11.13	+ 8	.44	10.80	−14
1702–1703....	.793	.800	356	363	11.13	− 1	.52	10.52	0
1704–1705....	.806	.812	369	375	10.90	− 1	.47	10.33	− 1
1706–1707....	.820	.827	6	13	10.41	− 5	+ .23	9.92	+16
1708–1709....	.834	.841	20	27	9.95	− 7	−0.02	9.72	+15

systematic divergence; for example, those for J.D. 7660, 7667, which are prevailingly positive, and for J.D. 7994, 8118, 8120, 8122, 8131, which are negative. These can scarcely be attributed to variability in the comparison stars, nor is it likely that they are the result of systematic errors of observation. The residuals are larger than the errors normally associated with observations made with the wedge photometer. Moreover, many of the measures, those for J.D. 7660, 7667, 7994, among others, were referred simultaneously to two comparison stars, one equal to or a little brighter, the other appreciably fainter than the variable, with results sensibly the same in both cases. Another peculiarity is found in the fact that divergences of a given sign tend to persist for some time; thus during the interval J.D. 7660–6716 they were positive, while for J.D. 8118–8131 they were negative. The explanation is probably to be found in abnormal fluctuations in the light of the variable, a phenomenon now apparently well established for several of the cluster-type variables.[1]

COMPARISON WITH SPECTROSCOPIC RESULTS

The large change in color—more than 0.6 mag. in the index—led naturally to an inquiry into the spectroscopic behavior of RS Boötis. Mr. Pease kindly placed the star on his program, and quickly found a change in spectral type[2] occurring simultaneously with the fluctuations in light and color which very nearly equals that to be inferred from the well-known relation between color-index and spectrum.

A detailed comparison of Pease's spectra with the color-curve of RS Boötis is given in Table XII. The date, G.M.T. of the middle of exposure, and the spectrum are from his data. The color-indices in the fourth column were interpolated from the curve in Table V, with phases counted from instants for which $v=b$. Those in the sixth column were derived from the spectra by the formula C.I.$=0.4\,S$, in which for Bo, Ao, Fo, etc., S has the

[1] Shapley and Shapley, *Mt. Wilson Contr.*, No. 104; *Astrophysical Journal*, **42**, 159, 160, 1915; Shapley, *Mt. Wilson Contr.*, No. 112; *Astrophysical Journal*, **43**, 217, 1916; also various references cited in the latter of these investigations.

[2] *Publications of the Astronomical Society of the Pacific*, **26**, 257, 1914.

numerical values −1, 0, +1, etc.[1] The differences in the two series of results—the color-excess[2]—are in the last column of the table. Ten of the thirteen values are less than a tenth of a magnitude. The amplitude of the color-variation derived from spectra is somewhat smaller than that indicated by the color-curve; but the spectra near minimum are few in number and less accordant than the others, so that conclusive results are not to be expected. Notwithstanding the slight excess of negative signs, the mean color-excess is sensibly zero.

TABLE XII

COMPARISON OF COLOR-CURVE WITH SPECTRA

Date	G.M.T. Middle of Exposure	Sp.	C.I. from Curve	Phase	C.I. from Spectrum	Color-Excess
1914						
July 20......	$17^h 16^m$	B9	−0.05	0.018	−0.04	−0.01
20......	17 57	A0	− .05	.047	.00	− .05
20......	18 40	A5	+ .11	.077	+ .20	− .09
21......	16 35	A6	+ .45	.235	+ .24	+ .21
21......	18 2	F0	+ .49	.296	+ .40	+ .09
21......	19 17	A5	+ .52	.349	+ .20	+ .32
22......	16 28	A7	+ .21	.099	+ .28	− .07
22......	18 5	A8	+ .39	.167	+ .32	+ .07
23......	17 22	A4	+ .22	.004	+ .16	+ .06
23......	18 7	B8	− .11	.036	− .08	− .03
23......	18 52	A6	+ .06	.067	+ .24	− .18
Aug. 17......	15 57	A0	− .08	.042	.00	− .08
17......	16 52	A5	+0.12	0.079	+0.20	−0.08

The negligible character of the color-excess was found from a preliminary reduction, based upon approximate zero-point determinations made in 1914, and was then a result of special interest as one of the earliest indications that the coefficient of space-absorption must be extremely small. Hertzsprung[3] and Russell[4] had found the Cepheids, as a class, to be very bright, and, in general, stars with early type spectra were known to be highly luminous

[1] Seares, *Mt. Wilson Communication*, No. 16; *Proceedings of the National Academy of Sciences*, 1, 481, 1915.

[2] *Ibid.*

[3] *Zeitschrift für wissenschaftliche Photographie*, 5, 107, 1907; *Astronomische Nachrichten*, 196, 201, 1913.

[4] *Science*, N.S., 37, 652, 1913.

and to show only a small dispersion in their absolute magnitudes. Since the median apparent brightness of RS Boötis is only 10.3, this meant that its distance must be relatively great.[1] The color, however, is only that normally associated with similar spectra in the case of much less distant stars, thus leaving no appreciable excess to be accounted for by scattering of the star's light in its passage to the earth.[2] This conclusion disregarded the influence of the star's absolute magnitude on its color; but the indications were that for stars of early-type spectra this must be very small, and, moreover, that for a highly luminous object like RS Boötis any effect of absolute magnitude would probably increase the color-excess and thus make the estimated upper limit for the absorption coefficient too large.[3]

Subsequent observations by Shapley[4] have revealed variations similar to those of RS Boötis in the spectra of other objects, including several of the longer-period variables which by themselves were formerly supposed to constitute the Cepheid class, as well as a number of the short-period cluster-type variables now also commonly classed with the Cepheids. The spectrograms by Pease and Shapley, however, were of low dispersion. For those of Pease $H\gamma$ to K was $2\frac{1}{4}$ mm and their classification could be based only on the hydrogen lines and to some extent on the K line of calcium. Thus the presence of types B8 and B9 in Table XII is not to be interpreted as meaning that the helium lines characteristic of these types were actually observed in the spectra. The classifications by Shapley are also based mainly on the lines of hydrogen, although calcium lines and the G band were used to some extent for types F and G. Still later observations by Adams and Joy,[5] with higher

[1] A recent determination by Shapley gives absolute magnitude $M = -0.3$, parallax $\pi = 0\overset{''}{.}00076$. *Mt. Wilson Contr.*, No. 153; *Astrophysical Journal*, 48, 1918.

[2] The same principle and others analogous to it have recently been applied by Shapley to the enormously distant globular clusters, with the result that the coefficient of absorption must be far below the upper limit fixed by this early inference.

[3] See summary of results by Adams and by van Rhijn given by Kapteyn in *Mt. Wilson Contr.*, No. 83; *Astrophysical Journal*, 40, 187, 1914.

[4] *Mt. Wilson Contr.*, No. 124; *Astrophysical Journal*, 44, 273, 1916, and elsewhere.

[5] *Mt. Wilson Communication*. No. 53; *Proceedings of the National Academy of Sciences*, 4, 130, 1918.

dispersion, confirm the foregoing results, so far as changes in the hydrogen lines are concerned, for a number of stars with periods greater than a day, but show little or no corresponding change in the less conspicuous features of the spectra, which are also characteristic of the various spectral classes.

Presumably this peculiar behavior occurs also in the case of the cluster-type variables. But, in spite of the many similarities connecting the two subclasses of Cepheids, the singularities in their periods and distribution, and possibly also in their motions,[1] suggest some reservation of judgment in the case of stars such as RS Boötis which thus far have not been observed with the dispersion necessary for a detailed study of their spectral characteristics. For the present, the significant result is the suggestion that the color of the Cepheid variables is more closely correlated with the behavior of their hydrogen lines than with other spectral phenomena, even though the latter may be important criteria for classification.

For several of the longer-period Cepheids the question thus raised can be put to a direct test. Adams and Joy have given spectral classifications corresponding to maximum and minimum light, based both on the hydrogen lines and on the "general spectrum"; and from various sources we can derive values of the color-variation for comparison with the spectral changes. Supplementary evidence indicating the intimate connection of color and spectral type as derived from the hydrogen lines is also available for various stars whose general spectrum, in the sense used by Adams and Joy, has not yet been observed.

The data are in Table XIII. The results by Adams and Joy for the third, fourth, and fifth stars are from *Mt. Wilson*

[1] The frequency-curve of the periods shows two maxima, one for twelve hours and one for seven days, corresponding to the two subclasses of Cepheids. The longer-period stars are closely confined to the plane of the Galaxy, while the cluster-type variables are widely scattered throughout space. So far as known, the proper motions and radial velocities of the long-period Cepheids are small or moderate; of the short-period Cepheids, relatively large. Two recent spectrograms of RS Boötis by Adams give $V = -52$ km per sec. See Hertzsprung, *Astronomische Nachrichten*, **179**, 376, 1909; **192**, 262, 1912; **196**, 205, 1913; and Shapley, *Mt. Wilson Contr.*, No. 153; *Astrophysical Journal*, **48**, December, 1918.

Communication, No. 53;[1] those for η Aquilae and δ Cephei are from spectrograms specially selected by Adams on account of their close agreement with the instants of maximum and minimum brightness, while those for XZ Cygni are unpublished results from two low-dispersion spectra, one at minimum, the other near but not actually coinciding with the maximum. The limiting spectral types by Shapley are from the curves given in *Mt. Wilson Contribution*, No. 124.[2] The changes in color given in the last three columns, corresponding to the observed spectral variations, were obtained as usual from the relation C.I. = 0.4 S.

TABLE XIII

CHANGES IN COLOR AND SPECTRUM FOR VARIOUS CEPHEIDS

Star	Amplitude		Obsd. Change in C.I.	General Spectrum Adams and Joy	Hydrogen Lines		Change in C.I. from Sp.		
	Pg.	Vis.			Adams and Joy	Shapley	General Spectrum	Hydrogen Lines A. and J.	Hydrogen Lines Sh.
η Aquilae*	1.09	0.67	0.42	F9–G1	F1–F9	F0–G1	0.08	0.32	0.44
δ Cephei†	1.25	.76	.49	G0–G0	F0–G0	F0–G2	.00	.40	.48
ζ Gemin.‡	1.00	.62	.38	F9–G0	F2–F3	....	(.04)	(.04)	.....
RT Aurigae§	1.20	.85	.35	F7–G0	F0–F8	A9–F9	.12	.32	.40
SU Cass.‖	0.47	.33	.14	F7–F8	F3–F6	A9–F4	0.04	0.12	.20
SU Cygni¶	1.15	.80	.35	....	....	A7–F6	......	......	.36
T Vulpec.¶	1.20	.80	.40	....	....	A9–G0	......	......	.44
S Sagitt.¶	1.30	.77	.53	....	....	F4–G2	......	......	.32
U Vulpec.¶	1.1	.68	.4	....	....	F8–G4	......	......	(0.24)
XZ Cygni**	1.62	1.04	0.58	....	A0–A8	....	......	(0.32)	.....

* Kohlschütter, Pg.; Luizet, Vis. *Astronomische Nachrichten*, **183**, 265, 1910.

† Wirtz, Pg. *Astronomische Nachrichten*, **154**, 327, 1901; Stebbins, Vis., *Astrophysical Journal*, **27**, 188, 1908.

‡ Wirtz, Pg. *Loc. cit.;* Vis. is the value from Argelander's observations as given by Wirtz. The only Cepheid of normal amplitude thus far observed whose hydrogen lines do not indicate an appreciable change of type.

§ Kiess, *Laws Observatory Bulletin*, **2**, 99, 1915.

‖ Parkhurst, Pg., *Astrophysical Journal*, **28**, 279, 1908; Müller and Kempf, Vis., *Astronomische Nachrichten*, **173**, 307, 1907.

¶ Pg. data by Wilkins; Vis., by various observers, collected by Wilkins, *Astronomische Nachrichten*, **172**, 316, 1906. The comparison for U Vulpeculae is unreliable; the star is probably subject to large fluctuations (cf. *Laws Observatory Bulletins*, **1**, 150, 1907); moreover, the spectra are difficult and not well distributed over the curve.

** Photometric data by Martha Betz Shapley, *Mt. Wilson Contr.*, No. 128; *Astrophysical Journal*, **45**, 182, 1917. The spectrum A0 corresponds to an instant following the maximum by 0.03 day. Since the period is only 0.47 day, the change in color derived from the spectrum is probably too small.

For the general spectrum there is little change in passing from maximum to minimum and the inferred color-changes in the

[1] *Proceedings of the National Academy of Sciences*, **4**, 130, 1918.

[2] *Astrophysical Journal*, **44**, 273, 1916.

eighth column are very small, whereas those derived from the hydrogen lines (ninth and tenth columns) are sensibly equal to the color-variation found from the photometric data (fourth column). Disregarding values in parentheses because of the peculiarities and uncertainties mentioned in the notes, the mean results are:

High-dispersion, four stars, Adams and Joy:

Color-change from general spectrum	+0.06 mag.
Color-change from hydrogen lines	+0.29

Low-dispersion, same stars, Shapley:

Color-change from hydrogen lines	+0.38 mag.
Observed change in color-index	+0.35

For the seven stars observed by Shapley which have trustworthy results the means are

Color-change from hydrogen lines	+0.38 mag.
Color-change from color-index	+0.38

The intimate connection between color and spectral type as derived from the hydrogen lines is shown clearly enough by these results; in fact, the closeness of the agreement for individual stars is rather surprising in view of the heterogeneous character of the photometric data and the uncertainties of classification, especially for low-dispersion spectrograms. On the other hand, it is equally clear that there is little or no correlation between the color of Cepheids and numerous other features of their spectra, which, for the stars in general, commonly follow the hydrogen lines in their changes throughout the normal succession of spectral types.

SUMMARY

Two independent series of observations on the cluster-type variable RS Boötis—one visual, the other photographic and photovisual—made with different instruments and separated by an interval of several years, have been discussed for the revision of the elements and the determination of mean light- and color-curves. There is some evidence of irregular fluctuations in the light, but no certain indication of a variation in the period, the adopted value for which is based on nearly 7000 cycles of variation.

The photo-visual observations are well satisfied by the mean curve based on the visual observations. The amplitude of the photographic variation is 1.78 times that of the visual. The color-index varies continuously from -0.15 mag. at maximum to $+0.52$ mag. at minimum.

The change in color agrees closely with that to be inferred from spectrograms by Pease. Bearing in mind certain anomalies in the spectroscopic behavior of other Cepheids, this result indicates that the color of these variables is more closely correlated with the characteristics of their hydrogen lines than it is with less conspicuous phenomena of the general spectrum also used for spectral classification. An examination of all the data available confirms this conclusion for a number of Cepheids.

Mount Wilson Solar Observatory
July 1918

ARC AND SPARK SPECTRA AND THE PERIODIC SYSTEM

By INGO W. D. HACKH

The purpose of this paper is twofold: first, to give a precise statement of the present status of the periodic system, and, second, to point to some very striking regularities in the arc and spark spectra of the elements. The necessity of a clear-cut statement of the modern form of the periodic system is evident from a consideration of the rapid development of chemistry and increase of our knowledge which has made the classical table of Mendeléeff obsolete. The second reason is perhaps more important, as it forms additional evidence for some modern theories concerning the structure of the atom. The regularities in the total number of lines and their intensity, connected with the electro-potential and polar numbers, are remarkable, and the following figures and tables are the result of an extensive survey of the arc and spark spectra, which perhaps may lead to a clearer understanding of subatomic phenomena. It is needless to say that the present work, so far as it concerns the spectra, has been very selective, that is, for obvious reasons very few publications from the very large amount of literature on the subject have been made the basis for this paper.

Beginning with the periodic system we may assume that under the present cosmical conditions of our earth 92 elements (with isotypes) are able to exist, and of these 92 elements only 5 remain still to be discovered, namely, the elements with atomic numbers 43, 61, 75, 85, and 87.

With these five unknown elements the periodic system is complete, for the limits are determined on the one hand by the radioactive substances, that is, those elements in which the structure has become so large and the number of electrons so great that it is more or less unstable under present cosmical conditions and causes atomic disintegration, forming smaller atoms. On the other hand we have the lightest elements, hydrogen and helium,

and although there are some indications of elements lighter than helium, it does not matter, for the first period of the present system begins with helium, and any new elements lighter may form with hydrogen a separate period (e.g., the hypothetical coronium, nebulium, protofluorine, etc.).

Having defined the limits to be H-1 and He-2, respectively, and U-92, the next task will be a suitable graphical representation of

TABLE I

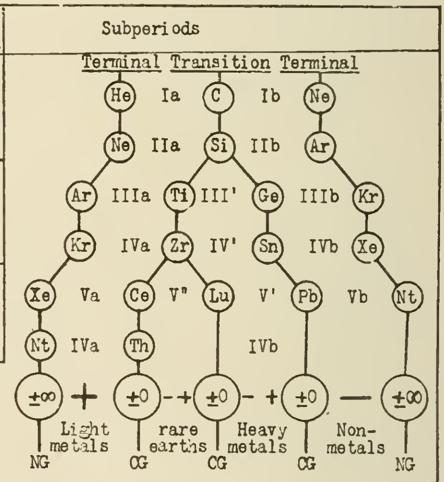
The Periods and Subperiods

Periods are determined by the noble gases (inert elements), the subperiods by the elements of the carbon group

Period	Elements from to	No. of elements
I	He - F	8
II	Ne - Cl	8
III	Ar - Br	18
IV	Kr - I	18
V	Xe - 85	32
VI	Nt - U	7

electro motive force

classes of elements
NG = noble gases
CG = carbon group



the system, which, when arranged according to increasing atomic weights, forms a continuous line, with periodic changes in the properties of its members. Mendeléeff and Meyer wrote the series simply in rows, and at that time it was the best representation. But today we know that the periods are not of equal length but become longer with higher atomic weights. We may take the noble gases as terminals for each period and have then the periods shown in Table I. In this table a further distinction of the periods into subperiods has been made, necessitated by the properties of the elements: thus the elements in I*a*, II*a*, III*a*, IV*a*, and V*a* are analogous to each other, also the elements in III′, IV′, V′, etc.

The different length of the periods is of fundamental importance, for by it alone can we explain a great number of phenomena. Assuming, for example, the noble gases, or inert elements, as having an electromotive force of $\pm\infty$, that is, having no free electrons because their electrons form a stable system of eight (or multiple of eight), and the electromotive force of the carbon-group elements as ± 0, that is, having four (or a multiple of four) free electrons, we may take the noble gases as the terminals of the periods and the carbon elements as the transition points. In each period we proceed then from $\pm\infty$ through ± 0 to $\pm\infty$. The first two periods, containing only eight elements, show therefore well-marked and characteristic electro-potentials, the curve falling from positive (Li and Na) through zero (C and Si) to negative (F and Cl). In the next two periods the beginning and end are analogous to the first two periods, the curve falling from positive (K and Rb) to zero (Ti and Zr) at the beginning, and from zero (Ge and Sn) to negative (Br and I) at the end. But between the two zero-points in each period (Ti–Ge and Zr–Sn) there is the subperiod coming in, the curve falling at first from zero (Ti and Zr) to negative (Cr and Mo) and then rising to positive (Ga and In), when it again falls to zero (Ge and Sn) and, as indicated above, to negative (Br and I), and is then connected with the following period by the noble gases (Kr and X). In the very long period of 32 elements we have similar curves of electromotive forces, for example, from positive Cs to amphoteric Ce, then follow the rare earths, and finally from negative Ta to positive Tl and through amphoteric Pb to negative Eka-iodine and the inert Nt.

Nearly all of the newer proposals for a modification of the periodic system try to embody this fundamental distinction of the different periods,[1] and the fact that the periods are unequal in length gave rise to the many different graphic representations in the form of spirals or helix.[2] All of these are more or less suitable,

[1] Batschinsky, *Zeitschrift für physikalische Chemie*, 43, 372, 1893; Werner, *Berichte*, 38, 914, 1905; Adams, *Journal of American Chemical Society*, 33, 648, 1911; Harkins, *ibid.*, 38, 169, 1916; Hackh, *Weltwissen*, 3, 63, 1915; *American Journal of Science*, 46, 481, 1918.

[2] Emerson, *American Chemical Journal*, 45, 411, 1911; Rayleigh, *Proceedings of Royal Society*, 85, 471, 1911; Scheringa, *Chem. Weekblad*, 8, 389, 1911; Rydberg, *Chemical Abstracts*, 9, 540, 1915; Soddy, *Le Radium*, 11, 6, 1914; "Chemistry of the Radioactive Elements," Part 2, 11, 1915; Loring, *Chemical News*, 111, 157, 1915.

and their acceptance depends mainly upon the taste of the individual. I myself have modified the spiral of Emerson in the way shown in Fig. 1, differing somewhat from the spirals of Soddy and Harkins. While there is no essential difference but only a different

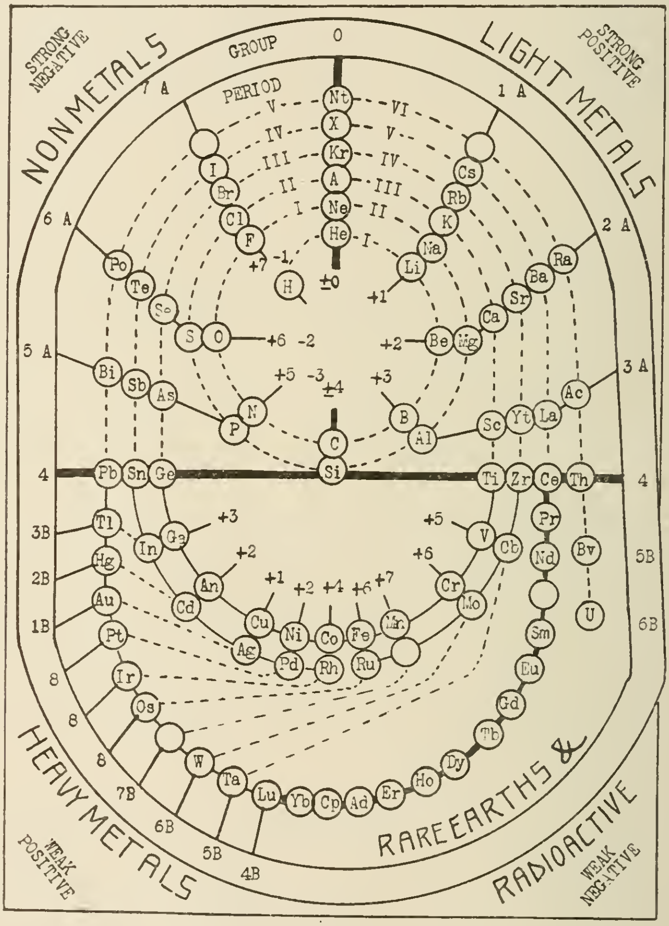


FIG. 1

appearance of the same principle, I have selected this spiral, for it makes possible the construction of a table. The great drawback of all spirals is the difficulty of reproducing them, and my aim was to embody in a table the advantages of a spiral. Thus I have

treated the upper part of the spiral in relation to the lower half as an object and its mirror-image, and constructed Table II. In this table the Roman numerals indicate the periods, while the Arabic

TABLE II

THE PERIODIC SYSTEM OF THE ELEMENTS

Atomic numbers, symbols, groups, periods

	Carbon Group 4	Phosphorus Group 5A	Sulphur Group 6A	Halogen Group 7A		0		Alkali Group 1A	Earth Alkalies 2A	Earth Metals 3A	Carbon Group 4	
V*b*	82 Pb	83 Bi	84 Po	85		86 Nt		87	88 Ra	89 Ac	90 Th	VI
IV*b*	50 Sn	51 Sb	52 Te	53 I		54 Xe		55 Cs	56 Ba	57 La	58 Ce	V*a*
III*b*	32 Ge	33 As	34 Se	35 Br		36 Kr		37 Rb	38 Sr	39 Y	40 Zr	IV*a*
II*b*	14 Si	15 P	16 S	17 Cl		18 Ar		19 K	20 Ca	21 Sc	22 Ti	III*a*
I*b*	6 C	7 N	8 O	9 F		10 Ne		11 Na	12 Mg	13 Al	14 Si	II*a*
	.			1 H		2 He		3 Li	4 Be	5 B	6 C	I*a*
III′	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	III′
IV′	40 Zr	41 Cb	42 Mo	43	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	IV′
V″	58 Ce, 59 Pr	60 Nd	61	62 Sa, 63 Eu	64 Gd	65 Tb, 66 Dy	67 Ho	68 Er	69 Ad	70 Cp, 71 Yb	72 Lu	V″
V′	72 Lu	73 Ta	74 W	75	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	V′
VI	90 Th	91 Bv	92 U									
	4	5B	6B	7B		8		1B	2B	3B	4	

numerals show the respective groups. which are identical with the main and subgroups of Mendeléeff's table. The advantages of the table are manifold. In the first place its compactness is remarkable, and by elimination of the many empty spaces of the old

table the elements have come closer together. Also similar elements are not separated by the members of the "subgroups," and still the relationship of main group A to subgroup B is preserved. A new and striking relationship is established in the fact that the similarity among the properties of the elements in (*a*) the upper half of the table is more pronounced in the *vertical direction*, and (*b*) the lower half of the table is more pronounced in the *horizontal direction.*

If we call the relationship in the vertical direction "group relation" and the relationship in the horizontal direction "period relation," it becomes apparent that, for example, the group relation of gold refers to its relation with Ag and Cu, while the period relation of gold refers to Hg and Pt. It furthermore becomes apparent that in the subperiods I*a*, II*a*, III*a*, IV*a*, V*a*, and I*b*, II*b*, III*b*, IV*b*, V*b*, the *group relation* is predominant, while in the subperiods III′, IV′, V′, and V″ the *period relation* prevails.

This fact can be explained by the constitution of the atom: that is, the excess of free electrons is either too small or too large to form the stable system of eight or a multiple of eight in the case of group relationship, while in the case of period relationship the number of free electrons is large enough to form two systems which are in equilibrium, an idea similar to the one developed by Parson[1] in the case of the "magneton."

It will be sufficient to mention only a few of the general properties of the elements prevailing in certain areas of the table.

The elements in the upper half of the table have the highest electro-potential and form mainly colorless ions and soluble compounds; they possess mostly a single valence and produce simple salts.

The elements in the lower half of the table have a weaker electro-potential and form mainly colored ions and many insoluble compounds; they possess mostly more than one valence and produce complex double salts.

On the left side of the table are the electro-negative elements which form acids and oxysalts.

[1] "A Magneton Theory of the Structure of the Atom," *Smithsonian Misc. Coll.*, **65**, No. 11 (Publication No. 2371), 1915.

On the right side of the table are the electro-positive elements which form bases and sulphides.

In the center of the lower half are the amphoteric elements, forming weak acids and weak bases.

The values for specific gravity, melting- and boiling-points, conductivity, compressibility, etc., when written in the table, show clearly that the periodicity and the relationship of these constants is seen much better than in the old table, for the reasons explained above.

Coming now to the spectra of the elements my aim was to select from the vast amount of literature such data as would enable a good comparison, that is, such results as were obtained by employing as far as possible the same method of experimentation. It is of little value to compare, for instance, the spectrum of an element obtained by a short photographic exposure with the spectrum of an element obtained by a longer exposure, for in the latter case naturally many more lines will appear. The resolving power of the grating or prism, the density of current employed, the physical condition of the substance—all these affect the appearance of the spectrum, as is well known. Then after the photograph of the spectrum has been made, the personal equation, that is, the judgment of the intensity of a line, is subject to many variations. All these factors must be considered, and a comparison of the spectrum obtained by different observers is therefore extremely difficult and of doubtful value. After a survey of the extensive literature I came to the conclusion that the work of Exner and Haschek,[1] covering a period of several decades and published in the *Proceedings of the Vienna Academy*, furnishes a good basis of comparison. In addition to this I have also consulted Eder and Valenta,[2] whose beautiful atlas is a valuable contribution to the physical literature.

The work of Exner and Haschek embraces 77 elements, of which the arc and spark spectra were photographed. The plates were projected upon a screen on which the measurements were made. The range extends from 2200 to 700 A for the arc spectra and from

[1] *Die Spektren der Elemente bei normalen Druck*, Leipzig and Wien, 1911–1912, 3 vols.

[2] *Atlas typischer Spektren*, Wien, 1911.

2100 to 6800 A for the spark spectra. The same Rowland grating and quartz lens were used throughout the investigation, and the experimental conditions were as far as possible kept within certain limits. For this reason their work represents a valuable basis for comparison. In the atlas of Eder and Valenta there are flame, arc and spark spectra, and their work is more of an encyclopedic character, for in preparing their tables they used not only their own results but also those of other investigators to supplement their own data.

The total number of lines measured by Exner and Haschek is:

61,580 lines in arc spectra
60,252 lines in spark spectra

and in Tables III and IV the total number of lines in the arc or spark spectra for each element is given, while *L* of Fig. 2 shows the periodicity of the total number of lines.

Omitting the two first periods, it will be observed that the curve rises in subperiod II*a*, III*a*, IV*a*, V*a*, that the highest peak is reached in III′, IV′, V′, respectively, and drops very low in III*b*, IV*b*, V*b*. In the figure the noble gases are indicated by a heavy line, thus starting and ending a period, while the elements of the carbon group are indicated by a broken line, thus showing the transition points.

Excepting the rare earths (subperiod from 58–72) it is evident that the curve is similar in the periods

III*a*, III′, III*b*
IV*a*, IV′, IV*b*
V*a*, V′, V*b*

and in Fig. 3 the average curve for these three periods has been constructed. The number of lines in the arc spectra (indicated by unbroken line and circle) differs little from the number of lines in the spark spectra (indicated by broken line and points), and this difference may prove of importance in a study of the physical difference of experimentation in relation to the disturbance of electrons in the atom.

The curve at the bottom of Fig. 2 is the respective intensity-curve of the arc and spark spectra, that is, the brightest line is

TABLE III

Number of Lines in Arc Spectra of the Elements

Lower section, the intensity of the brightest line or lines (Exner and Haschek)

Pb 46 1000	Bi 48 500			Nt		Ra 50 100		Th 2316 10
Sn 44 100	Sb 38 30	Te 4 5	I 0	Xe	Cs 14 200	Ba 207 1000	La 512 20	Ce 2894 10
Ge 27 50	As 18 10	Se 0	Br	Kr	Rb 19 500	Sr 146 1000	Y 684 50	Zr 1070 15
Si 40 30	P 15 5	S	Cl	Ar	K 18 200	Ca 114 1000	Sc 342 50	Ti 1123 20
C 1 3	N	O	F	Ne	Na 25 1000	Mg 52 500	Al 28 1000	Si 40 30
			H	He	Li 13 1000	Be 9 20	B 2 20	C 1 3

Ti 1123 20	V 1642 30	Cr 1697 50	Mn 865 100	Fe 2392 100	Co 1830 30	Ni 976 50	Cu 368 1000	Zn 35 200	Ga 14 30	Ge 27 50
Zr 1070 15	Cb 1770 50	Mo 3390 50		Ru 1948 50	Rh 1002 100	Pd 268 200	Ag 27 500	Cd 38 500	In 28 300	Sn 44 100

Ce 2894 10	Pr 2490 30	Nd 2762 20	Sm 1679 20	Eu 857 100	Gd 1687 20	Tb 2487 20	Dy 3312 30	Ho 1482 30	Er 2321 20	Tm′ 1007 50	Tm″	Yb	Lu

Lu	Ta 1285 15	W 3254 20		Os 1340 30	Ir 806 15	Pt 461 50	Au 35 15	Hg 78 500	Tl 22 500	Pb 46 1000
Th 2316 10		U 4940 10								

charted. It will be seen that the intensity-curve is nearly reciprocal to the curve of the number of lines, that is, as a general rule

elements whose spectrum contain few lines show a high intensity of these lines, while elements with many lines in their spectrum have

TABLE IV

NUMBER OF LINES IN SPARK SPECTRA OF THE ELEMENTS

Lower section, the intensity of brightest line or lines (Exner and Haschek)

Pb 84 500	Bi 121 100			Nt		Ra 10 50		Th 2298 10
Sn 103 30	Sb 200 50	Te 111 20	I 172 20	Xe	Cs 66 20	Ba 148 1000	La 356 50	Ce 1758 19
Ge 62 50	As 69 10	Se 63 15	Br 153 20	Kr	Rb 62 20	Sr 89 1000	Y 430 100	Zr 1529 30
Si 49 15	P 85 20	S 44 10	Cl 101 30	Ar	K 61 20	Ca 84 1000	Sc 204 100	Ti 1705 100
C 28 20	N 142 50	O 113 10	F 69 5	Ne	Na 13 20	Mg 58 500	Al 115 100	Si 49 15
			H 1 1	He	Li 12 200	Be 10 20	B 3 20	C 28 20

Ti 1705 100	V 2837 50	Cr 1806 50	Mn 1215 30	Fe 1838 20	Co 1360 30	Ni 623 15	Cu 328 200	Zn 134 500	Ga 14 20	Ge 62 50
Zr 1529 30	Cb 2086 15	Mo 3248 30		Ru 1659 20	Rh 948 20	Pd 532 50	Ag 380 100	Cd 129 200	In 30 50	Sn 103 30

Ce	Pr 1732 15	Nd 2540 15	Sm 1085 10	Eu 1508 100	Gd 1411 20	Tb 1409 20	Dy 1464 20	Ho 1222 30	Er 1785 10	Tm′ 667 20	Tm″	Yb	Lu

Lu	Ta 1560 10	W 3912 20		Os 867 10	Ir 1400 10	Pt 618 15	Au 370 20	Hg 99 200	Tl 18 30	Pb 84 500
Th 2298 10		U 5655 5								

a low intensity of these lines. It appears that number of lines (L) and intensity (I) are proportional to each other: $\frac{L}{I}=k$; but there

is the general exception that if the element is negative the intensity of the lines falls much below what should be expected.

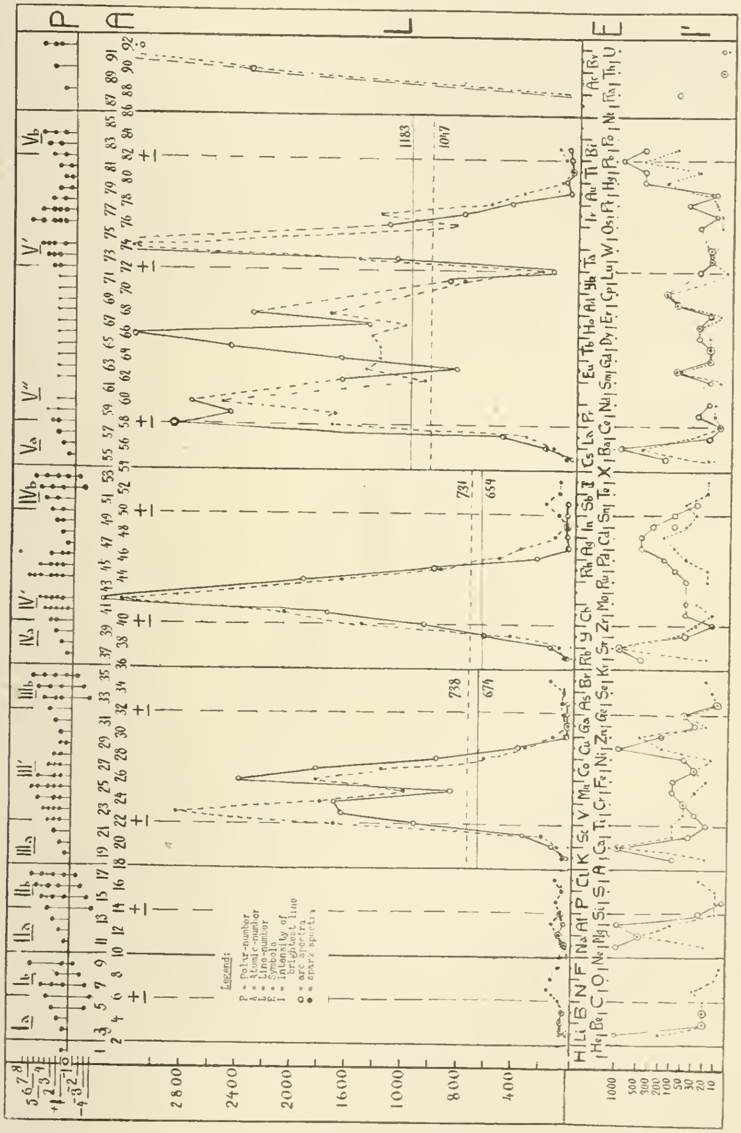


FIG. 2

The polar number of the elements is shown in the upper part of Fig. 2. The periodic character and the regular rise and fall are

very clearly represented. Each dot represents a valence; thus bromine forms typical compounds with the polar numbers -1, 1, 3,

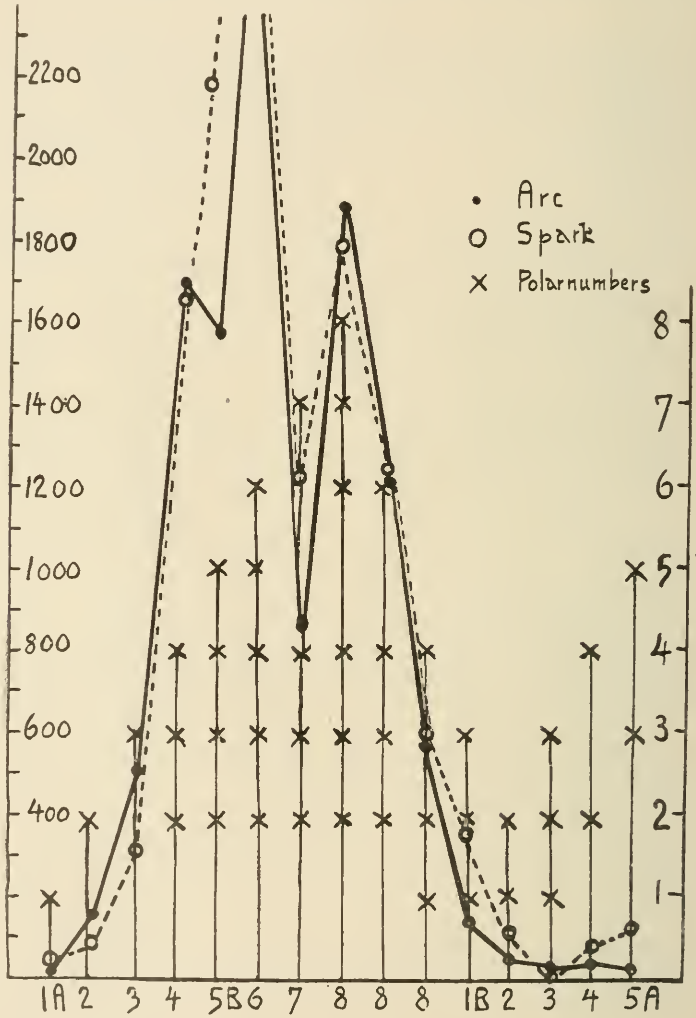


FIG 3

5, and 7, while iron compounds with the polar numbers 2 (ferrous), 3 (ferric), and 6 (ferrates) are known.

Comparison of the polar numbers with the number of lines seems to indicate a displacement in the peaks by two elements: thus V

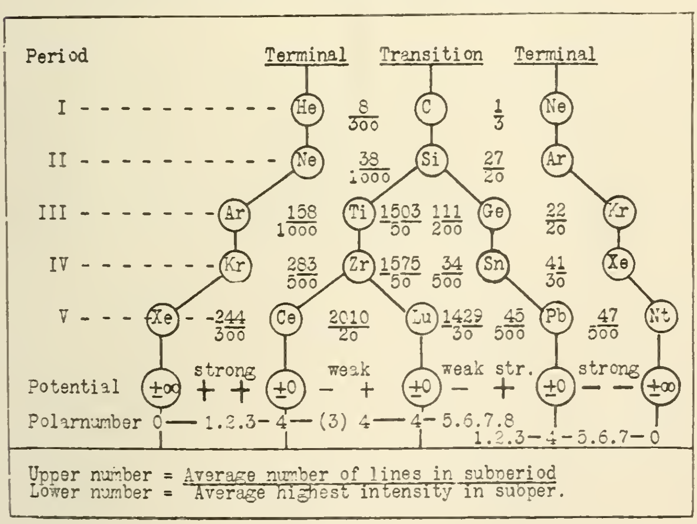
and Mn, Mo and Ru, W and Os, the peak of the number of lines preceding the peak of the polar numbers.

While the curve of the number of lines resembles the curve of the polar numbers, the curve of the maximum intensity resembles the curve of electro-potentials (which is not shown in Fig. 2), and the generalization is justified that *elements of high electro-potential give few lines* and *elements of weak electro-potential give many lines*, to which as a modifying factor comes the fact that *elements with a low polar number show very intense lines* and *elements with a high polar number show lines of weak intensity*.

This regularity is best expressed in Table V, in which L indicates the average total number of lines for a subperiod, and I the

TABLE V

The Average Number of Lines of the Arc Spectra and the Average Highest Intensity in the Subperiods



This table shows the relationship between:

a) Few lines in spectra—strong electromotive force

b) Lines of high intensity—low polar numbers (1–3)

average intensity for the brightest line. The values refer to the arc spectra, while a similar table of the spark spectra would show the same regularity. This fact supports the theory that the

spectrum of an element is not the result of a great number of homogeneous atoms capable of forming many centers of vibration, but the average result of a number of different atoms with characteristic centers of vibration. Only by this assumption at present can the relationship between number of lines and intensity be explained. Accordingly an element does not consist of uniform or homogeneous molecules or atoms, but may exist in a variety of molecules or atoms. Each single molecule or atom emits its characteristic frequencies, and we observe the characteristic lines or

TABLE VI

THE TOTAL NUMBER OF LINES IN THE ARC AND SPARK SPECTRA OF THE ELEMENTS GIVEN FOR EACH PERIOD

The numbers in the last column are approximately 8:40:320 or as 1:5:40

Period	Elements	No.	Total No. of Lines per Period in		Average No. of Lines in		Arc	Spark
			Arc	Spark	Arc	Spark		
		n	ΣL_a	ΣL_s	$\frac{\Sigma L_a}{n}$	$\frac{\Sigma L_s}{n}$	$\frac{\Sigma L_a}{n^2}$	$\frac{\Sigma L_s}{n^2}$
I........	Li-F	7	25	376	3.57	53.71	.51	7.67
II.......	Na-Cl	7	160	376	22.88	53.71	3.27	7.67
III......	K-Br	17	11461	12557	674.18	738.65	39.65	43.45
IV......	Rb-I	16	10476	11708	654.75	731.75	40.92	45.73
V.......	Cs-Bi	26	30769	27231	1183.42	1047.35	45.52	40.28
VI......	Ra-U	5	7306	7963	1461.20	1592.60	292.24	318.52

series of lines. Different lines or different series of lines of an element are emitted by different atoms, that is, atoms with a different arrangement of the electrons. A spectrum containing only a few bright lines would indicate that the atoms of the element occur only in a few species. This is the case with the alkali metals, where apparently only a few modifications of the atom with one electron exist. That is, this one electron can form only a few systems of different frequencies, and the spectrum shows only a few lines of bright intensity. A spectrum containing many weak lines would indicate that the atoms of the element occur in many species. Iron, for example, has a number of free electrons which may form many different systems, each system having its char-

acteristic vibration or vibrations mathematically connected, and the net result would be many lines of weak intensity.

This theory would not explain the non-homogeneity of the lines and their displacement by different physical agencies but would be supported by the valence theory and magneton theory of Parson.

A mathematical relationship, for which there is at present no explanation, is shown in Table VI. In this table the total number of lines of a period is given. If this sum is divided by the number of the elements, of which the lines have been counted, we get for the first two periods values of about 60 (lower values in the table must be due to incomplete spectra, which is a result of the physical form of these elements, many of them being gases), for the following two periods about 700, and for the last two periods about 1100 and 1500. If these average numbers of lines are again divided by n, the approximate values 8, 40 (320) are obtained.

CONCLUSIONS

Elements of strong E.M.F. (e.g., halogens, alkali metals) have an arc and a spark spectrum composed of few lines, while elements of weak E.M.F. (e.g., iron group, rare earth metals) have an arc and a spark spectrum of many lines.

Elements with polar numbers 1, 2, 3 have lines of great intensity, while elements with polar numbers over 5 have lines of weak intensity.

For the majority of elements (non-metals excepted) the constant $K=\frac{L}{I}$ is nearly equal, where L is the total number of lines of an element and I the arithmetical sum of the intensities of these lines.

The theory is advanced and supported that the spectrum is the average result of a certain number of different atom "species" of the same element; elements with strong E.M.F. having few species, those with weak E.M.F. having many species. These species manifest themselves by the different valences of elements.

An unexplained relationship exists among the six periods when the sum of the total number of lines of a period ΣL is divided by the number of elements n or n^2.

Berkeley, Cal.
June 1918

MINOR CONTRIBUTIONS AND NOTES

CORRECTION OF OPTICAL SURFACES

In the June, 1918, number of the *Astrophysical Journal* Professor Michelson pointed out that the method described by me in the *Philosophical Magazine* (6), **35**, 49, 1917, was unsuitable for the correction of large mirrors and lenses, since it would require for that purpose an interferometer having mirrors of equally large dimensions.

The criticism is just, as applied to the use for large lenses of the particular form of apparatus there described.

The form preferred by us for large lenses is as shown in Fig. 1 below, which is self-explanatory. Full descriptions, both of this

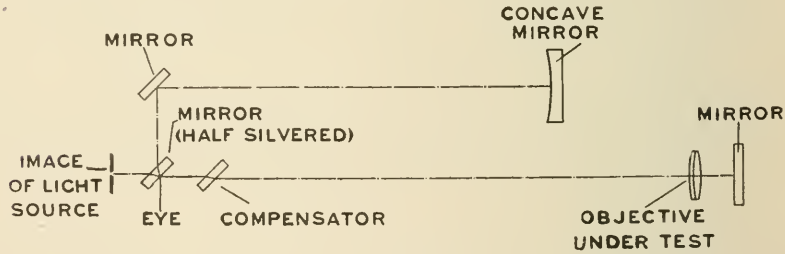


FIG. 1

form and that described in the *Philosophical Magazine*, are given in the British Patent 103832 of 1916 and United States of America Patent 1253308 of 1918. The form used for microscope lenses is given in Fig. 2, while Fig. 3 shows the method applied to a complete optical instrument, in this case a microscope.

Admittedly the method described by Professor Michelson has, when applied to the special case of a concave mirror, the undoubted advantage that no large optical element other than that under test is needed. Whether this advantage is outweighed by the fact that the test consists of a series of observations on one point after

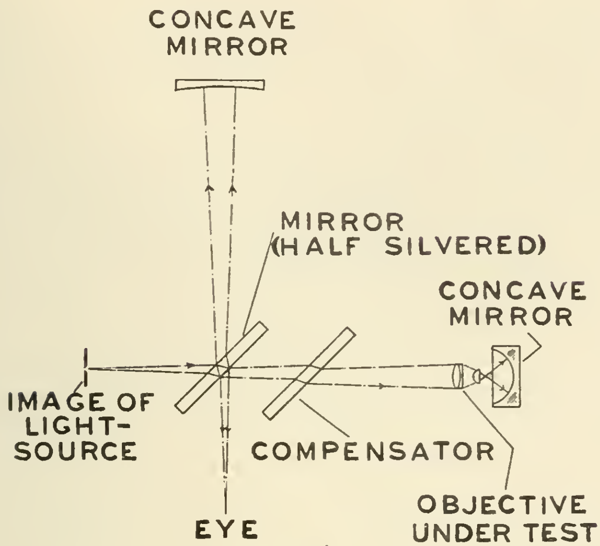

CONCAVE
MIRROR
MIRROR
(HALF SILVERED)
CONCAVE
MIRROR
IMAGE OF
LIGHT-
SOURCE
COMPENSATOR
OBJECTIVE
UNDER TEST
EYE

FIG. 2

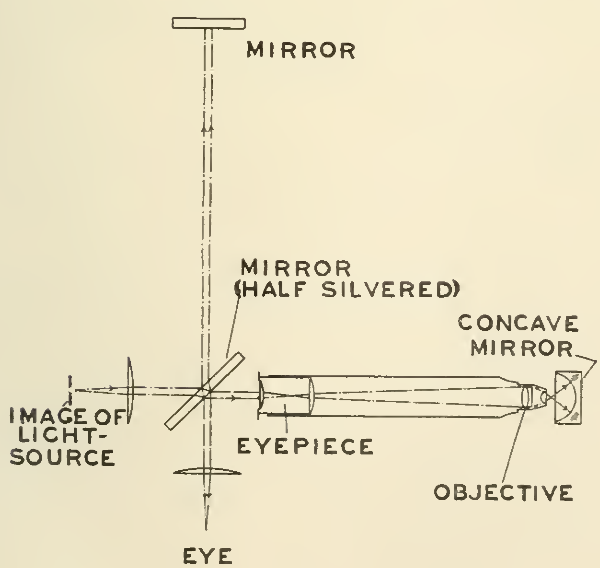

MIRROR
MIRROR
(HALF SILVERED)
CONCAVE
MIRROR
IMAGE OF
LICHT-
SOURCE
EYEPIECE
OBJECTIVE
EYE

FIG. 3

another of the element under examination (whereas in the forms of interferometer I have described the entire wave-front coming from the optical element is seen at a glance) can best be decided by experience of both methods.

Some years ago we tried a method of Dr. Chalmers which in principle is similar to that of Professor Michelson,[1] and it would seem from our experience that my own method is far more convenient for optical elements of such size as we have had occasion to correct recently. At the present time we have in our workshops ten of these interferometers in constant use, but the largest is only designed for a 5-inch beam, since recent requirements have not given us an opportunity of using the method for really large work.

I take this opportunity of acknowledging what is very obvious, namely, my indebtedness in this matter to Professor Michelson, whose original interferometers, besides serving many more important purposes, were the origins of these forms I have described. I must also not fail to make known the great part taken in developing these interferometers and their uses by Mr. Alfred Green, chief optical foreman of Adam Hilger, Ltd. It is to his fertility of suggestion that the many forms we have tried are in great part due.

F. Twyman

Adam Hilger, Ltd., London
Research Department
August 20, 1918

RADIAL VELOCITY OF 2 ω LEONIS

This well-known visual binary ($\alpha = 9^h 23^m$, $\delta = +9°30'$, $5^M.6$, spectrum G0) was stated by the writer to have a variable radial velocity over ten years ago.[2] My preliminary measures gave a change of velocity between the first and second plates of over 20 km, but no such range was found in later spectrograms, of which nineteen have been gradually obtained. With the omission of the second plate, No. IB 525, the resulting values of the radial velocity are as follows. The measures of 17 plates were made by Mr. E. P. Hubble in

[1] For description see *Proceedings of the Optical Convention* (1912), p. 156. Published by the University of London Press.

[2] *Astrophysical Journal*, 25, 61, 1907.

familiarizing himself with the use of the Hartmann spectrocomparator early in 1916. Two plates taken later were recently measured by Miss Evelyn W. Wickham, who used the Gaertner machine, with reduction by the Cornu-Hartmann formula.

Plate	Date		G.M.T.	Taken By	Velocity	Quality
IB 501......	1905	Feb. 3	$21^h 50^m$	B, S	− 7.6 km	Good
715......	1906	Mar. 30	16 10	F, S	− 3.6	Good
925......		Dec. 14	20 17	B, S	− 5.3	Good
934......		Dec. 17	23 00	B, S	− 0.4	Good
940......	1907	Jan. 4	23 00	B, S	− 6.8	Good
1540......	1908	Mar. 20	18 09	L, B, S	− 6.6	Good
1560......		Apr. 11	17 19	L, S	− 5.9	Good
1568......		Apr. 12	16 23	L, S	− 6.0	Very weak
1575......		Apr. 13	15 40	B, S	− 4.8	Weak
1583......		Apr. 18	14 38	L, S	− 0.8	Weak
1589......		Apr. 20	15 01	B, S	− 1.8	Weak
1940......		Dec. 28	22 20	B, S	− 7.2	Good
1966......	1909	Jan. 25	20 05	B, S	−10.2	Good
4081......	1915	Mar. 30	17 13	F, S	− 4.4	Fair
4352......	1916	Jan. 7	20 50	Cr, S	− 3.4	Fair
4363......		Jan. 17	21 28	Cr, S	− 1.6	Fair
4372......		Jan. 18	20 30	H, S	+ 1.4	Very good
4455......		Apr. 14	15 55	H, S	− 8.0	Weak
4859......	1917	Apr. 6	14 59	M, S	− 3.3	Good

B = Barrett; Cr = Crump; F = Frost; H = Hubble; L = Lee; M = Monk; S = Sullivan.

While examining the plates of this star, Mr. George S. Monk applied to the second plate, IB 525, the criteria for type in use at Mount Wilson. He found its spectrum to be about K3, while the other plates of 2 ω Leonis yielded G0 to G2. He suggested that this plate was not of 2 Leonis but of the neighboring star, 3 Leonis ($5^M.8$), which has the same right ascension and is almost 1° south in declination. The spectrum is classed as K0 in *Harvard Annals*, **50.** We then arranged to take a plate of 3 Leonis, which was measured by Mr. Monk with the result $\rho = +20$ km. This is in agreement with my preliminary measure of +18 km for IB 525, for which Mr. Hubble obtained +24.4 with the Hartmann spectrocomparator. I therefore believe that Mr. Monk's suggestion is correct, that IB 525 is a plate of 3 Leonis, an error having been made in picking up the star in the finder. The basis for the original statement as to 2 ω Leonis is thus removed. The range of 11 km

in the measures of one-prism plates given above leaves the star still open to suspicion, but hardly establishes a variation. If considered constant, the mean radial velocity from the period 1905 to 1918 may be taken as -4 km.

The data for 3 Leonis are:

Plate	Date	G.M.T.	Taken By	Velocity	Quality
IB 525	1905 March 3	17^h53^m	F, B, S	$+24$ km	Good
4855	1917 April 2	14 53	M, S	$+20$	Good

EDWIN B. FROST

YERKES OBSERVATORY
November 1918

PRELIMINARY NOTE ON 66 ERIDANI

This star, for which $\alpha = 5^h2^m$, $\delta = -4^\circ 47'$, Mag. $= 5.2$, Spectrum = A0, is an interesting spectroscopic binary having the lines of the two components about equally strong.

The lines happened to be single on the first spectrogram, taken on October 1, 1915, but their appearance aroused my suspicions, and the next plate showed a wide separation, corresponding to a relative velocity of about 240 km per second, the components yielding $+150$ km and -90 km. These are in fact the extreme values so far obtained. After I had measured the third plate, the sudden detachment of the right retina postponed indefinitely the use of the measuring machine, and the sixty spectrograms since secured have been measured by various assistants, as will be specified when the orbit is published.

Considerable time was spent in discovering the period, in which work I was assisted by Mr. Julius Lemkowitz, then computer here. It is closely 5.52 days, and if a few plates are obtained this season at the most suitable phase, it should be determinate to the third or fourth decimal, so that we shall have the necessary material for a good orbit. The period during which the components cannot be separately measured is something over six hours. The preliminary velocity-curve implies a small eccentricity. The motion of the system is not far from $+30$ km.

EDWIN B. FROST

YERKES OBSERVATORY
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THE RADIAL VELOCITIES OF 119 STARS OBSERVED AT THE CAPE

By JOSEPH LUNT

In continuation of radial velocity work carried on with the 24-inch refractor (Victoria telescope) and the four-prism stellar spectrograph with long camera up to May, 1908, published in the *Annals of the Royal Observatory, Cape of Good Hope*, **10,** Parts 1 and 3, a new working list containing 365 stars was brought into use in July of that year.

The program extended the magnitude down to 4.5 and embraced both northern and southern stars of types A to M*b*. The brighter stars down to about magnitude 3.7 were observed with a narrower slit than were the fainter ones, and the present paper gives the radial velocities of 119 of these brighter stars in a condensed form pending a more detailed account of the work contemplated when conditions become more normal.

A considerable number of these stars (33) classified as of types A and F were abandoned, as the spectra with the dispersion employed proved to be unsuitable for accurate measurement. A list of these is given in Table VI. The velocities of the fainter stars which, with the wider slit, gave more diffuse spectra will be dealt with in a subsequent communication. As the instrument and methods of work have been fully described in the publications

referred to and in Sir David Gill's *History and Description of the Cape Observatory*, it is unnecessary to say more than that an iron spark, diffused by a ground-glass screen placed in front of the slit, was employed throughout as the source of the comparison spectrum, and that the measures were made with a Hartmann spectrocomparator. The region measured was between the iron lines:

λλ 4118.7–4528.8	in the case of	α Can. Min., standard 2147
4315.3–4584.0	" " " "	α Tauri, standard 2145
4337.2–4528.8	" " " "	Solar standards
4427.5–4603.1	" " " "	α Boötis, standard 1191
4427.5–4603.1	" " " "	α Tauri, standard 1110

The photographic work was completed at the end of 1916.

The measures were made by the writer except where otherwise stated. In some cases plates remeasured by Halm or measured by him alone are included in the mean velocities given in Table III.

The stars here dealt with are divided into two classes: 76, which appear to have constant velocities, and 43, which are either known or suspected to be spectroscopic binaries showing variable radial velocities.

Eighteen stars in the first class were very frequently observed with a view to a continuation of the solar parallax work, and as these give the best criterion of the accuracy attained, the results for these stars are given separately in Table I.

Out of the total of 1269, 561 plates of the spectra of these 18 stars were measured and 9 were rejected for abnormal discordance, the discordant velocities being given in the last column of Table I.

The table shows that for the stars of early type (α Can. Min. and π Sagittarii), in which it was possible to extend the measures into the region of shorter wave-length and larger scale, the probable error of the velocity given by a single plate was about half a kilometer and that for the stars of later type (α Arietis and α Tauri) and the solar stars 0.75 km.

The probable error of the derived velocities was 0.1 km for the early-type stars and 0.15 km for solar and later-type stars.

Of these 552 plates, 98 per cent show discordances from the means of the groups of 2.5 km or less; 94 per cent show a discordance from 0.0 to 2.0 km. It is difficult to fix the point where these

TABLE I

Cape List Number	Star	Extent of Period in Days	Middle Date	No. of Plates Included	Radial Velocity in Km	Probable Errors Km: Of Velocity R	Probable Errors Km: Of 1 Plate r	Lick Radial Velocity Km	Lick *minus* Cape Km	Plates Re-jected	Rejected Velocities
9	β Ceti	1583	1910.80	49	+12.8	±0.11	±0.77	+14.6	+1.8	2	+17.5, 1909, June 21; 9.4, 1912, July 7
24	α Arietis*	1622	" .84	35	−15.3	.14	.84	−14.0	+1.3	0	
56	α Tauri*	1633	" .58	56	+54.0	.09	.65	+55.1	+1.1	1	+56.2, 1911, Feb. 5
108	α Can. Min.*	1577	" .99	45	− 3.6	.06	.39	− 3.5	+0.1	0	
112	β Geminorum	1085	1911.32	27	+ 3.2	.15	.76	+ 3.9	+0.7	0	
143	α Hydrae	1104	1910.86	32	− 4.6	.16	.92	− 3.5	+1.1	0	
187	ϵ Corvi	1281	" .73	28	+ 5.6	.15	0.82	+ 4.8	−0.8	1	+ 0.5, 1912, Jan. 15
192	β Corvi	1272	" .77	26	− 7.8	.20	1.02	− 7.1	+0.7	0	
199	ϵ Virginis	1281	" .82	22	−14.3	.12	0.55	−13.2	+1.1	1	−18.3, 1912, Jan. 24
212	θ Centauri	1269	" .79	36	+ 1.4	.13	.80	+ 1.5	+0.1	0	
214	α Boötis	1485	" .61	33	− 5.3	.12	.69	− 3.9	+1.4	0	
234	α Serpentis	1299	" .90	26	+ 2.9	.15	.75	+ 3.4	+0.5	2	− 0.9, 1909, Feb. 15; 1.7, 1909, Mar. 1
253	ϵ Scorpii	1138	1911.20	24	− 1.9	.12	.88	− 2.2	−0.3	0	
283	δ Sagittarii	1429	1910.71	21	−20.1	.17	.77	−20.2	−0.1	0	
287	λ Sagittarii	1327	" .48	15	−43.4	.11	.52	−43.1	+0.3	1	−46.5, 1908, Aug. 31
299	π Sagittarii*	1381	" .48	21	−10.1	.12	.53	−10.5	−0.4	0	
327	β Aquarii	1486	" .80	30	+ 5.6	.15	.81	+ 6.9	+1.3	0	
335	α Aquarii	1499	" .82	26	+ 6.8	0.14	0.73	+ 7.5	+0.7	1	+ 9.7, 1912, Nov. 13
Total	18 stars	....	....	552	Means.	±0.13	±0.73	....	+0.6	9	....

* Solar (daylight) standard plates used for all stars except 24 and 56 (α Tauri, standard 2145) and 108 and 229 (α Can. Min., standard 2147).

results for radial velocity should be considered to show a real variation, but we may fix 5 km as a "range," that is, difference between highest and lowest values, which is on the border line. The mean

TABLE II

(The subscript after the velocities denotes the number of plates used)

H. R. No.	Star	Rad. Vel. (1) Cape	Rad. Vel. (2) D. O. Mills	Diff. (2)−(1)	Range	
					Cape	D. O. Mills
		km	km	km	km	km
98	β Hydri	$+21.7_6$	$+23.0_5$	+1.3	1.9	0.6
322	β Phoenicis	-0.8_6	-1.2_6	−0.4	3.2	1.8
1208	γ Hydri	$+17.6_3$	$+15.7_4$	−1.9	3.7	0.4
1829	β Leporis	-14.9_7	-13.7_5	+1.2	3.7	2.3
2040	β Columbae	$+89.3_7$	$+89.2_5$	−0.1	2.3	3.5
2326	α Argus	$+19.7_{13}$	$+20.5_{10}$	+0.8	3.6	1.1
3307	ϵ Carinae	$+11.2_8$	$+10.8_4$	−0.4	4.8	3.3
3347	β Volantis	$+28.8_5$	$+27.0_4$	−1.8	4.2	1.6
3634	λ Velorum	$+18.5_8$	$+18.2_5$	−0.3	2.6	1.9
3699	ι Carinae	$+13.5_6$	$+13.3_6$	−0.2	1.7	2.3
3803	N Velorum	-14.2_4	-14.2_5	0.0	2.0	1.7
3890	υ Carinae	$+13.5_3$	$+13.9_5$	+0.4	0.3	2.3
4630	ϵ Corvi	$+5.6_{28}$	$+4.8_6$	−0.8	5.6	1.4
4763	γ Crucis	$+20.4_7$	$+20.6_4$	+0.2	2.2	2.0
4786	β Corvi	-7.8_{26}	-7.5_4	+0.3	5.3	2.1
5287	π Hydrae	$+27.8_5$	$+27.4_4$	−0.4	2.4	2.9
5288	θ Centauri	$+1.4_{36}$	$+2.0_4$	+0.6	4.7	2.1
5463	α Circini	$+7.6_6$	$+7.7_4$	+0.1	2.5	0.9
5649	ζ Lupi	-8.9_3	-10.0_4	−1.1	1.7	1.8
5705	ϕ' Lupi	-28.1_3	-30.4_4	−2.3	2.2	2.2
6217	α Triang. Aust.	-4.0_8	-3.2_4	+0.8	2.7	1.2
6241	ϵ Scorpii	-1.9_{24}	-2.2_4	−0.3	4.5	2.4
6285	ζ Arae	-5.6_3	-6.3_4	−0.7	0.9	1.3
6461	β Arae	$+0.9_3$	-1.2_4	−2.1	1.6	1.1
6630	G Scorpii	$+25.0_3$	$+24.5_4$	−0.5	0.4	2.3
6832	η Sagittarii	0.0_3	$+0.2_4$	+0.2	1.8	2.2
6859	δ Sagittarii	-20.1_{21}	-19.7_2	+0.4	3.6	0.2
6913	λ Sagittarii	-43.4_{15}	-43.2_2	+0.2	2.0	0.6
7264	π Sagittarii	-10.1_{21}	-11.9_2	−1.8	3.1	1.6
7665	δ Pavonis	-22.8_3	-22.6_5	+0.2	2.7	3.6
7869	α Indi	$+0.1_6$	-1.6_4	−1.7	3.0	1.0
7986	β Indi	-6.2_2	-6.6_4	−0.4	0.4	2.0
8636	β Gruis*	$+1.2_8$	$+0.6_6$	−0.6	5.1	3.7
Total	33 stars	No. of plates 311	No. of plates 147		Mean range 2.8	Mean range 1.9

*Strongly suspected of variability by Wright (see *Publications of the Lick Observatory*, **9**, 315).

"range" for the 76 stars is 2.9 km; the highest values are 5.6, 5.3 (two), 5.1 (two) (see Table III).

As the period of observation is extended, some of these 76 stars may prove to be binaries. It would be easy to pick out series of

TABLE III

Cape List No.	H.R. No.	Star	α 1900	δ 1900	Mag.	Type	Standard Plates Used	No. of Plates	Mean Epoch	Radial Velocities Km: Lick* (1)	Radial Velocities Km: Cape (2)	Range Km	Diff. (1)-(2) Km	Remarks
2....	74	ι Ceti........	0^h14^m3	− 9° 23′	3.75	K	⊙	6	1910.4	+18.8	+18.8	3.5	0.0	
4....	98	β Hydri.......	20.5	−77 49	2.90	G	⊙	6	1910.5	+22.8	+21.7	1.9	+1.1	
9....	188	β Ceti........	38.6	−18 32	2.24	K	⊙	49	1910.8‡	+14.6	+12.8	4.1	+1.8	$c+13.8_4$
12....	322	β Phoenicis....	1 1.6	−47 15	3.35	K	⊙	6	1910.6	− 0.5	− 0.8	3.2	+0.3	
13....	334	η Ceti........	3.6	−10 43	3.60	K	⊙	5	1909.8	+11.6	+12.0	2.0	−0.4	
16....	437	η Piscium......	26.1	+14 50	3.72	G5	⊙	5	1909.2	+15.5	+15.5	4.4	0.0	(6) $\begin{cases}e+14.8_4\\c+13.8_4\end{cases}$
17*a*...	509	τ Ceti........	39.4	−16 28	3.65	K	2145	4	1913.7	−15.5	−16.4	2.5	+0.9	
24....	617	α Arietis†......	2 1.5	+22 59	2.23	K2	2145	35	1910.8‡	−14.0	−15.3	5.3	+1.3	(1) (16)
25....	681	ο Ceti (Mira)..	14.3	− 3 26	var.	Md	2145	5	1916.0	+62.3	+63.4	5.1	−1.1	(9*a*) $c+65.1_3$
31....	911	α Ceti†........	57.1	+ 3 42	2.82	Ma	2145	10	1909.0	−25.1	−25.4	2.6	+0.3	$c-24.4_4$
37....	1084	ε Eridani......	3 28.2	− 9 48	3.81	K	⊙	5	1908.9	+16.5	+14.8	2.0	+1.7	
42....	1208	γ Hydri.......	48.8	−74 33	3.17	Ma	2145	3	1912.9	+16.	+17.6	3.7	−1.6	
43....	1231	γ Eridani......	53.4	−13 48	3.19	K5	⊙	7	1909.8	+62.5	+62.0	1.5	+0.5	
56....	1457	α Tauri†.......	4 30.2	+16 19	1.06	K5	2145	56	1910.6‡	+55.1	+54.0	4.2	+1.1	(7*a*) $\begin{cases}a+54.1_5\\c+55.1_7\end{cases}$
64....	1654	ε Leporis......	5 1.2	−22 30	3.29	K5	2145	5	1909.6	+ 1.1	+ 1.1	2.7	0.0	
66....	1829	β Leporis†.....	24.0	−20 50	2.96	G	⊙	7	1910.5	−13.7	−14.9	3.7	+1.2	$b-12.4_1$
68....	1865	α Leporis......	28.3	−17 54	2.69	F	2147	10	1909.2	+24.9	+24.3	3.6	+0.6	
76....	2040	β Columbae†...	47.4	−35 48	3.22	K	1191	7	1910.3	+89.2	+89.3	2.3	−0.1	(3*a*)
84....	2286	μ Geminorum..	6 16.9	+22 34	3.19	Ma	2145	2	1913.2	+54.6	+56.5	1.3	−1.9	$c+56.0_3$
86....	2326	α Argus†......	21.7	−52 38	0.86ⁿ	F	2147	13	1909.2	+20.8	+19.7	3.6	+1.1	(15)
101....	2773	π Puppis†.....	7 13.6	−36 55	2.74	K5	2145	7	1909.9	+16.4	+16.2	1.5	+0.2	
108....	2943	α Can. Min.†...	34.1	+ 5 20	0.48	F5	2147	45	1911.0‡	− 3.5	− 3.6	2.1	+0.1	$c-4.6_7$
111....	2985	κ Geminorum..	38.4	+24 38	3.68	G5	⊙	3	1913.8	+20.3	+21.3	0.3	−1.0	$\begin{cases}a+19.9_5\\c+20.7_4\end{cases}$
112....	2990	β Geminorum†	39.2	+28 16	1.21	K	⊙	27	1911.3‡	+ 3.9	+ 3.2	3.9	+0.7	(17)
124....	3307	ε Carinae†.....	8 20.5	−59 11	1.74	Kp	⊙	8	1910.0	+12.	+11.2	4.8	+0.8	
127....	3347	β Volantis.....	24.6	−65 48	3.65	K	⊙	5	1911.2	+28.	+28.8	4.2	−0.8	
135....	3547	ζ Hydrae......	50.1	+ 6 20	3.30	K	⊙	6	1910.4	+23.1	+22.7	4.6	+0.4	$c+23.2_3$
139....	3634	λ Velorum†....	9 4.3	−43 2	2.22	K5	2145	8	1910.4	+19.2	+18.5	2.6	+0.7	
142....	3699	ι Carinae†.....	14.4	−58 51	2.25	F	2147	6	1910.7	+13.3	+13.5	1.7	−0.2	
143....	3748	α Hydrae†.....	22.7	− 8 14	2.16	K2	⊙	32	1910.9‡	− 3.5	− 4.6	5.1	+1.1	$c-3.5_4$
145....	3803	N Velorum†....	28.2	−56 36	3.04	K5	2145	4	1910.9	−13.5	−14.2	2.0	+0.7	
149....	3873	ε Leonis	40.2	+24 14	3.12	Gp	⊙	3	1912.5	+ 5.0	+ 5.0	2.8	0.0	$c+4.9_5$
150....	3890	υ Carinae†.....	44.6	−64 36	3.15	F	2147	3	1912.8	+13.8	+13.5	0.3	+0.3	
168....	4216	μ Velorum†....	10 42.5	−48 54	2.84	G5	⊙	5	1910.0	+ 7.4	+ 7.1	2.3	+0.3	(10)
169....	4232	ν Hydrae......	44.7	−15 40	3.32	K	2145	3	1912.2	− 1.1	− 0.3	1.9	−0.8	
182....	4540	β Virginis......	11 45.5	+ 2 20	3.80	F8	⊙	5	1911.2	+ 4.9	+ 4.1	1.9	+0.8	$\begin{cases}a+6.4_5\\c+3.9_4\end{cases}$

TABLE III—*Continued*

Cape List No.	H.R. No.	Star	α 1900	δ 1900	Mag.	Type	Standard Plates Used	No. of Plates	Mean Epoch	Radial Velocities Km: Lick* (1)	Radial Velocities Km: Cape (3)	Range Km	Diff. (1)–(2) Km	Remarks
187....	4630	ε Corvi........	12 5.0	−22 4	3.21	K	☉	28	1910.7‡	+ 4.8	+ 5.6	5.6	−0.8	
190....	4763	γ Crucis†......	25.6	−56 33	1.61	Mb	2145	7	1910.2	+22.	+20.4	2.2	+1.6	
192....	4786	β Corvi†.......	29.1	−22 51	2.84	G5	☉	26	1910.8‡	− 7.1	− 7.8	5.3	+0.7	
199....	4932	ε Virginis......	57.2	+11 30	2.95	K	☉	22	1910.8†	−13.2	−14.3	3.2	+1.1	$\begin{cases} a-12.2_5 \\ c-13.5_8 \end{cases}$
201....	5020	γ Hydrae......	13 13.5	−22 39	3.33	G5	☉	6	1910.4	− 5.6	− 4.5	3.4	−1.1	
211....	5287	π Hydrae......	14 0.7	−26 12	3.48	K	☉	6	1910.3	+27.3	+27.8	2.4	−0.5	
212....	5288	θ Centauri†....	0.8	−35 53	2.26	K	☉	36	1910.8‡	+ 1.5	+ 1.4	4.7	+0.1	
214....	5340	α Boötis†......	11.1	+19 42	0.24	K	☉	33	1910.6‡	− 3.9	− 5.3	4.4	+1.4	(7) (18)
218....	5463	α.Circini.......	34.4	−64 32	3.41	F	2147	6	1911.5	+ 7.0	+ 7.6	2.5	−0.6	
221....	5506	ε Boötis.......	40.6	+27 30	2.70	K	☉	3	1911.8	−16.4	−16.1	3.2	−0.3	(5) $\begin{cases} a-17.3_5 \\ c-15.0_6 \end{cases}$
223....	5603	σ Librae.......	58.2	−24 53	3.41	Mb	2145	3	1913.2	− 3.5	− 4.9	1.1	+1.4	
224....	5649	ζ Lupi........	15 5.1	−51 43	3.50	K	☉	3	1911.0	− 9.4	− 8.9	1.7	− 0.5	
226....	5705	φ¹ Lupi........	15.5	−35 54	3.59	K5	2145	3	1913.2	−30.2	−28.1	2.2	−2.1	
234....	5854	α Serpentis....	39.3	+ 6 44	2.75	K	☉	26	1910.9‡	+ 3.4	+ 2.9	4.6	+0.5	$\begin{cases} a+4.1_5 \\ c+3.7_6 \end{cases}$
245....	6075	ε Ophiuchi.....	16 13.0	− 4 27	3.34	K	☉	5	1910.9	− 9.2	− 9.3	2.3	+0.1	(11)
251....	6217	α Triang. Aust.†	38.1	−68 51	1.88	K2	2145	8	1910.6	− 3.6	− 4.0	2.7	+0.4	(1*a*)
253....	6241	ε Scorpii†.....	43.7	−34 7	2.36	K	☉	24	1911.2‡	− 2.2	− 1.9	4.5	−0.3	
255....	6285	ζ Arae........	50.3	−55 50	3.06	K5	2145	3	1913.8	− 6.6	− 5.6	0.9	−1.0	
259....	6299	κ Ophiuchi.....	52.9	+ 9 32	3.42	K	2145	3	1914.3	−55.9	−54.7	4.3	−1.2	(8) $c-54.7_4$
261....	6406	α Herculis.....	17 10.1	+14 30	3.48	Mb	2145	3	1914.3	−32.2	−32.4	1.1	+0.2	
262....	6461	β Arae........	17.0	−55 26	2.80	K2	2145	3	1913.5	− 1.2	+ 0.9	1.6	−2.1	
270....	6603	β Ophiuchi†....	38.5	+ 4 37	2.94	K	2145	6	1909.4	−11.8	−11.2	2.8	−0.6	(9) $\begin{cases} b-11.1_3 \\ c-11.4_6 \end{cases}$
272....	6623	μ Herculis.....	42.5	+27 47	3.48	G5	2145	3	1916.7	−15.6	−15.9§	2.9	+0.3	$c-15.5_4$
273....	6630	G Scorpii......	43.0	−37 1	3.25	K2	2145	3	1913.2	+24.5	+25.0	0.4	−0.5	
274....	6698	ν Ophiuchi.....	53.5	− 9 46	3.50	K	2145	6	1912.5	+12.9	+12.7	2.2	+0.2	(1*b*)
281....	6832	η Sagittarii....	18 10.9	−36 47	3.16	Mb	2145	3	1912.0	0.0	0.0	1.8	0.0	
283....	6859	δ Sagittarii....	14.6	−29 52	2.84	K	☉	21	1910.7‡	−20.2	−20.1	3.6	−0.1	
287....	6913	λ Sagittarii....	21.8	−25 29	2.94	K	☉	15	1910.5‡	−43.1	−43.4	2.0	+0.3	
291....	7150	ξ² Sagittarii....	51.8	−21 14	3.61	K	☉	4	1911.5	−19.9	−19.4	1.2	−0.5	(2)
299....	7264	π Sagittarii....	19 3.8	−21 11	3.02	F2	2147	21	1910.5‡	−10.5	−10.1	3.1	−0.4	
304....	7525	γ Aquilae......	41.5	+10 22	2.80	K2	☉	4	1909.2	− 1.9	− 3.0	2.6	+1.1	
310....	7635	γ Sagittae.....	54.3	+19 13	3.71	K5	2145	3	1916.4	−33.8	−31.6§	2.9	−2.2	
311....	7665	δ Pavonis......	58.9	−66 26	3.64	G5	☉	3	1913.0	−21.7	−22.8	2.7	+1.1	$\begin{cases} b-1.8_3 \\ c-1.6_7 \end{cases}$

315....	7869	α Indi.........	20 30.5	−47 38	3.21	K	⊙	6	1911.6	− 1.7	+ 0.1	3.0	−1.8	
321....	7986	β Indi.........	47.0	−58 50	3.72	K	⊙	2	1910.1	− 6.2	− 6.2	0.4	0.0	
327....	8232	β Aquarii......	21 26.3	− 6 1	3.07	G	⊙	30	1910.8‡	+ 6.9	+ 5.6	4.1	+1.3	(12)
330....	8308	ε Pegasi......	39.3	+ 9 25	2.54	K	1191	10	1909.3	+ 5.0	+ 4.8	3.4	+0.2	(14) $c+5.7_4$
335....	8414	α Aquarii......	22 0.6	− 0 48	3.19	G	⊙	26	1910.8‡	+ 7.5	+ 6.8	4.2	+0.7	(3, 19)
342....	8636	β Gruis†.......	36.7	−47 24	2.24	Mb	1110	8	1910.1	+ 1.2	+ 1.2	5.1	0.0	$c+5.9_4$
356....	8812	c² Aquarii......	23 4.1	−21 43	3.80	K	⊙	5	1908.9	+21.2	+21.4	2.0	−0.2	a)
		76 stars					Total..	852			Means...	2.9	+0.09	

* W. W. Campbell, "The Radial Velocities of 915 Stars," *Lick Observatory Bulletins*, **7**, 113.

† Previously observed at the Cape; see *Annals of the Cape Observatory*, **10**, Parts I and III.

‡ Middle date.

§ Measures by Halm.

a Emerson McMillin (correction −1.8 km applied), Lord and Maag, 12.5-inch refractor, *Astrophysical Journal*, **21**, 313, 1905.

b Yerkes, Frost and Adams, 40-inch refractor, *Astrophysical Journal*, **18**, 273, 1903.

c Bonn (correction −1.0 km applied), Küstner and Zurhellen, 30 cm refractor, *Astrophysical Journal*, **27**, 301, 1908.

(1), (1*a*), and (1*b*), 7, 2, and 3 plates measured with 1191 standard included respectively.

(2) and (4), 2 and 3 plates measured with 2145 standard included respectively.

(3) and (3*a*), 3 and 2 plates measured with solar standard included respectively.

(3) and (3*a*), 1 plate measured with 2145 standard included in each case.

(4*a*), Strongly suspected of variability by Wright (see *Publications of Lick Observatory*, **9**, 315, 19 plates, range, 5.8 km.)

(5), ε Boötis is suspected to be variable. Zurhellen's measures, 5 plates, range, 6.4 km (see *Astrophysical Journal*, **27**, 313, 1908), but it has not been included in Campbell's second list of binaries and is not noted in his list of 915 stars.

(6), η Piscium is suspected to be variable. Maag's measures, 4 plates, range, 6.6 km (see *Astrophysical Journal*, **21**, 313, 1905).

(7), α Boötis, 31 plates at Mount Wilson (Adams) give −4.3 km (see *Astrophysical Journal*, **42**, 186, 1915, and *ibid.*, **31**, 379, 1910). (Frost.)

(7*a*), α Tauri, 16 plates at Mount Wilson (Adams) give +54.0 km (see *Astrophysical Journal*, **42**, 186, 1915).

(8), κ Ophiuchi, 1 plate measured with solar standard gives for 1910, August 1, −57.3 km (included).

(9), β Ophiuchi, 3 plates measured with solar standards and 3 with 1191; duplicate measures of four plates by Halm with standard 2145 included.

(9*a*) ο Ceti (Mira) velocity from absorption lines. See also *Astrophysical Journal*, **18**, 354, 1903 (+62.7 km) and *Jour. of Roy. Astro. Soc. Canada*, **1**, 50, 1907 (+65.5 km).

(10), rejected plates, Star No. 168, μ Velorum, Plate 2942, 1911, January 2, +12.0.

(11), rejected plates, Star No. 245, ε Ophiuchi, Plate 3638, 1912, June 1, −20.5.

(12), rejected plates, Star No. 321, β Indi, Plate 3223, 1911, July 10, −0.2 faint star spectrum.

Particulars of nine other rejected plates are given in Table I.

(14), β Aquarii, 7 plates measured in duplicate with solar and α Can. Min. 2147 standards give +6.5 km and +6.3 km, respectively. Dates, 1912, June 4 to November 1.

(15), duplicate measures of 7 plates by Halm and Lunt give respectively +19.2 and +19.6 km.

$(16)\begin{cases} a\ -14.2_5 \\ b\ -13.7_3 \\ c\ -13.8_6 \end{cases}$ α Arietis

$(17)\begin{cases} a\ +3.5_5 \\ b\ +3.4_3 \\ c\ +3.8_{10} \end{cases}$ β Geminorum

$(18)\begin{cases} a\ -5.5_7 \\ b\ -4.8_5 \\ c\ -4.9_6 \end{cases}$ α Boötis

$(19)\begin{cases} a\ +4.3_5 \\ b\ +6.2_3 \\ c\ +5.0_5 \end{cases}$ ε Pegasi

The subscript after the velocity indicates the number of plates measured.

plates showing a very small range over a limited period, but these give an erroneous idea of the consistency of results which would be obtained when the period is extended over several years, and this is the experience of other observatories. The stars in Table I have been observed under severe conditions, being taken at quadratures, at opposite seasons of the year, often with the telescope first on one side of the pillar and then the other, and at very various hour angles, first east and then west, and for periods extending over from 1104 to 1583 days; and moreover ten different observers took part at various times in the guiding during the exposure of the plates. These conditions were imposed mainly by the nature of the solar-parallax problem. Had the object been only radial velocities, more uniform conditions could have been employed and the consistency of results probably enhanced.

At the same time the possibility of convection in stellar atmospheres, noted by Evershed[1] and Campbell,[2] causing variations in displacements of lines, should not be overlooked.

If we regard the wide, hazy absorption lines in stellar spectra as due to convection effects in stellar atmospheres, as suggested by Campbell, it follows that any star in which the uprush of gases predominates in the hemisphere turned toward us at the time of taking a spectrogram will show an abnormal velocity of approach and vice versa.

These convection effects may thus introduce slight apparent variations in velocity which do not belong to the star as a whole, and which would be irregular in character and may account for unexplained discordances in determinations of radial velocity extending over long periods.

Summarizing from Table I, we have the following means of the probable errors, R and r being the probable errors of a mean velocity and of a single plate respectively:

			R	r
66 plates of	2 stars	α Can. Min., standard 2147	±0.09	±0.46 km
91 plates of	2 stars	α Tauri, standard 2145	±0.12	±0.75 km
395 plates of	14 stars	Solar standards	±0.14	±0.77 km
552 plates of	18 stars	Mean	±0.13	±0.73 km

[1] *Kodaikanal Observatory Bulletin*, No. 36. [2] *Lick Observatory Bulletin*, 8, 82.

The radial velocities measured with stellar plates as standards have the further uncertainty in the assumed value of the shift of the spectrum on the standard plate, which is for

α Can. Min.	2147	Shift	− 1.22	±0.14 km
α Tauri	2145	"	+77.38	±0.14 km
α Tauri[1]	1110	"	+84.41	±0.09 km
α Boötis	1191	"	+19.03	±0.09 km

the shift being measured by comparison with 20 plates of the solar spectrum (daylight spectra) duly corrected for the earth's radial velocity with respect to the sun.

Comparison with velocities obtained at other observatories.—Radial velocities for the 76 stars of Table III have been published by Campbell in *Lick Observatory Bulletin* 7, 114–128, representing the best values obtained from the results available from all sources. The systematic difference is +0.08 km (Lick *minus* Cape). The individual differences may be divided as follows:

	km km	
Lick *minus* Cape	0.0 to 0.5	36 stars
	0.6 to 1.0	16
	1.1 to 1.5	15
	1.6 to 2.0	6
	2.1 to 2.2	3
		76 stars

The range of velocities for individual stars may be thus classified:

	km km	
Range	0.0 to 2.0	23 stars
	2.1 to 3.0	21
	3.1 to 4.0	13
	4.1 to 5.0	13
	5.1 to 5.6	6
		76 stars

Thirty-three stars common to Table III and to the program of the D. O. Mills Expedition to Chile show a systematic difference of −0.3 km (Chile *minus* Cape).

[1] *Annals of the Cape Observatory*, 10, 45 and 49.

TABLE IV

KNOWN AND SUSPECTED SPECTROSCOPIC BINARIES

Cape List No.	H.R. No.	Star	α (1900)	δ (1900)	Mag.	Type	No. of Plates 852+	Observed Range km/sec.	Published Range km/sec.	L.O.B. Reference
5	99	α Phoenicis*	0h 21m.3	−42° 51′	2.44	K	39	11.4	5.2	3 110¶
8	165	δ Andromedae	34.0	+30 19	3.5	K	3	4.8	9.0	7 102
20	566	χ Eridani†	1 52.0	−52 7	3.73	G5	5	8.2	....	7 117
48	1336	α Reticuli†	4 13.1	−62 43	3.36	G5	5	6.3	....	7 117
61	1543	π^3 Orionis†	44.4	+ 6 47	3.31	F8	6	6.6	....	7 114
78	2061	α Orionis*	5 49.8	+ 7 23	0.92	Ma	63	8.7	....	......¶
86a	2421	γ Geminorum*	6 31.9	+16 29	1.93	A	3	22.1	13.4	......¶
87	2473	ε Geminorum†	37.8	+25 14	3.18	G5	3	6.1	....	7 118
88a	2491	α Canis Maj.	40.7	−16 35	−1.58	A	9	2.1	6.8	3 81¶
91	2553	τ Puppis	47.5	−50 30	2.83	K	8	12.6	7.3	5 60
97	2693	δ Canis Maj.*‡	7 4.3	−26 14	1.98	F8p	10	3.3	3.0	5 176
98	2736	γ^2 Volantis†	9.6	−70 20	3.87	K	4	5.7	....	6 145
105	2878	σ Puppis	26.1	−43 6	3.27	K5	33	40.9	16.6	3 111¶
116	3045	ξ Puppis†	45.1	−24 37	3.47	G	6	8.2	....	7 118
117	3080	a Puppis	48.8	−40 19	3.76	G5	7	17.5	12.	3 111¶
119	3185	ρ Puppis*	8 3.3	−24 1	2.88	F5	9	5.9	8.4	2 29
130	3445	b Velorum	37.3	−46 18	4.06	F5p	3	3.3	14.	8 72
133	3482	ε Hydrae	41.5	+ 6 47	3.48	F8	3	6.8	16.1	1 23¶
157	4057	γ Leonis§	10 14.5	+20 21	2.61	K	4	2.4	....	6 24¶
178	4450	ξ Hydrae†	11 28.1	−31 18	3.72	G5	3	8.1	....	7 118
194	{4825	γ Virginis	12 36.6	− 0 54	{3.65	F	{1	....	....	......
	4826}				3.68}		2	0.0	....	......
209	5235	η Boötis	13 49.9	+18 54	2.80	G	8	14.4	16.7	3 85¶
216	5459	α_1 Centauri*	14 32.8	−60 25	{0.33	G	22	2.3	....	3 3¶
217	5460	α_2 Centauri*			1.70}	K5	12	3.7	....	3 3¶
243	6056	δ Ophiuchi†	16 9.1	− 3 26	3.03	Ma	5	10.5	....	......
247	6134	α Scorpii*	23.3	−26 13	1.22	Map	50	5.9	....	......¶
257	6148	β Herculis	25.9	+21 42	2.81	K	3	27.3	26.5	5 24¶
268	6561	ξ Serpentis	17 31.9	−15 20	3.64	A5	3	13.4	23.	3 85

271	6615	ι^1 Scorpii†	40.6	−40 5	3.14	F5p	4	6.1	....	7 127
277	6746	γ Sagittarii	59.4	−30 26	3.07	K	19	4.5	6.4	6 153
284	6869	η Serpentis†	18 16.1	− 2 55	3.42	K	6*	15.1	....	7 125
295	7234	τ Sagittarii	19 0.7	−27 49	3.42	K	6	11.6	26.	3 86
303	7417	β Cygni	26.7	+27 45	3.24	Kp	3	0.7	....	7 125¶
313	7754	α_2 Capricorni†	20 12.5	−12 51	3.77	K	3	8.8	....	....
314	7776	β Capricorni	15.4	−15 6	3.25	Gp	17	42.4	44.1	6 6¶
316	7882	β Delphini	32.9	+14 15	3.72	F5	3	9.1	14.1	6 148
322	8115	ζ Cygni	21 8.7	+29 49	3.40	K	4	1.7	7.1	4 97
325	8204	ζ Capricorni	21.0	−22 51	3.86	Gp	3	1.2	9.2	5 176
329	8278	γ Capricorni	34.6	−17 7	3.80	Fp	3	5.2	8.6	6 148
337	8502	α Toucani	22 11.7	−60 45	2.91	K2	8	13.5	8.7	6 152
343	8650	η Pegasi	38.3	+29 42	3.10	G	3	3.7	29.7	1 29¶
354	8775	β Pegasi†	58.9	+27 32	2.61	Mb	3	6.1	....	7 128
Total	....	43 stars				Total	1269	plates		

* Previously observed at the Cape; see *Annals of the Cape Observatory*, **10**, Plates I and III.

† The binary character of δ Ophiuchi, η Serpentis, and β Pegasi appears not to have been recorded previously, but is regarded as established. The ranges are 10.5, 15.1, and 6.1, respectively. In the last case the velocities show a further variation from the Lick and Yerkes values, total range 12.5 km. The four stars χ Eridani ξ Puppis, ξ Hydrae, and α_2 Capricorni, with ranges over 8 km per second are regarded as probably variable, while α Reticuli, π^3 Orionis, ϵ Geminorum, γ^2 Volantis, and ι^1 Scorpii, with ranges 5.7 to 6.6 are put on the suspected list. Further observations of these twelve stars will be made as opportunity occurs.

‡ Wright regards δ Canis Majoris as a binary having a period of nine months and a range of three kilometers, but it may be doubted whether observations have as yet reached sufficient accuracy to establish this conclusion, especially as this star gives unusually broad lines.

§ Burnham shows that the apparent motion of the companion is rectilinear and uniform (*Monthly Notices*, 58, 387). γ Leonis, therefore, is not a visual binary.

¶ For other references see the following. The figures in parentheses are Cape List numbers.

(5) Orbit (Lunt) *Astrophysical Journal*, **47**, 196, 1918.
(78) Orbit (Bottlinger) *Astronomische Nachrichten*, **187**, 33, and *Astrophysical Journal*, **44**, 250.
(86*a*) Orbit (Harper) *Jour. Roy. Astron. Soc. Canada*, **6**, 185.
(88*a*) Orbit (Zwiers) *Astrophysical Journal*, **21**, 178.
(105) Orbit (Lunt) *Astrophysical Journal*, **44**, 260; Orbit (Wilson) *Lick Obs. Bull.*, **9**, 117.
(117) *Astrophysical Journal*, **21**, 373.
(133) Visual orbit (Aitken) *Lick Obs. Bull.*, **2**, 55.
(157) *Astron. Nach.*, **147**, 92; *Astrophysical Journal*, **21**, 315, and **27**, 310.
(194) *Astron. Nach.*, **147**, 89.
(209) Orbit (Harper) *Jour. Roy. Astron. Soc., Canada*, **4**, 194.
(216, 217) Visual orbits and velocity ephemeris, etc. (Lunt) *Astrophysical Journal*, **48**, 182.
(247) Orbit (Halm) *Cape Annals*, **10**, 56C, and *Astrophysical Journal*, **44**, 250.
(257) Orbit (Plummer) *Lick Obs. Bull.*, **5**, 24.
(303) Recorded as a binary, *Lick Obs. Bull.*, **7**, 125.
(314) Orbit (Merrill) *Lick Obs. Bull.*, **6**, 6.
(343) Orbit (Crawford) *Lick Obs. Bull.*, **1**, 29.

TABLE V

H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	O.−C. km
99	......	α *Phoenicis*[a]	$0^h21^m.3$	....	
165	......	δ *Andromedae*	$0^h34^m.0$	....	
	4664	1915 Dec. 2	2 4	−9.6*	....
	4853	1916 Nov. 23	1 20	−13.8*	....
	4854	Nov. 24	1 7	−9.0*	....
566	......	χ *Eridani*[b]	$1^h52^m.0$	....	
	2060	1908 Dec. 4	2 53	−6.4	....
	2597	1909 Dec. 7	2 55	−5.8	....
	3356	1911 Nov. 16	1 3	−2.2	....
	3400	Dec. 19	3 48	−6.1	....
	3907	1912 Nov. 27	2 46	−9.9	....
	3907	Nov. 27	....	−10.4*	....
1366	......	α *Reticuli*[c]	$4^h13^m.1$	....	
	2612	1910 Jan. 10	4 54	+33.3	....
	3344	1911 Nov. 7	1 50	+37.0	....
	3361	Nov. 26	5 55	+36.2	....
	3432	1912 Jan. 24	5 39	+39.6	....
	3897	Nov. 15	2 9	+35.2	....
	3897	Nov. 15	2 9	+34.9*	....
1543	......	π^3 *Orionis*[d]	$4^h44^m.4$	....	
	1966	1908 Oct. 21	4 10	+24.3	....
	2109	1909 Jan. 9	5 20	+26.2	....
	2176	Jan. 28	5 47	+24.6	....
	2857	1910 Nov. 4	5 4	+22.6	....
	3277	1911 Sept. 13	3 48	+19.6	....
	3289	Oct. 2	3 54	+21.1	....
2061	......	α *Orionis*[e]	$5^h49^m.8$	....	
2421	......	γ *Geminorum*[f]	$6^h31^m.9$	....	
	4483	1915 Jan. 26	7 6	−17.0	....
	4715	1916 Mar. 3	6 45	+3.3*	....
	4717	Mar. 4	6 36	+5.1*	....

H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	O.−C. km
2473	......	ε *Geminorum*[g]	$6^h37^m.8$	....	
	2293	1909 Mar. 26	8 7	+7.5	....
	4702	1916 Feb. 18	6 59	+12.9*	....
	4706	Feb. 23	7 23	+13.6*	....
2491	......	α *Canis Majoris*[h]	$6^h40^m.7$	....	
	1956	1908 Oct. 13	5 10	−9.0	−1.5
	2230	1909 Feb. 16	6 39	−8.2	−0.6
	2325	Apr. 17	10 11	−8.2	−0.6
	2330	Apr. 22	9 42	−8.6	−1.0
	3501	1912 Feb. 26	6 28	−8.2	−0.2
	3502	Feb. 26	6 45	−9.1	−1.1
	3529	Mar. 11 *	7 4	−7.0	+1.0
	3535	Mar. 15	7 7	−7.2	+0.8
	3539	Mar. 16	6 49	−8.5	−0.5
2553	......	τ *Puppis*	$6^h47^m.5$	....	
	2015	1908 Nov. 16	6 0	+34.3	....
	2231	1909 Feb. 16	7 50	+32.8	....
	2263	Mar. 3	7 11	+29.7	....
	2953	1911 Jan. 18	7 54	+31.9	....
	3018	Feb. 21	8 28	+34.5	....
	3326	Oct. 26	4 35	+34.5†	....
	3865	1912 Oct. 23	5 40	+42.3	....
	4256	1914 Mar. 14	7 23	+33.1	....
2693	......	δ *Canis Majoris*	$7^h4^m.3$	....	
	1937	1908 Oct. 7	4 12	+32.2	....
	2089	Dec. 18	8 6	+32.0	....
	2196	1909 Feb. 3	6 50	+31.2 †	....
	2237	Feb. 19	8 3	+34.1	....
	2252	Feb. 27	8 34	+32.8	....
	2283	Mar. 19	7 46	+32.0	....
	2854	1910 Oct. 25	6 38	+34.5	....

	2869	1910 Nov. 13	6 14	+33.3	....
	2917	Dec. 13	7 15	+33.4	....
	2980	1911 Feb. 6	7 32	+33.3	....
2736	......	γ^2 *Volantis*	$7^h 9^m.6$	....	
	3086	1911 Apr. 1	8 56	+ 0.9	....
	3403	Dec. 22	7 53	+ 2.6	....
	3563	1912 Mar. 28	8 32	+ 2.3	....
	4692	1916 Jan. 27	5 55	+ 6.6	....
2878	......	σ *Puppis*[i]	$7^h 26^m.1$	....	
3045	......	ξ *Puppis*[j]	$7^h 45^m.1$	....	
	2104	1909 Jan. 5	8 52	+ 2.3	....
	2126	Jan. 15	5 52	+ 0.2	....
	3418	1912 Jan. 18	5 11	+ 3.4	....
	3454	Feb. 6	7 7	+ 4.2	....
	3460	Feb. 8	6 22	+ 8.4	....
	3471	Feb. 12	8 51	+ 3.8	....
3080	......	*a Puppis*	$7^h 48^m.8$	....	
	3103	1911 Apr. 15	8 53	+28.2	....
	3560	1912 Mar. 27	9 32	+15.5	....
	4553	1915 Apr. 20	10 9	+31.5	....
	4745	1916 Apr. 13	9 40	+31.8	....
	4749	Apr. 17	9 11	+31.2 *	....
	4760	May 2	9 16	+30.9	....
	4761	May 3	10 5	+33.0	....
3185	......	ρ *Puppis* (ι *Argus*)	$8^h 3^m.3$	....	
	2090	1908 Dec. 18	8 52	+45.9	....
	2127	1909 Jan. 15	8 47	+49.9 †	....
	2179	Jan. 29	9 20	+47.4	....
	2281	Mar. 16	9 26	+46.2	....
	3382	1911 Dec. 8	7 1	+49.1	....
	3420	1912 Jan. 19	6 20	+49.0	....
	3441	Jan. 29	6 25	+48.4	....

3185	4186	1913 Dec. 29	9 59	+46.5	....
	4747	1916 Apr. 14	11 14	+44.0*	....
3445	....	*b Velorum*	$8^h 37^m.3$	....	
	4286	1914 Apr. 17	10 2	+21.6	....
	4555	1915 May 3	10 30	+20.3*	....
	4557	May 4	10 16	+23.6*	....
3482	....	ϵ *Hydrae*	$8^h 41^m.5$	....	
	2318	1909 Apr. 15	9 22	+38.6	....
	2333	Apr. 26	9 50	+38.6	....
	4734	1916 Mar. 29	9 24	+31.8*	....
4057	....	γ *Leonis*	$10^h 14^m.5$	....	
	2046	1908 Nov. 30	7 30	−38.1	....
	2066	Dec. 6	8 46	−35.7	....
	2340	1909 May 4	10 45	−36.0	....
	2933	1910 Dec. 23	8 4	−36.9	....
4450	....	ξ *Hydrae*[k]	$11^h 28^m.1$	....	
	2246	1909 Feb. 25	13 6	− 9.4	....
	3150	1911 May 23	13 12	− 3.5	....
	3633	1912 May 30	12 50	− 1.3	....
4825–6	....	γ *Virginis*	$12^h 36^m.6$	....	
	4556 Np	1915 May 3	12 22	−17.2	....
	4567 Sf	May 25	13 33	−19.3 *	....
	4569 Sf	June 3	12 32	−19.3	....
5235	....	η *Boötis*[l]	$13^h 49^m.9$	....	
	2157	1909 Jan. 22	12 22	+ 1.9	....
	2214	Feb. 10	12 9	+ 4.0	....
	2421	July 17	13 50	− 4.2	....
	3039	1911 Mar. 3	13 6	− 7.4	....
	3200	June 27	13 38	+ 7.0	....
	3634	1912 May 30	14 46	− 5.2	....
	3683	July 2	14 17	− 3.9‡	....
	3714	July 16	15 24	− 3.7‡	....

* Measures by Halm.

† Mean of measures by Halm and Lunt.

‡ Wide slit.

a See *Astrophysical Journal*, **47**, 196.

b *Lick Obs. Bull.* **7**, 117, gives −5.7

c *Lick Obs. Bull.* **7**, 117, gives +35.4.

d *Lick Obs. Bull.* **7**, 114, gives +25.0.

e See *Astrophysical Journal*, **44**, 250.

f Velocities do not fit Harper's orbit.

g *Lick Obs. Bull.*, **7**, 118, gives +9.6.

h Computed values from Campbell's formula, *Astrophysical Journal*, **21**, 181.

i See *Astrophysical Journal*, **44**, 260, and *Lick Obs. Bull.* **9**, 117.

j *Lick Obs. Bull.*, **7**, 118, gives +4.2.

k *Lick Obs. Bull.*, **7**, 118, gives −4.4.

l Cape, Bonn, and Chile results show large residuals from Harper's orbit.

TABLE V—*Continued*

H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	O.−C. km
5459		α_1 *Centauri*[a]	$14^h32^m.8$		
5460		α_2 *Centauri*[a]	$14^h32^m.8$		
6056		δ *Ophiuchi*[b]	$16^h\ 9^m.1$		
	1790	1908 July 31		−25.3*‡	
	1791	July 31		−25.3	
	2799	Aug. 10		−24.7	
	3108	1911 Apr. 21		−14.8	
	3177	June 15		−18.1	
6134		α *Scorpii*[c]	$16^h23^m.3$		
6148		β *Herculis*[d]	$16^h25^m.9$		
	2337	1909 May 3		−26.4	+2.4
	3719	1912 July 18		− 5.7	+1.0
	4786	1916 June 16		−33.0*	−1.9
6561		ξ *Serpentis*	$17^h31^m.9$		
	4091	1913 July 29		−30.1	
	4807	1916 Aug. 21		−41.3*	
	4808	Aug. 21		−43.5*	
6615		ι' *Scorpii*[e]	$17^h40^m.6$		
	2289	1909 Mar. 22		−27.4†	
	2485	Sept. 4		−30.7	
	2492	Sept. 8		−31.7	
	3109	1911 Apr. 21		−25.6	
6746		γ *Sagittarii*	$17^h59^m.4$		
	2506	1909 Sept. 17		+24.1	
	2516	Sept. 25		+19.6	
	3031	1911 Feb. 27		+23.3	
	3037	Mar. 2		+22.7	
	3044	Mar. 9		+22.7	
	3074	Mar. 22		+19.6	
	3281	Sept. 29		+20.5	
	3288	Sept. 30		+20.4	
	3312	Oct. 20		+22.6	

H.R. No.	Plate No.	Star and Date	Sid. Time and R.A.	Rad. Vel. km	O.−C. km
6746	3524	1912 Mar. 6		+22.0	
	3575	Apr. 10		+21.2	
	3577	Apr. 11		+22.2	
	3582	Apr. 14		+21.3	
	3585	Apr. 15		+23.2	
	3587	Apr. 16		+20.9	
	3594	Apr. 18		+21.5	
	3786	Aug. 31		+21.2	
	3832	Oct. 4		+21.8	
	3852	Oct. 16		+21.9	
6869		η *Serpentis*[f]	$18^h16^m.1$		
	2321	1909 Apr. 15		+16.9	
	2465	Aug. 26		+ 5.0	
	2476	Sept. 2		+ 4.8	
	2841	1910 Oct. 7		+ 1.8	
	3250	1911 Aug. 19		+ 6.2	
	4103	1913 Aug. 13		+ 6.1	
7234		τ *Sagittarii*	$19^h0^m.7$		
	1939	1908 Oct. 8		+52.2	
	2493	1909 Sept. 8		+50.0	
	2523	Oct. 2		+51.4	
	2525	Oct. 7		+49.5	
	3822	1912 Sept. 26		+40.6	
	3831	Oct. 1		+43.4	
7417		β *Cygni*	$19^h26^m.7$		
	4619	1915 Sept. 24		−22.0 } *	
	4812	1916 Aug. 28		−22.1 } *	
	4813	Aug. 29		−22.7 } *	
7754		α_2 *Capricorni*	$20^h12^m.5$		
	1957	1908 Oct. 15		+ 0.7	
	2544	1909 Oct. 21		− 8.0	
	4620	1915 Sept. 28		+ 0.8*	

7776	......	*β Capricorni*[g]	**20^h15^m4**	....	....
	1930	1908 Oct. 6	....	− 7.4	
	1934	Oct. 7	....	− 7.7	−0.2
	1943	Oct. 9	....	− 8.5	
	1963	Oct. 18	....	− 8.2†	−0.9
	2335	1909 Apr. 29	....	− 2.4	+0.3
	2354	May 16	....	− 2.1	+0.4
	2482	Sept. 3	....	− 3.9	+0.2
	2490	Sept. 7	....	− 1.0	
	2517	Sept. 25	....	− 4.6 †	−0.1
	2554	Oct. 30	....	− 3.2	
	2844	1910 Oct. 14	....	−43.4	−0.9
	2846	Oct. 21	....	−41.3	
	3153	1911 May, 26	....	−25.2	0.0
	3182	June 16	....	−24.3	−0.3
	3191	June 22	....	−25.6	−1.9
	3291	Oct. 3	....	−19.7	−1.2
	3823	1912 Sept. 26	....	− 4.8	+0.8
7882	......	*β Delphini*	**20^h32^m9**	....	
	4630	1915 Oct. 18	....	−21.6	....
	4821	1916 Sept. 15	....	−22.1 *	....
	4825	Sept. 22	....	−30.7	....
8115	......	*ζ Cygni*	**21^h 8^m7**	....	
	4834	1916 Oct. 9	....	+20.0	....
	4835	Oct. 12	....	+20.4 *	....
	4836	Oct. 13	....	+20.0	....
	4837	Oct. 16	....	+21.7	....

8204	....	*ζ Capricorni*	**21^h21^m0**	....	
	1976	1908 Oct. 29	....	+ 4.3	....
	1980	Oct. 30	....	+ 4.5	....
	2011	Nov. 14	....	+ 3.3	....
8278	....	*γ Capricorni*	**21^h34^m6**	....	
	1971	1908 Oct. 22	....	−34.2†	....
	1989	Nov. 4	....	−31.1†	....
	3828	1912 Sept. 30	....	−29.0	....
8502	....	*α Toucani*	**22^h11^m7**	....	....
	1960	1908 Oct. 16	....	+49.7	....
	2823	1910 Sept. 20	....	+42.4	....
	2847	Oct. 21	....	+41.6	....
	3224	1911 July 16	....	+36.5	....
	3232	July 26	....	+36.7	....
	3292	Oct. 3	....	+37.4	....
	3343	Nov. 7	....	+36.2	....
	3702	1912 July 9	....	+38.2	....
8650	....	*η Pegasi*	**22^h38^m3**	....	
	4642	1915 Oct. 30	....	+ 5.3	....
	4840	1916 Oct. 26	....	+ 6.7 *	....
	4842	Oct. 27	....	+ 9.0	....
8775	....	*β Pegasi*[h]	**22^h58^m9**	....	
	4143	1913 Oct. 13	....	− 1.9	....
	4432	1914 Oct. 26	....	− 1.3	....
	4444	Nov. 13	....	+ 4.2	....

* Measures by Halm.

† Mean of measures by Halm and Lunt.

‡ Wide slit.

a See *Astrophysical Journal*, **48**, 182.

b *Lick Obs. Bull.*, **7**, 127, gives −19.5; Adams gives −17.5.

c See *Astrophysical Journal*, **44**, 257.

d Computed values from Plummer's orbit, *Lick Obs. Bull.*, **5**, 25.

e *Lick Obs. Bull.* **7**, 116, gives −27.8.

f *Lick Obs. Bull.*, **7**, 125, gives +9.5; Bonn gives +11.1 to +8.3.

g Computed values from Merrill's orbit, *Lick Obs. Bull.*, **6**, 6.

h *Lick Obs. Bull.*, **7**, 128, gives +8.4; Adams gives +10.6.

TABLE VI

Cape List No.	H. R. No.	Star	α 1900	δ 1900	Mag.	Type	Rad. Vel. (Lick)
18......	553	β Arietis*.......	1 49.1	+20 19	2.72	A5	− 1.
22......	591	α Hydri†........	55.6	−62 3	3.02	F	− 5.
54......	1412	θ^2 Tauri*........	4 22.9	+15 39	3.62	A5	....
65......	1666	β Eridani†......	5 2.9	− 5 13	2.92	A2	− 8.
72......	1998	ζ Leporis†......	42.4	−14 52	3.67	A2	....
88......	2484	ξ Geminorum*...	6 39.7	+13 0	3.40	F5	+27.
89......	2540	θ Geminorum‡...	46.2	+34 5	3.64	A2	+ 8.
90......	2550	α Pictoris§†.....	47.2	−61 50	3.30	A5	....
100......	2763	λ Geminorum*...	7 12.3	+16 43	3.65	A2	− 9.
102......	2777	δ Geminorum*...	14.2	+22 10	3.51	F	....
144......	3786	ψ Velorum*......	9 26.8	−40 2	3.64	F5	....
155......	4031	ζ Leonis*.......	10 11.1	+23 55	3.65	F	....
174......	4357	δ Leonis†.......	11 8.8	+21 4	2.58	A2	−18.
181......	4534	β Leonis*.......	44.0	+15 8	2.23	A2	+ 1.3
202......	5028	ι Centauri*§....	13 15.0	−36 11	2.91	A2	+ 2.0
205......	5107	ζ Virginis†......	29.6	− 0 5	3.44	A2	....
222......	5531	α^2 Libræ*........	14 45.3	−15 38	2.90	A2	....
237......	5897	β Triang. Aust.*.	15 46.3	−63 7	3.04	F	....
248......	6095	γ Herculis*......	16 17.5	+19 23	3.79	F	−39.
258......	6380	η Scorpii*.......	17 5.0	−43 6	3.44	F2	−28.
260......	6445	ξ Ophiuchi*.....	15.0	−21 1	4.46	F5	− 8.6
266......	6553	θ Scorpii*.......	30.1	−42 56	2.04	F	+ 5.
267......	6556	α Ophiuchi†.....	30.3	+12 38	2.14	A5	....
292......	7194	ζ Sagittarii¶.....	18 56.3	−30 1	2.71	A2	+22.
302......	7377	δ Aquilae*......	19 20.5	+ 2 55	3.44	F	....
306......	7557	α Aquilae†......	45.9	+ 8 36	0.89	A5	−33.
317......	7913	β Pavonis*......	20 36.0	−66 34	3.60	A5	+ 9.4
332......	8322	δ Capricorni*....	21 41.5	−16 35	2.98	A5	....
339......	8558–9	ζ Aquarii(du)*...	22 23.7	− 0 32	3.81	F5	+27.1
346......	8675	ϵ Gruis†........	42.5	−51 51	3.69	A2	....
350......	8709	δ Aquarii†......	49.3	−16 21	3.51	A2	+22.
352......	8728	α Piscis Aust.†...	52.1	−30 9	1.29	A3	+ 6.7
355......	8787	θ Gruis*........	23 1.2	−44 4	4.35	F5	+10.0

* Faint diffuse lines.
† Hα and Hδ (Mg 4481).
‡ Not observed.
§ Type stars.
¶ Plate missing.

The results for individual stars are given in Table II. They may be divided as follows:

	km km	
Chile *minus* Cape	0.0 to 0.5	18 stars
	0.6 to 1.0	6
	1.1 to 1.5	3
	1.6 to 2.0	4
	2.1 to 2.3	2
		33 stars

The mean range for Chile is 1.9 km and for the Cape 2.8 km, but the Cape results depend on 311 plates as compared with 147 plates taken in Chile.

Eleven of the stars have a range equal to or smaller than that shown by the Chile plates and 22 larger.

The 18 stars in Table I show a more consistent positive difference, viz., +0.6 km, Lick − Cape, when compared with Campbell's values in *Lick Observatory Bulletin*, No. 229.

In Table III 24 stars have also been observed at Bonn[1] by Küstner and Zurhellen. The greatest difference in velocity is −1.7 km Bonn − Cape in the case of η Piscium, a star suspected to be variable by Lord and Maag.[2] The Cape and Lick velocities agree. Only 5 stars show a difference over a kilometer. The Bonn velocities were corrected −1.0 km in accordance with the observations made on the isolated peaks near the terminator of the moon. The mean difference Bonn − Cape is 0.14 km per sec.

Eleven stars could be compared with the results of Lord and Maag[3] at the Emerson McMillin Observatory at Columbus. These show a positive difference throughout amounting to +2.1 km in the mean. Their observations of Venus taken as a check indicate that their velocities are too high (positive) by 1.8 km, and if their results are corrected by this amount the difference Columbus − Cape becomes only +0.3 km.

We have therefore the following corrected

SYSTEMATIC DIFFERENCES

76 stars	Lick − Cape	+0.1 km	(Table III)
33	Chile − Cape	−0.3	(Table II)
24	Bonn − Cape	+0.1	Correction applied −1.0 km
18	Lick − Cape	+0.6	(Table I)
11 stars	Columbus − Cape	+0.3	Correction applied −1.8 km

Mean results for each of the 76 stars considered as constant in velocity are given in Table III. Table IV gives the particulars relating to the 43 stars known or suspected to be spectroscopic binaries. It is self-explanatory. Table V gives the results of the

[1] *Astrophysical Journal*, **27**, 301, 1908. [2] *Ibid.*, **21**, 313, 1905. [3] *Ibid.*

measures of individual plates of the stars of Table IV. Thirty-three stars of the A and F types, which were included in the list for observation with a narrow slit, and which were found to have spectra unsuitable for inclusion in the present series either because of the faint and diffuse nature of the lines with the dispersion employed or the paucity of lines, are shown in Table VI.

These stars are not comparable with the type stars Sirius (A), Canopus (F), and α Can. Min. (F5), which have much heavier and sharper absorption lines well suited for measurement of velocities.

Those stars marked † generally show only the hydrogen lines $H\gamma$ and $H\delta$ and in some cases the Mg line at 4481 in the limited field of spectrum photographed. Those marked * show numerous hazy lines of feeble absorption. These stars are better dealt with by using a smaller dispersion.

Velocities for 20 of the stars in Table VI, in most cases approximate, have been published in *Lick Observatory Bulletin*, 7, 20 and 114, and are inserted in the table.

Royal Observatory, Cape of Good Hope
May 16, 1918

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

EIGHTH PAPER: THE LUMINOSITIES AND DISTANCES OF 139 CEPHEID VARIABLES

By HARLOW SHAPLEY

Aside from the error involved in the determination of the zero-point of the luminosity-period curve,[2] most of the error affecting the absolute parallax of an isolated Cepheid variable is due to the uncertainty of apparent magnitude, since the period of light-variation is usually known with an accuracy that for this work is superfluous. Scores of observers, employing numerous methods and systems of magnitudes, have participated in the discovery and observation of these stars; but observations have often been made solely for the determination of periods, little attention being given to light-curves or accurate values of maximum and minimum light. Frequently the magnitudes are referred to the system of the *Bonner Durchmusterung*, and only occasionally, for the brighter or best known stars, to the Potsdam or Harvard photometric scales.

Adding to the uncertainty in the systems of magnitude the occasional difficulty, arising from changes in the form of light-curves, in obtaining accurate mean values of maximum and minimum brightness, we must expect an average probable error in the adopted median magnitude of the order of 0.4 mag. The consequent average probable error computed for the parallaxes of stars that are normal Cepheids is about 20 per cent of the tabulated values; for a number of the brighter stars and for well-observed cluster-type variables the error is only a little more than half that amount, but possibly it attains a maximum of 50 per cent for some of the faintest stars that are scantily observed on uncertain magnitude systems. Accurate determinations of the apparent visual or

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 153.

[2] *Mt. Wilson Contr.*, No. 151, sec. III.

photovisual magnitude should reduce the probable error of the parallax of any Cepheid to less than 15 per cent.

The estimated range of 10 to 50 per cent in the probable error of the parallaxes allows for a variation in the probable error of apparent median brightness from a tenth to a whole magnitude—an amount which is shown by an examination of the underlying observational material to be sufficient. We note, therefore, that for parallaxes obtained with the period-luminosity curve the accuracy appears to surpass that of direct measures on any object for which the parallax is less than 0."01, and is essentially independent of distance. About two-thirds of the Cepheids now known have parallaxes smaller than 0."001.

The greatest chance for serious error in the work lies in the unintentional inclusion of some stars that are not typical Cepheids. For a star showing periodic continuous variation which simulates certain Cepheid characteristics but exhibits peculiarities such as double maxima or minima, we have found from cluster studies that the absolute brightness usually is less than for typical variables having the same period. The mean periods of such peculiar stars are often long; and, conversely, Cepheids of very long period are frequently abnormal. Until the normality of period and light-curve is established, we must, therefore, look with some doubt upon the enormously great absolute luminosity (and distance) obtained for galactic variables with periods in excess of forty days. Accordingly such stars are relegated to a supplementary table, which also contains provisional parallaxes and luminosities of variables that for various other reasons seem uncertain.

The list of variables in Table I is essentially complete for typical stars with definitely determined periods less than forty days. The various Harvard compilations and Hartwig's annual catalogue are the principal sources of observational data.[1] Names and posi-

[1] *Note added to proof, April, 1918.*—Hartwig's catalogue and ephemeris of variable stars for 1915 was the last number of that annual publication available when the tables for this paper were compiled. The issue for 1918, which has now been received, contains not only numerous revisions of the periods of the older variables but also some additions to the list. The corrections demanded by the revised periods have been applied to the values in all the tables, the text has been modified where necessary, and the statistical results in Tables II–VI now include the data for the twelve additional stars in Table I*a*.

tions have been taken, whenever possible, from Table VIII of *Harvard Annals*, **56**. For a majority of the stars the median visual magnitude of the eighth column is the mean of the maximum and minimum magnitudes given in this Harvard list, although for a number of variables improved magnitudes are obtainable from recent literature. All photographic magnitudes have been reduced to the visual system with the aid of a mean color-curve. The periods are mainly from Hartwig's catalogues, but occasionally recent publications afford better values. The parallaxes have been computed from the absolute magnitudes, which were read directly from the luminosity-period curve. A representation of the curve appears in Fig. 1 of *Contribution* No. 151. For the sake of uniformity the first decimal place is retained for all the distances in the last three columns, though for the larger values it is generally meaningless.

The 35 stars in Table II include: (1) all with periods greater than forty days, some of which, RS Puppis, for instance, seem to show typical Cepheid variation, although others, with M-type spectra, may be classed more correctly with the long-period variables; (2) a few with period or type somewhat uncertain; (3) 7 stars that appear to belong to the RV Tauri type of variation,[1] and (4) a number known to be otherwise peculiar or irregular. Further observation is sure to place some of these stars with those of Table I. While it is very probable that the absolute brightness of all of them is high, in many cases the luminosity-period relation may not hold rigorously. Moreover, the high galactic latitude of some variables with periods in excess of forty days suggests, as strongly as the frequent peculiarities of period and amplitude, that these stars differ too greatly from the typical Cepheid to make the estimated distances of much value.

[1] The periods of RV Tauri, R Sagittae, and V Vulpeculae are taken as thirty-nine, thirty-five, and thirty-eight days, respectively; these values, representing approximately the cycle of the principal variations, are more likely to give correct absolute luminosities. Cf. van der Bilt, *Recherches Astronomiques de l'Observatoire d'Utrecht*, VI, 1916. On the authority of Enebo similar treatment is accorded TV Andromedae, RY Lacertae, SW Persei, and RX Ursae Majoris. L_2 Puppis is a bright southern variable that may belong to this interesting type; a critical examination of its spectrum is very desirable. The study of the secondary variations of such typical stars as RR Lyrae (*Astrophysical Journal*, **43**, 217, 1916) shows that the difference between the RV Tauri type and the ordinary Cepheids is not so great as appears superficially.

TABLE I

PARALLAXES AND DISTRIBUTION IN SPACE OF 127 CEPHEID VARIABLES*

No.	Name	Position in 1900 R.A.	Position in 1900 Decl.	Galactic Long.	Galactic Lat.	Period in Days	Med. Mag. Visual	Abs. Mag.	Parallax	Distance (Unit Is 100 Parsecs) Radial	Projected	From Plane
1...	SY Cassiopeiae	0h 9m.8	+57°52′	85°	− 4°	4.071	9.6	−1.8	0″.00052	19.2	19.2	− 1.3
2...	SW Andromedae	0 18.4	+28 51	85	−33	0.442	9.3	−0.3	0.0012	8.3	7.0	− 4.5
3...	TU Cassiopeiae	0 20.9	+50 44	88	−11	2.137	7.9	−1.2	0.0015	6.6	6.5	− 1.3
4...	α Ursae Minoris	1 25.4	+88 46	91	+26	3.968	2.12	−1.81	0.016	0.6	0.5	+ 0.3
5...	RR Ceti	1 27.0	+ 0 50	114	−59	0.553	8.6	−0.4	0.0016	6.3	3.2	− 5.4
6...	RW Cassiopeiae	1 30.7	+57 15	97	− 4	14.80	9.5	−3.8	0.00022	45.4	45.3	− 3.2
7...	U Trianguli	1 49.7	+33 17	106	−27	0.447	11.6	−0.3	0.00042	23.8	21.2	−10.8
8...	UX Persei	2 6.1	+57 37	102	− 4	4.6	10.6	−2.0	0.00030	33.3	33.2	− 2.3
9...	UY Persei	2 27.1	+58 26	104	− 2	5.5	9.6	−2.2	0.00044	22.7	22.7	− 0.8
10...	SU Cassiopeiae	2 43.0	+68 28	101	+ 9	1.950	5.9	−1.2	0.0038	2.6	2.6	+ 0.4
11...	TU Persei	3 1.8	+52 48	111	− 5	0.607	11.7	−0.4	0.00038	26.3	26.2	− 2.3
12...	RW Camelopardalis	3 46.2	+58 22	112	+ 5	16.402	8.8	−3.9	0.00029	34.5	34.4	+ 3.0
13...	SX Persei	4 10.2	+41 29	126	− 5	4.290	10.8	−1.9	0.00029	34.5	34.4	− 3.0
14...	SZ Tauri	4 31.4	+18 20	147	−17	3.149	7.1	−1.6	0.0018	5.5	5.2	− 1.6
15...	SV Persei	4 42.8	+42 7	130	0	11.128	8.9	−3.3	0.00036	27.8	27.8	0
16...	RX Eridani	4 45.2	−15 54	182	−33	0.587	9.2	−0.4	0.0012	8.2	6.9	− 4.5
17...	SU Aurigae	4 49.6	+30 24	140	− 7	0.470	8.7	−0.3	0.0016	6.3	6.2	− 0.8
18...	U Leporis	4 52.0	−21 23	188	−33	0.581	9.5	−0.4	0.0010	9.5	8.0	− 5.2
19...	RX Aurigae	4 54.5	+39 49	134	0	11.626	7.6	−3.4	0.00063	15.9	15.9	0
20...	SY Aurigae	5 5.5	+42 43	133	+ 4	10.138	9.2	−3.2	0.00033	30.3	30.2	+ 2.1
21...	Y Aurigae	5 21.5	+42 21	134	+ 6	3.859	9.6	−1.8	0.00052	19.2	19.1	+ 2.0
22...	RZ Geminorum	5 56.6	+22 15	155	+ 1	5.529	9.4	−2.2	0.00048	20.8	20.8	+ 0.4
23...	RS Orionis	6 16.5	+14 44	164	+ 2	7.566	8.6	−2.7	0.00055	18.2	18.2	+ 0.6
24...	T Monocerotis	6 19.8	+ 7 8	171	− 1	27.012	6.2	−4.7	0.00066	15.2	15.2	− 0.3
25...	RT Aurigae	6 22.1	+30 34	151	+11	3.728	5.3	−1.8	0.0038	2.6	2.6	+ 0.5
26...	RZ Camelopardalis	6 23.7	+67 6	115	+23	0.480	12.0	−0.3	0.00035	28.6	26.3	+11.2
27...	W Geminorum	6 29.2	+15 24	165	+ 5	7.916	7.1	−2.8	0.0010	9.5	9.5	+ 0.8
28...	ζ Geminorum	6 58.2	+20 43	164	+13	10.154	4.0	−3.2	0.0036	2.8	2.7	+ 0.6
29...	TZ Aurigae	7 4.6	+40 56	143	+23	0.392	11.8	−0.3	0.00038	26.3	24.2	+10.3
30...	RR Geminorum	7 15.1	+31 4	155	+21	0.397	10.2	−0.3	0.00079	12.7	11.9	+ 4.5

31...	X Puppis	7 28.4	−20 42	204	0	25.953	8.5	−4.6	0.00024	41.7	41.7	0
32...	V Carinae	8 26.7	−59 47	242	−12	6.695	7.8	−2.5	0.00087	11.5	11.2	− 2.4
33...	T Velorum	8 34.4	−47 1	233	− 3	4.639	8.0	−2.0	0.0010	10.0	10.0	− 0.5
34...	V Velorum	9 10.2	−55 32	244	− 4	4.371	7.8	−1.9	0.0012	8.7	8.7	− 0.6
35...	*l* Carinae	9 42.5	−62 3	251	− 7	35.523	4.3	−5.1	0.0013	7.6	7.5	− 0.9
36...	RR Leonis	10 2.1	+24 29	176	+54	0.452	9.6	−0.3	0.0010	9.5	5.6	+ 7.7
37...	UW Carinae	10 23.3	−59 10	254	− 1	5.346	9.0	−2.2	0.00058	17.2	17.2	− 0.3
38...	YZ Carinae	10 24.6	−58 51	253	− 1	18.158	8.6	−4.1	0.00029	34.5	34.5	− 0.6
39...	UX Carinae	10 25.4	−57 6	252	+ 1	3.682	8.2	−1.7	0.0010	9.5	9.5	+ 0.2
40...	UY Carinae	10 28.5	−61 16	255	− 3	5.544	8.8	−2.2	0.00063	15.9	15.9	− 0.8
41...	Y Carinae	10 29.4	−57 59	253	0	3.640	8.4	−1.7	0.00096	10.4	10.4	0
42...	UZ Carinae	10 32.6	−60 30	255	− 2	5.205	9.3	−2.1	0.00052	19.2	19.2	− 0.7
43...	SV Velorum	10 40.9	−55 46	253	+ 3	14.097	8.7	−3.7	0.00033	30.3	30.3	+ 1.6
44...	VY Carinae	10 40.9	−57 2	254	+ 2	18.984	7.3	−4.1	0.00052	19.2	19.2	+ 0.7
45...	WW Carinae	10 47.6	−58 51	256	0	4.676	9.6	−2.0	0.00048	20.8	20.8	0
46...	WZ Carinae	10 51.4	−60 24	257	0	23.00	7.8	−4.4	0.00036	27.8	27.8	0
47...	XX Carinae	10 53.4	−64 36	258	− 4	15.725	8.2	−3.8	0.00040	25.0	25.0	− 1.8
48...	U Carinae	10 53.7	−59 12	256	0	38.740	7.4	−5.3	0.00029	34.5	34.5	0
49...	XY Carinae	10 58.3	−63 43	258	− 3	12.434	8.9	−3.5	0.00033	30.3	30.3	− 1.6
50...	XZ Carinae	11 0.1	−60 26	258	0	16.644	8.4	−3.9	0.00035	28.6	28.6	0
51...	ST Centauri	11 5.5	−51 57	256	+ 8	3.151	10.0	−1.6	0.00048	20.8	20.6	+ 2.9
52...	SU Draconis	11 32.2	+67 53	100	+49	0.660	9.2	−0.4	0.0012	8.3	5.4	+ 6.3
53...	UZ Centauri	11 36.3	−62 8	262	0	3.334	9.2	−1.6	0.00069	14.5	14.5	0
54...	S Muscae	12 7.4	−69 36	266	− 7	9.657	6.8	−3.1	0.0010	9.5	9.4	− 1.2
55...	SW Draconis	12 12.8	+70 4	94	+48	0.570	10.0	−0.4	0.00083	12.0	8.0	+ 8.9
56...	SX Centauri	12 15.9	−48 39	266	+14	16.50	8.2	−3.9	0.00038	26.3	25.5	+ 6.4
57...	T Crucis	12 15.9	−61 44	267	+ 1	6.732	7.2	−2.5	0.0012	8.7	8.7	+ 0.2
58...	R Crucis	12 18.1	−61 4	267	+ 1	5.825	7.3	−2.3	0.0012	8.3	8.3	+ 0.1
59...	R Muscae	12 36.0	−68 52	270	− 7	0.882	7.0	−0.6	0.0030	3.3	3.3	− 0.4
60...	Z Canum Venat.	12 45.0	+44 19	88	+73	1.890	10.2	−1.1	0.00055	18.2	5.3	+17.4
61...	SS Hydrae	13 25.0	−23 8	284	+38	8.20	7.8	−2.8	0.00076	13.2	10.4	+ 8.1
62...	VW Centauri	13 27.1	−63 33	275	− 2	15.037	8.9	−3.8	0.00029	34.5	34.5	− 1.2
63...	RV Ursae Majoris	13 29.4	+54 31	74	+62	0.468	9.8	−0.3	0.00096	10.4	4.9	+ 9.2
64...	ST Virginis	14 22.4	− 0 26	315	+53	0.411	10.8	−0.3	0.00060	16.7	10.0	+13.3
65...	V Centauri	14 25.4	−56 27	284	+ 3	5.494	7.1	−2.2	0.0014	7.2	7.2	+ 0.4
66...	RS Boötis	14 29.2	+32 11	18	+66	0.377	10.3	−0.3	0.00076	13.2	5.3	+12.0

* Table I*a*, which is appended to this table, contains twelve additional Cepheids. Cf. n. 1, p. 2.

TABLE I—*Continued*

No.	Name	Position in 1900		Galactic		Period in Days	Med. Mag. Visual	Abs. Mag.	Parallax	Distance (Unit Is 100 Parsecs)		
		R.A.	Decl.	Long.	Lat.					Radial	Projected	From Plane
67...	RU Boötis	14h41m5	+23°44′	358°	+63°	0.493	13.5	−0.3	0″.00017	58.8	26.7	+52.4
68...	RY Boötis	14 45.2	+23 27	356	+62	9.0	7.2	−3.0	0.00091	11.0	5.2	+ 9.7
69...	R Trianguli Aust.	15 10.8	−66 8	285	− 9	3.389	7.0	−1.7	0.0018	5.5	5.4	− 0.9
70...	U Normae	15 34.6	−54 59	293	− 1	12.641	8.9	−3.5	0.00033	30.3	30.3	− 0.5
71...	S Trianguli Aust.	15 52.2	−63 30	290	− 9	6.323	6.9	−2.4	0.0014	7.2	7.1	− 1.1
72...	U Trianguli Aust.	15 58.4	−62 38	291	− 9	2.568	8.1	−1.4	0.0013	7.9	7.8	− 1.2
73...	RW Draconis	16 33.8	+58 1	54	+40	0.443	10.4	−0.3	0.00072	13.9	10.6	+ 8.9
74...	RV Scorpii	16 51.8	−33 27	319	+ 4	6.062	7.1	−2.4	0.0013	7.9	7.9	+ 0.6
75...	SW Herculis	16 54.2	+21 42	9	+33	0.493	13.5	−0.3	0.00017	58.8	49.3	+32.0
76...	ST Ophiuchi	17 28.8	− 1 0	350	+15	0.450	11.6	−0.3	0.00042	23.8	23.0	+ 6.2
77...	X Sagittarii	17 41.3	−27 48	329	− 1	7.012	4.7	−2.6	0.0035	2.9	2.9	− 0.0
78...	RY Scorpii	17 44.3	−33 40	324	− 4	20.32	8.2	−4.2	0.00033	30.3	30.2	− 2.1
79...	Y Ophiuchi	17 47.3	− 6 7	347	+ 8	17.121	6.3	−4.0	0.00087	11.5	11.4	+ 1.6
80...	S Arae	17 51.4	−49 25	311	−13	0.452	9.3	−0.3	0.0012	8.3	8.1	− 1.9
81...	W Sagittarii	17 58.6	−29 35	330	− 5	7.595	4.7	−2.7	0.0033	3.0	3.0	− 0.3
82...	W Serpentis	18 4.1	−15 34	342	+ 2	14.153	9.0	−3.7	0.00029	34.5	34.5	+ 1.2
83...	WZ Sagittarii	18 11.1	−19 6	340	− 2	21.7	8.4	−4.4	0.00028	35.7	35.7	− 1.2
84...	Y Sagittarii	18 15.5	−18 54	340	− 4	5.773	5.8	−2.3	0.0024	4.2	4.2	− 0.3
85...	XX Sagittarii	18 19.0	−16 51	343	− 4	6.43	9.0	−2.7	0.00046	21.7	21.7	− 1.5
86...	U Sagittarii	18 26.0	−19 12	341	− 6	6.745	6.9	−2.5	0.0013	7.6	7.6	− 0.8
87...	Y Scuti	18 32.6	− 8 27	351	− 3	10.347	9.0	−3.2	0.00036	27.8	27.8	− 1.4
88...	Y Lyrae	18 34.2	+43 52	40	+20	0.503	11.8	−0.3	0.00038	26.3	24.7	+ 9.0
89...	RU Scuti	18 36.7	− 4 13	356	− 1	19.7	8.9	−4.2	0.00024	41.7	41.7	− 0.7
90...	RZ Lyrae	18 39.9	+32 41	29	+15	0.511	10.4	−0.3	0.00072	13.9	13.4	+ 3.6
91...	YZ Sagittarii	18 43.7	−16 50	346	− 8	9.553	7.4	−3.1	0.00079	12.7	12.6	− 1.8
92...	RT Scuti	18 44.0	−10 30	351	− 5	0.496	9.4	−0.3	0.0012	8.7	8.7	− 0.8
93...	SZ Aquilae	18 59.6	+ 1 9	3	− 3	17.136	8.7	−4.0	0.00029	34.5	34.5	− 1.8
94...	TT Aquilae	19 3.2	+ 1 9	4	− 5	13.753	7.6	−3.6	0.00058	17.2	17.1	− 1.5
95...	RR Lyrae	19 22.3	+42 36	43	+12	0.567	7.25	−0.35	0.0030	3.3	3.2	+ 0.7
96...	U Aquilae	19 24.0	− 7 15	359	−13	7.024	6.6	−2.6	0.0014	6.9	6.7	− 1.6
97...	XZ Cygni	19 30.4	+56 10	56	+16	0.467	9.7	−0.3	0.0010	10.0	9.6	+ 2.8

98...	U Vulpeculae	19 32.3	+20 7	24	− 2	7.990	7.0	−2.8	0.0011	9.1	9.1	− 0.3
99...	SU Cygni	19 40.8	+29 1	32	+ 2	3.846	6.6	−1.8	0.0021	4.8	4.8	+ 0.2
100...	η Aquilae	19 47.4	+ 0 45	8	−14	7.176	4.05	−2.62	0.0046	2.2	2.1	− 0.5
101...	S Sagittae	19 51.5	+16 22	23	− 7	8.382	5.8	−2.9	0.0018	5.5	5.5	− 0.7
102...	X Vulpeculae	19 53.3	+26 17	32	− 3	6.319	8.8	−2.4	0.00058	17.2	17.2	− 0.9
103...	XX Cygni	20 1.3	+58 40	60	+13	0.135	11.8	−0.4	0.00036	27.8	27.0	+ 6.2
104...	RW Aquilae	20 7.3	+15 46	24	−11	7.87	8.8	−2.8	0.00048	20.8	20.4	− 4.0
105...	SZ Cygni	20 29.6	+46 16	52	+ 3	15.113	9.2	−3.8	0.00025	40.0	40.0	+ 2.1
106...	X Cygni	20 39.5	+35 14	44	− 5	16.385	6.5	−3.9	0.00083	12.0	12.0	− 1.0
107...	T Vulpeculae	20 47.2	+27 52	40	−11	4.436	5.8	−2.0	0.0028	3.6	3.5	− 0.7
108...	UY Cygni	20 52.3	+30 3	43	−10	0.561	10.0	−0.4	0.00083	12.0	11.8	− 2.1
109...	VX Cygni	20 53.6	+39 48	50	− 5	20.125	9.7	−4.2	0.00017	58.8	58.6	− 5.1
110...	RV Capricorni	20 55.9	−15 37	1	−37	0.468	10.0	−0.3	0.00087	11.5	9.2	− 6.9
111...	TX Cygni	20 56.4	+42 12	52	− 3	14.726	9.1	−3.7	0.00028	35.7	35.7	− 1.9
112...	VY Cygni	21 0.4	+39 34	51	− 5	7.859	9.1	−2.8	0.00042	23.8	23.7	− 2.1
113...	SW Aquarii	21 10.2	− 0 20	20	−33	0.459	10.4	−0.3	0.00072	13.9	11.7	− 7.6
114*..	β Cephei	21 27.4	+70 7	75	+14	0.190	3.3	−0.4	0.018	0.6	0.5	+ 0.1
115...	VZ Cygni	21 47.7	+42 40	59	− 9	4.864	8.8	−2.0	0.00069	14.5	14.3	− 2.3
116...	Y Lacertae	22 5.2	+50 33	66	− 5	4.325	9.0	−1.9	0.00066	15.2	15.1	− 1.3
117...	δ Cephei	22 25.4	+57 54	73	+ 1	5.366	4.14	−2.19	0.0054	1.8	1.8	0
118...	Z Lacertae	22 36.9	+56 19	73	− 1	10.89	8.9	−3.3	0.00036	27.8	27.8	− 0.5
119...	RR Lacertae	22 37.4	+55 55	73	− 2	6.412	9.0	−2.4	0.00052	19.2	19.2	− 0.7
120...	V Lacertae	22 44.6	+55 48	74	− 3	4.983	9.0	−2.1	0.00060	16.7	16.7	− 0.9
121...	SW Cassiopeiae	23 2.9	+58 1	77	− 1	5.44	9.4	−2.2	0.00048	20.8	20.8	− 0.4
122...	RS Cassiopeiae	23 32.6	+61 53	82	+ 1	6.295	9.6	−2.4	0.00040	25.0	25.0	+ 0.4
123...	UU Cassiopeiae	23 45.8	+60 21	83	− 2	4.314	9.6	−1.9	0.00050	20.0	20.0	− 0.7
124...	RY Cassiopeiae	23 47.1	+58 12	83	− 2	12.328	9.8	−3.5	0.00022	45.4	45.4	− 1.6
125...	VW Andromedae	23 59.6	+34 12	80	−28	0.517	10.3	−0.3	0.00076	13.2	11.7	− 6.2
126†..	—Hydrae	12 25.2	−25 30	266	+37	0.479	10.9	−0.3	0.00057	17.5	14.0	+10.5
127†..	—Herculis	16 26.2	+18 36	3	+37	0.365	10.4	−0.3	0.00072	13.9	11.1	+ 8.4

* The possible uncertainty in the length of period suspected by Guthnick (*Astronomiche Nachrichten*, **196**, 357, 1913; **205**, 97, 1917) does not affect the adopted absolute magnitude appreciably.

† These stars were added to the tables after receiving (February 1, 1918) the list of new variables in *Harvard Circular* No. 201.

TABLE I*a*

ADDITIONS TO TABLE I

No.	Name	Galactic		Period in Days	Med. Mag. Visual	Parallax	Distance (Unit Is 100 Parsecs)		
		Long.	Lat.				Radial	Projected	From Plane
128	169. 1907 Leonis	149°	+55°	0.685	12.0	0".00033	30.3	17.4	+24.8
129	RR Canum Venaticorum	110	+82	0.558	11.0	0.00052	19.2	2.7	+19.0
130	S Comae	181	+88	0.587	10.9	0.00055	18.2	0.6	+18.2
131	SX Ursae Majoris	77	+60	0.307	10.8	0.00060	16.7	8.4	+14.5
132	RU Canum Venaticorum	20	+73	0.364	11.1	0.00052	19.2	5.6	+18.4
133	W Canum Venaticorum	37	+70	0.552	10.3	0.00072	13.9	4.8	+13.1
134	172. 1907 Boötis	28	+66	0.622	10.8	0.00058	17.2	7.0	+15.7
135	X Scuti	346	− 3	4.187	9.8	0.00046	21.7	21.7	− 1.1
136	Z Scuti	355	− 2	12.9	9.2	0.00029	34.5	34.5	− 1.2
137	SX Aquarii	26	−35	0.536	11.8	0.00036	27.8	22.8	−15.9
138	VV Pegasi	46	−32	0.488	11.2	0.00050	20.0	17.0	−10.6
139	RZ Cephei	77	+ 6	0.309	9.4	0.0012	8.7	8.6	+ 0.9

The period of any star in Table II may be found, if desired, by reading its logarithm from the luminosity-period curve for the corresponding tabulated absolute magnitude; and the adopted median apparent magnitude can be readily computed from the parallax and absolute magnitude. The last five stars of the table were added from Hartwig's 1918 catalogue; see note 1, p. 280. The variables of Table II are not used in the diagrams or in the following discussion.

TABLE II

Supplementary List of Variables

No.	Name	Absolute Magnitude	Parallax
1.....	RX Andromedae	−5.5	0″.00010
2.....	SZ Cassiopeiae	−5.6	0.00008
3.....	SW Persei	−6.5	0.00008
4.....	SW Aurigae	−4.7	0.00004
5.....	RV Tauri	−5.3	0.00009
6.....	SS Geminorum	−5.5	0.00018
7.....	V Lyncis	−6.2	0.00009
8.....	RU Camelopardalis	−4.4	0.00019
9.....	RS Puppis	−5.3	0.00029
10.....	Z Cancri	−6.2	0.00010
11.....	RX Ursae Majoris	−6.0	0.00003
12.....	S Antliae	−0.3	0.0042
13.....	Z Leonis	−5.8	0.00012
14.....	S Crucis	−2.0	0.0016
15.....	W Virginis	−4.0	0.00014
16.....	V Ursae Minoris	−6.2	0.00015
17.....	UV Draconis	−6.3	0.00010
18.....	TX Scorpii	−0.6	0.0021
19.....	κ Pavonis	−3.0	0.0032
20.....	S Vulpeculae	−6.1	0.00009
21.....	TX Aquilae	−5.1	0.00010
22.....	R Sagittae	−5.1	0.00013
23.....	V Vulpeculae	−5.2	0.00017
24.....	TW Pegasi	−6.3	0.00018
25.....	RY Pegasi	−4.6	0.00010
26.....	RY Lacertae	−6.0	0.00003
27.....	W Cephei	−2.4	0.00091
28.....	X Lacertae	−2.2	0.00076
29.....	TV Andromedae	−6.0	0.00007
30.....	RU Aquarii	−6.1	0.00009
31.....	TZ Persei	−4.0	0.00004
32.....	U Monocerotis	−5.5	0.00044
33.....	UU Herculis	−6.0	0.00010
34.....	TX Ophiuchi	−6.2	0.00005
35.....	AP Sagittarii	−3.3	0.00083

There can be no doubt of the intimate relationship of cluster-type variables to the longer-period Cepheids, but in a few

characteristics there appear wide differences—discontinuities in the usual progressive connections. One conspicuous discontinuity appears in the frequency of periods, which is illustrated in Table III for the 139 variables of Tables I and I*a*. Attention has been previously called to this matter by Hertzsprung and others. The reason for the two maxima in the frequency-curve must be sought in the dynamics of the stars themselves.

TABLE III

FREQUENCY OF THE PERIODS OF 139 CEPHEID VARIABLES

Period in Days	Number of Stars	Period in Days	Number of Stars	Period in Days	Number of Stars
0.0–0.2	2	5.0– 5.5	5	13.0–14.0	1
0.2–0.4	7	5.5– 6.0	5	14.0–15.0	4
0.4–0.6	31	6.0– 6.5	6	15.0–16.0	3
0.6–0.8	4	6.5– 7.0	3	16.0–17.0	4
0.8–1.0	1	7.0– 7.5	3	17.0–18.0	2
1.0–1.5	0	7.5– 8.0	6	18.0–19.0	2
1.5–2.0	2	8.0– 8.5	2	19.0–20.0	4
2.0–2.5	1	8.5– 9.0	0	20.0–25.0	1
2.5–3.0	1	9.0– 9.5	1	25.0–30.0	2
3.0–3.5	4	9.5–10.0	2	30.0–35.0	0
3.5–4.0	6	10.0–11.0	4	35.0–40.0	2
4.0–4.5	7	11.0–12.0	2		
4.5–5.0	5	12.0–13.0	4		

Two other notable differences between the two groups are the space distribution and the space velocities. Hertzsprung has maintained in several notes that the cluster-type stars must be of low luminosity because of the large proper motion of their brightest representative, RR Lyrae ($\mu = 0''25$), and because of the position of many such stars in high galactic latitude. The closely comparable luminosity of the two groups, however, is established by the luminosity-period curve. Moreover, RR Lyrae has a much larger radial velocity than the ordinary Cepheids, $V_0 = 50$ km/sec., and its proper motion may be an indication of great peculiar velocity rather than of large parallax.

This supposition is strongly supported through spectroscopic observations by Adams of four other cluster-type variables. The smallest radial velocity is -52 km/sec. for RS Boötis; the greatest is -196 km/sec. for XZ Cygni, which has an annual proper motion

of 0.″1, according to a determination kindly made at the writer's request by Professor Tucker. If we adopt the parallaxes in Table I, the space velocities of RR Lyrae and XZ Cygni, 400 km/sec., are among the greatest known. The average radial velocity for the longer-period Cepheids is less than 10 km/sec. It is possible, therefore, that the marked contrast between the wide dispersion of isolated cluster-type stars and the galactic concentration of normal Cepheids arises solely from the fact that the velocities of the former are enormous while those of the latter are moderate, in keeping with the motions of most other red and yellow giant stars.

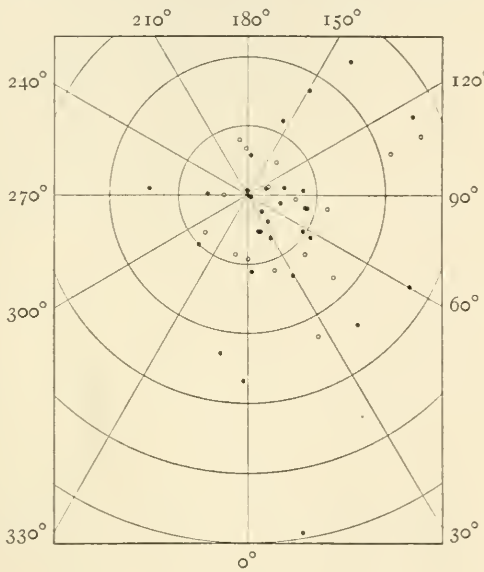

FIG. 1.—Projection on galactic plane of the position of Cepheid variables with periods shorter than one day. The circles are heliocentric, with radii of 1000 parsecs, 2000 parsecs, etc.; galactic longitudes are indicated in the margin. Many of these variables are so far above or below the plane that the diagram does not well represent the distribution in space.

Table IV contains a summary of the distribution of the variables with periods less than one day. The number of such stars is not large; the data are without doubt incomplete and there may be a preference for certain parts of the sky; hence no very definite quantitative conclusions should be based upon the material. The projection of these stars on the galactic plane is illustrated in Fig. 1; the distance from the plane is shown in Fig. 3. The great distances of RU Boötis and SW Herculis, and in particular the significance of the former's position so far from the galactic plane, have been remarked upon previously[1] and will be referred

[1] *Publications of the Astronomical Society of the Pacific*, **29**, 183, 1917.

to again in a subsequent paper. Six cluster-type variables, with $R \sin \beta$ greater than 1750 parsecs, are beyond the bounds adopted as defining the equatorial segment of the Galaxy (Fig. 3). The mean distance from the galactic plane is 960 parsecs for these 45 variables of Table IV; for Cepheids with periods greater than a day it is 150 parsecs—less than one-sixth as much. The parallaxes

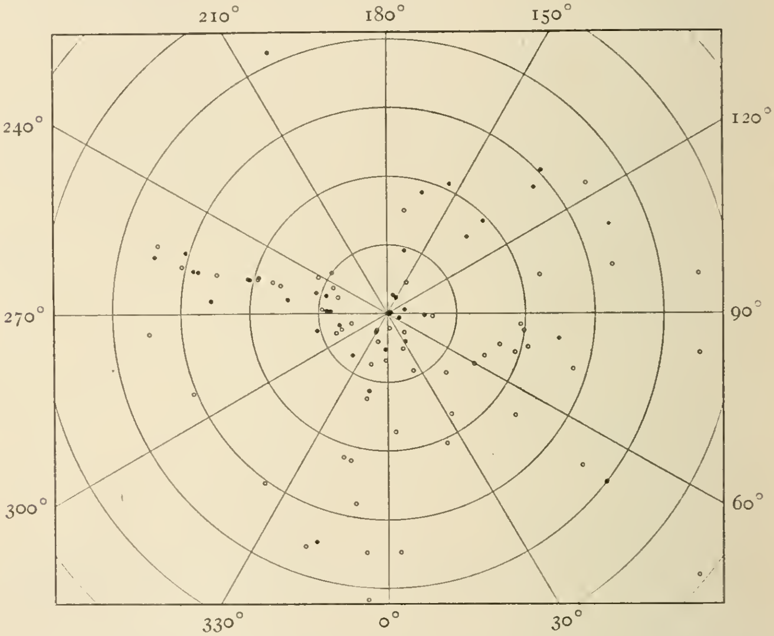

FIG. 2.—Projection on the galactic plane of the positions of Cepheid variables with periods greater than one day. The restriction to the Galaxy is so marked for these stars (see Fig. 3) that the diagram closely represents the distribution in space. The concentration of stars near longitude 255° reflects the systematic study at Harvard of the periods of all Cepheids in a restricted region (*Harvard Circular*, No. 170); it suggests the incompleteness of our data in other longitudes.

of RS Boötis, RU Boötis, XX Cygni, XZ Cygni, and RR Lyrae are the most accurate.

The distribution in the galactic plane of the 94 variables with periods longer than a day is shown in Fig. 2. The space co-ordinates are treated in some detail in Table V, showing the close restriction of ordinary Cepheids to the galactic plane. As a graphical test of

TABLE IV

Distance From Galactic Plane of 45 Cepheids with Periods Less Than a Day*

Variables North of Galactic Plane		Variables South of Galactic Plane	
Limits of Distance	Number	Limits of Distance	Number
>+1500 parsecs....	7	<−1000 parsecs..	3
+1500 to+1000...	7	−1000 to −500.	5
+1000 to+ 500...	9	− 500 to 0.	8
+ 500 to 0...	6		
Total..........	29	Total.........	16

* Allowance for the position of the sun north of the Milky Way plane would alter these values slightly but would not change the actual dispersion.

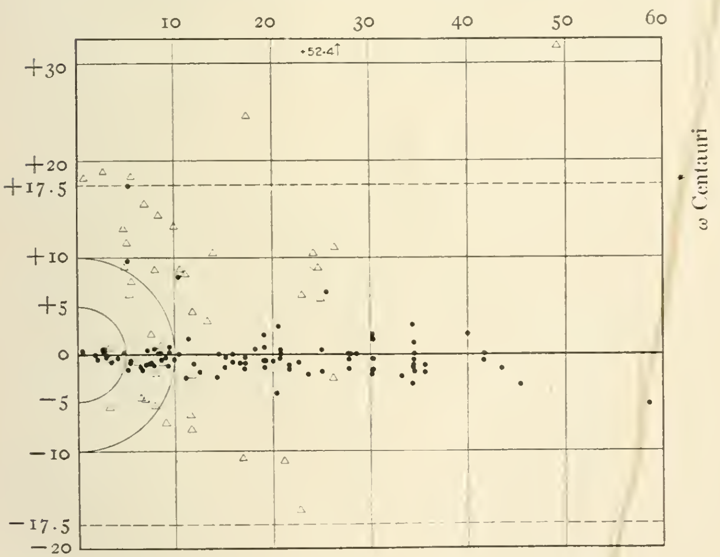

Fig. 3.—Distribution of Cepheid variables. The unit of distance is 100 parsecs. Ordinates are distances from the galactic plane; abscissae are projected distances in the plane; cf. Fig. 5 of the seventh paper. Open triangles and black dots designate respectively cluster-type variables and Cepheids with periods in excess of a day. The nearest globular cluster, ω Centauri, is just outside the boundary of the diagram on the right; RU Boötis, indicated by an arrow, is too far above the plane to fall within the figure. The semicircles with radii of 500 and 1000 parsecs ($\pi=0''.002$ and $0''.001$) indicate how very distant most of these variables are as compared with the average star of the tenth magnitude or brighter ($\pi>0''.004$, Kapteyn). Between the broken horizontal lines, ± 1750 parsecs, lies the equatorial galactic region devoid of globular clusters.

the relation of galactic concentration to distance from the sun, the values of $R \sin \beta$ are plotted in Fig. 3, against $R \cos \beta$, triangles for cluster-type variables, and dots for those with periods in excess of a day. It is important to note that for the latter the greatest distance from the plane is attained by the star of shortest period, Z Canum Venaticorum, with $R \sin \beta = +17.4$ and period $= 1.890$ days. Its space velocity may be high.

Table V and Fig. 3 reveal no conspicuous divergence from the galactic plane with increasing distance. The indications of a contrary result, obtained in the earlier study,[1] was due partly to the inclusion of a few stars of the RV Tauri type; these stars are probably of lower luminosity than typical Cepheids of like period, and in consequence the computed distances, both radial and from the plane, were much exaggerated.

TABLE V

SPACE DISTRIBUTION OF 94 CEPHEIDS WITH PERIODS GREATER THAN A DAY*

		Limits of $R \cos \beta$ (in Parsecs)			
		0 to 1000	1000 to 2000	2000 to 4000	>4000
North of Galaxy	No. of stars	14	6	11	2
	Mean $R \sin \beta$	+220	+220	+160	+100
	Max. $R \sin \beta$	+1740	+810	+640	+210
	Av. dev. from mean	±320	±200	±140	±100
South of Galaxy	No. of stars	17	17	23	4
	Mean $R \sin \beta$	−80	−100	−140	−260
	Max. $R \sin \beta$	−160	−240	−400	−510
	Av. dev. from mean	±40	±60	±80	±150
Total No. of Stars		31	23	34	6
Mean $R \sin \beta$	Without regard to sign	150	130	150	210
	With regard to sign	+60	−20	−40	−140

* Distances are given only to the nearest ten parsecs.

The distribution in longitude (Table VI) of variables with periods greater than one day shows some peculiarities which may be due to incompleteness of the data. There are, for instance, at least 50 stars thought to be Cepheid variables for which periods

[1] *Astrophysical Journal*, 40, 432, 1914.

have not been determined, and probably a great number for which types are not yet known also belong to this class.

The greater number of Cepheids south of the galactic plane is easily accounted for by assuming the sun to be slightly to the north. In fact the Cepheids should afford a good determination of the distance of the sun from the galactic plane when a more complete list becomes available.

TABLE VI

Distribution in Galactic Longitude—Number of Stars

	0° to 30°	30° to 60°	60° to 90°	90° to 120°	120° to 150°	150° to 180°	180° to 210°	210° to 240°	240° to 270°	270° to 300°	300° to 330°	330° to 360°	Total
North....	0	2	3	3	3	5	1	0	10	2	1	3	33
South....	6	7	9	3	3	1	0	1	13	5	2	11	61
Total....	6	9	12	6	6	6	1	1	23	7	3	14	94

SUMMARY

1. Through the use of the luminosity-period curve the absolute magnitudes and parallaxes have been determined for 174 variables, 139 of which are believed to be perfectly typical members of the Cepheid class (Tables I and I*a*). The average probable error is estimated at 20 per cent.

2. Forty-five variables belong to the cluster type, with absolute luminosities a little more than one hundred times the brightness of the sun. Ninety-four are ordinary Cepheids with periods longer than a day and with luminosities ranging from two hundred to ten thousand times that of the sun. For 35 stars either the type of variation, or the period, or the magnitude is not certain or regular enough to yield final parallaxes (Table II).

3. The distances of Cepheid variables are considerably greater than have been obtained heretofore for individual stars. Less than one-third of them have parallaxes greater than a thousandth of a second. The most distant Cepheids now known are nearly 20,000 light-years from the sun. The nearest globular cluster is at a distance of about 21,000 light-years. (Cf. ω Centauri in Fig. 3.)

4. The numerical evaluation of the distribution in space confirms both the well-known concentration toward the galactic plane of Cepheids with periods greater than a day and the indifference of cluster-type variables to that plane. A plausible explanation of the wide dispersion of the latter is to be found in their relatively very high velocities in space.

Mount Wilson Solar Observatory
December 1917

ON THE CAUSE OF THE DISTANCE-VELOCITY EQUATION IN STELLAR MOTIONS

SECOND PAPER

By C. D. PERRINE

Adams and Strömberg[1] object to the treatment of some of the data in my first paper[2] on the foregoing subject and reject the possibility that there can be a direct dependence upon distance from the sun. I am indebted to these gentlemen for sending me an advance copy of their comments.

My investigation rested very largely upon the assumption that there was a close relationship between the size of a star's proper motion and its distance from the sun, an assumption which had been generally accepted and furnished the chief basis of a knowledge of the distances of most of the stars. The conclusions were thought to be sufficiently guarded.

Recent investigations at Mount Wilson and at this observatory seem to throw considerable doubt upon proper motion as a measure of a star's distance. No useful purpose will be served, therefore, by discussing the details of my investigation until such time as this underlying one of distance is settled. Neither is it desirable, in my opinion, to discuss further the evidence in favor of a dependence of velocity upon absolute magnitude. If the suspicions which exist with regard to the parallaxes of the stars should be confirmed, such data will probably need revision also, for it is to be borne in mind that even spectroscopically determined parallaxes are based finally upon *relative* parallax displacements.

I shall therefore confine myself briefly to these doubts about distance and to considerations which are independent of such doubts.

In a recent investigation[3] Adams and Strömberg conclude that "for stars with proper motions less than 0."020 annually the

[1] *Astrophysical Journal*, 47, 189, 1918.

[2] *Ibid.*, 47, 179, 1918.

[3] *Ibid.*, 45, 297, 1917.

parallaxes appear to be nearly independent of proper motion and dependent only upon magnitude."

More recent preliminary results (unpublished) of an investigation here by First Astronomer Zimmer of peculiarities encountered in his work of determining fundamental places with our new Repsold meridian circle, not only tend to confirm the foregoing conclusion of Adams and Strömberg, but throw doubt upon the assumed absolute parallaxes of practically all stars. The details of these investigations will be given by Zimmer in another place. The evidence upon which the doubts rest in this case is too extensive to give here.

In Zimmer's list of stars there are 21 of type B and 2 of type O. If the observed quantities are parallax these stars have large parallaxes. The O and B stars were chosen partly because of their supposed great distance and consequently to furnish a better test of the possibility that the observed effect was parallax. The result has been a complete surprise. The proper motions of these O and B stars are vanishingly small, so that if large parallaxes for them are confirmed, grave doubt is thrown upon the whole question of the relation of parallax to size of proper motion. In other words, without some other criterion it will be impossible to consider proper motion as a reliable indication of distance.

An examination of Strömberg's investigation[1] of the parallax of 700 stars spectroscopically determined reveals some discordances which appear to me to be significant in this connection. The curves in his Fig. 1 show a marked deviation from the ordinary formula, as he points out. His new formula represents the stars of F and G types satisfactorily, but not so the K and M stars. In order better to show these peculiarities I have redrawn these two curves as Fig. 1 of this paper, omitting the broken, straight line. I cannot believe that such consistent divergences of the observations in two distinct cases are purely accidental. If they are not accidental, then one formula will not fit both giant and dwarf stars. We can, however, draw a curve through the giants in both groups which will also fit and include the dwarfs by *shifting it bodily in distance.* Such curves are given in Fig. 2. Is not this an

[1] *Astrophysical Journal*, 47, 10, 1918.

indication also that the derived parallaxes of these more distant stars may be too small?

From the way in which parallactic displacements are obtained it does not seem possible that the observed parallaxes of any stars can be too *large*. If the background of stars used in such investigations is, however, not so distant as has been assumed, then the

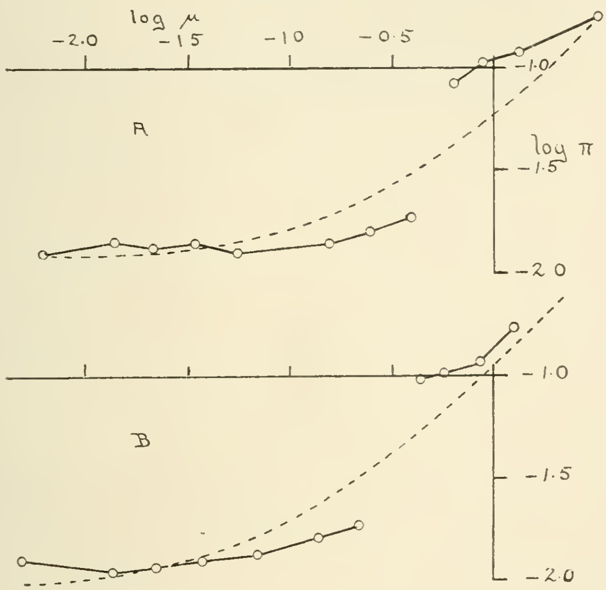

FIG. 1.—Relation between log π and log μ for K and M stars

A—Stars distant from sun's apices
B—Stars near sun's apices
Dotted curves represent formula
$\log \pi = \log A + \log (\mu + c)$

accepted absolute parallaxes may be too *small*. The evidence upon which the distance of this background of stars has been accepted as great is not lightly to be thrown aside. But in view of the difficulty of obtaining directly the distances of these stars, and the contrary evidence which is at hand, it is certainly permissible to consider the possibility of the absolute parallaxes in general being too small.

It may be noted also that Strömberg's curves of parallax for the F and G stars show some slight indications of the discontinuity which is so marked in the K and M stars. It would not attract attention, however, except for the large break in the K and M stars.

The discontinuity in the parallaxes of the K and M stars found by Strömberg, the conclusions of Adams and Strömberg regarding

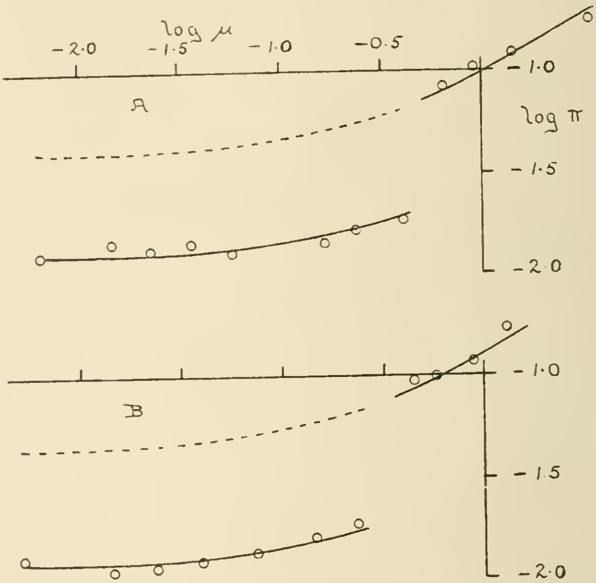

FIG. 2.—Relation between log π and log μ for K and M stars

Dotted portions of curves are extrapolations on the assumption that the small parallaxes may be too small.

the relation between parallax and proper motion in the case of the stars of small μ, but above all the suspicions raised by Zimmer's investigations and the consequent effects of a general change in parallax upon all absolute magnitudes, if confirmed, will necessitate a re-examination of practically all the absolute magnitudes after the question of parallax has been definitely answered. Should the present suspicions in the matter of parallax be confirmed, it seems certain that the corrections required would be greatest among the stars which are now classed as very bright.

The foregoing considerations seem to render useless any attempt at present to determine the distances of the different groups of stars of small μ of Table I of my former paper. To this extent the objection of Adams and Strömberg to that evidence appears to be upheld, at least for the present.

An examination of the frequency-curves[1] of absolute magnitudes by Adams and Joy is of some interest. The mean magnitudes of the giants are rather consistent and about $+1$ for all of the spectral classes given. On the other hand the magnitudes of the dwarfs decrease consistently from $+4\frac{1}{4}$ for the stars of type F to $+11$ for the stars of type M. A consideration of the absorption effects of their atmospheres seems to furnish a reason for the decline in brightness of the "dwarfs." The same consideration seems to require an explanation for the nearly uniform brightnesses of the "giants" of all spectral classes.

It is somewhat suggestive in this connection that the stars (K and M) which show the striking discontinuity in parallax are the same which show the greatest separation as to absolute magnitude.

An attempt was made to decide, by means of least-square solutions from 210 stars whose parallaxes had been measured spectroscopically, whether the dependence was upon absolute magnitude, or upon distance, or upon both. Ellipsoidal motion was directly allowed for. The results showed a larger dependence upon distance than upon absolute magnitude, but, on account of the small amount of data available and the suspicions which now attach to stellar parallax in general, little weight is attached to them.

With regard to the objection raised to the application to radial velocities of factors which will reduce all the stars to the same proper motion, the manifest uncertainties in the present case seem to render superfluous a discussion of my procedure. It is very often desirable and a common practice to attempt to render homogeneous groups of data within reasonable limits. If the size of μ is an index of velocity rather than of distance, as now seems possible, the effect of my reduction was to render the velocities of the groups of different apparent magnitudes more comparable (and perhaps

[1] *Astrophysical Journal*, 46, 335, 1917.

necessary), but had little bearing on the factor of distance. The classification according to apparent magnitude may be one according to distance also, but that this is strictly the case seems doubtful.

It seems to me that the effect of a large-distance dispersion in the stars of small μ is not necessarily of such great importance as suggested by Adams and Strömberg. I was dealing with the means of six groups, four of them containing from 169 to 317 stars each. Granting that there is such a dispersion as considered by them, 100 to 1 (which in the light of present suspicions seems doubtful), the *means* of such considerable groups can scarcely differ greatly. It seems to me doubtful if the dispersion in distance of these groups is likely to be much greater than their dispersions in velocity, in which case the errors would be comparable.

The entire matter of these dependencies may be looked at in another way. In practically all of these classifications the great majority of the velocities are low. This is also true to a considerable extent in the groups of fainter stars and those of large μ. It was found that if velocities over 30 km[1] were rejected in the investigation of a relation to spectral class no such relation remained. This and other investigations show that it is almost entirely the larger velocities which cause these different apparent relations. These large velocities appear to show a strong preference for the vertices of preferential motion. The direct inference from this is that the observed relations are in reality, largely at least, effects of stream-motion, thus corroborating the results of the direct classification in which stream-motion was one of the factors.

The possible unconscious selection of large velocities.—The fact that the absolutely faint stars in the lists of radial velocities have resulted from selections of stars largely because of their large proper motions raises the question whether such an unconscious selection has not also led to high velocities generally in these stars. A list of stars selected because of their large proper motions must lead, not only to nearness and absolute faintness, but also to unusually large velocities. If there is little or no relation of distance to size of proper motion, then variations in the size of proper motion become

[1] In the 1800 stars used by me only 12 per cent have radial velocities of 30 km or over.

in general indications of differences of velocity, and the possibility of an unusual proportion of large velocities among the absolutely very faint stars becomes accentuated. As practically all of such very faint stars have come into the lists because of their large (often excessive) proper motions, it seems to me very probable that such stars are not fair samples of their respective absolute magnitudes—that in general they may be the largest velocities of their classes, and that when stars of much fainter apparent brightness have been systematically investigated the average velocities now found may be greatly reduced.

Adams and Strömberg lay stress upon the consequence arising from the assumption of a dependence upon distance, that

> such a relationship would assign to the sun an extraordinary position in the stellar system which we have no reason for believing that it holds. It is a star of very moderate size and mass and is known to lie at a great distance from the center of the galaxy. It would seem far more probable that any relationship between velocity and distance would be one in which the latter was measured from some much more fundamental reference point than our sun.[1]

Such a consequence of a dependence upon distance was not overlooked in my first investigation, although it was not discussed. When my first paper was written I had in mind chiefly regions (an effect of cosmical matter which appears to exist in galactic regions), as is evidenced by the following expression:

> Some hypothesis in which distance is concerned, probably rather in the nature of regions, seems to me to offer a more satisfactory explanation of the observed facts. Taking all of these into consideration, I have no hesitancy in expressing the belief that distance in some manner plays a considerable part in the phenomena in question. If my hypothesis is correct the chief factor is distance or region, and the absolute magnitudes, spectral classes, and changes of velocity follow from the different conditions existing in the near and distant regions of our stellar system. This would explain also the very close relation of all of these factors.

For such a scheme it is not necessary that the observer should be at the center of the stellar system. It is necessary only that he should be somewhere near the middle of a region (not necessarily exactly circular) where the postulated cosmical matter is rather scarce, which region is surrounded in a general way by others richer

[1] *Astrophysical Journal*, 47, 191, 1918.

in such matter. It is possible to conceive of many such regions within a stellar system, whether it be of a general ring form or spiral.

There may be, however, an even closer relation to distance due entirely to stream-motion. For an observer situated in a stream of stars, especially if curved, there would be a tendency for the radial velocity to depend upon distance from the observer. A little consideration will make this plain. If, for example, the observer is situated in a stream of stars which is rather sharply curved, and the principal motion of the stars is in the direction of the stream, the stars will appear to have a preferential motion nearly in the direction of a *tangent to the stream at the position of the observer.* In such a case the near stars will show the greatest preferential motion and the largest radial motions, which will be in the direction of the stream-motion, and there will obviously be a real dependence of the phenomenon upon the position of the observer. Something similar might result if the systematic motions were radial.

In conclusion, I recognize, and so stated in my first paper, the necessity of confirming the conclusion that the near stars have, in general, larger individual velocities than the more distant ones. Such a dependence upon the position of the observer is theoretically at least possible. In my opinion the conclusion that the smaller stars are moving more rapidly than the large ones also requires confirmation, although I can see reasons for such a condition, and some evidence favors it. It has yet to be shown, however, what the effect would be of directly eliminating stream-motion from the absolute magnitudes. It may be that there are dependencies upon both magnitude and distance.

It is suggested that a possible unconscious selection of large velocities among the absolutely faint stars may be partly responsible for the apparent dependence upon magnitude as well as some others of the dependencies observed.

I look forward to the time when the present uncertainties as to parallax will be resolved and enough data will become available to enable me to continue the investigation with hope of a decisive result.

Observatorio Nacional Argentino, Cordóba
July 25, 1918

THE CHANGE IN BRIGHTNESS, SPECTRUM, AND TEMPERATURE OF NOVA AQUILAE NO. 3

By MENTORE MAGGINI

Using my Heterochrome Photometer[1] I have observed Nova Aquilae during the interval June 12—September 22. The observations were made at wave-lengths 645 $\mu\mu$, 558 $\mu\mu$, 412 $\mu\mu$. As 558 $\mu\mu$ is about the mean wave-length corresponding to the magnitudes in natural light, we can assume that comparisons at 558 $\mu\mu$ are equivalent to the visual magnitudes. Comparisons were executed according to the following scheme: The ocular tube is placed at the distance 30 mm from the focus and three comparisons (star No. 1, Nova, star No. 2) were made, then three (star No. 2, Nova, star No. 1). A complete set in three radiations requires about twenty-five minutes.

Magnitudes of comparison stars at 558 $\mu\mu$ are the Harvard magnitudes. At 645 and 412 $\mu\mu$ they were determined with the photometer as follows: I have adopted for stars of spectral type A the magnitude in all radiations equal to that of *Harvard Photometry;* by selecting a considerable number of stars from types Bo to Mo the scale of magnitude at 645 $\mu\mu$ and 412 $\mu\mu$ was obtained, measuring their difference from the Ao stars and making the difference in the three radiations. The values were then plotted against the types of these stars, and smooth curves were drawn through the several points. These curves provide the means of converting determinations of magnitude for the three radiations into spectral type. According to these curves the relation between spectral type and magnitude-differences can be regarded as linear, i.e.,

$$\begin{aligned} m_{558}-m_{645} &= +0.198\ s \\ m_{412}-m_{558} &= +0.353\ s \qquad (1) \\ m_{412}-m_{645} &= +0.526\ s \end{aligned}$$

[1] *Comptes Rendus*, **166**, 284, 1918; *Bulletin Astronomique*, **35**, 131, 1918; *Popular Astronomy*, **26**, 380, 1918.

in which s has the values $-1, 0, +1$, etc., according as the spectrum is Bo, Ao, Fo, etc.[1] Table I (p. 305) contains the observed magnitudes of Nova Aquilae.

These observations were plotted on Fig. 1. The curves show that the brightness of the Nova increases according as the refrangibility of radiation diminishes. During the greatest brightness its color was white, as α Lyrae; correspondingly the brightness at 412 $\mu\mu$ was greater than at 645 and 558 $\mu\mu$. Conspicuous changes were observed in the light. A species of pulsation is recorded with regular intervals on the dates June 18, 24, 30, July 11, 30, in which the brightness at 645 and 412 $\mu\mu$ has the least difference; hence the color of the Nova passed from the red toward the white. The period of these changes is about six days.

On the other hand if we consider the change at 558 $\mu\mu$, i.e., in the visual light, we find a pulsation having a period of about 10–13 days, and the minima fall at the following dates:

June	17	
		10^d
	27	
		12
July	9	
		12
	21	
		12
Aug.	2	
		13
	15	

This period can also be of 24–26 days with a secondary minimum.

[1] If we consider the ratios $\frac{645}{558}, \frac{558}{412}, \frac{645}{412}$, they show the energy-distribution in the different classes. We have the following table:

	$\frac{645}{558}$	$\frac{558}{412}$	$\frac{645}{412}$	Merrill
Bo	0.83	0.72	0.60	1.0
Ao	1.00	1.00	1.00	
Fo	1.20	1.39	1.62	2.5
Go	1.44	1.92	2.76	
Ko	1.73	2.65	4.58	6.0
Mo	2.07	3.67	7.61	6.5

The last column contains the ratio of red to blue light measured by Merrill ("Application of Dicyanin to the Photography of Stellar Spectra." *Scientific Papers of the Bureau of Standards*, No. 318) in good agreement with my results.

TABLE I

No.	1918	G.M.T.	Mag. 645 $\mu\mu$	Mag. 558 $\mu\mu$	Mag. 412 $\mu\mu$	Notes	No.	1918	G.M.T.	Mag. 645 $\mu\mu$	Mag. 558 $\mu\mu$	Mag. 412 $\mu\mu$	Notes
1..	June 12	8^h35^m	$0^M.26$	$0^M.20$	$-0^M.13$	II	39..	July 27	8^h46^m	$3^M.11$	$3^M.91$	$4^M.93$	III
2..	13	8 45	0.55	0.73	+1.33	II	40..	28	8 53	3.10	3.79	4.75	I
3..	14	8 45	1.00	1.14	1.52	II	41..	29	8 58	3.02	3.85	4.85	I
4..	15	9 30	1.02	1.10	1.75	II	42..	30	8 55	3.18	3.80	4.50	I
5..	16	8 55	1.09	1.70	2.35	II	43..	31	8 40	3.03	3.88	4.58	I
6..	17	9 0	1.73	1.91	2.36	II M	44..	Aug. 1	8 20	3.18	4.30	5.31	I
7..	18	9 10	1.79	1.75	1.90	II M	45..	2	8 0	3.20	4.34	5.39	I
8..	20	9 15	1.70	2.24	3.34	II M	46..	4	8 5	3.08	4.30	5.35	I
9..	21	9 45	1.82	2.45	3.50	III M	47..	7	8 17	3.10	4.10	5.35	I
10..	22	9 10	2.00	2.47	3.54	I M	48..	8	7 45	3.17	4.20	5.25	I
11..	23	9 30	2.25	2.92	3.32	II M	49..	10	8 0	3.02	4.20	5.48	I
12..	24	8 43	2.73	3.13	3.42	I M	50..	11	8 10	3.05	4.35	5.50	I
13..	25	8 40	2.51	3.28	4.62	I M	51..	12	8 43	3.28	4.37	5.63	I
14..	26	8 48	2.82	3.65	5.01	III	52..	13	8 35	3.32	4.45	5.52	III
15..	27	8 52	3.02	3.89	5.37	I	53..	14	8 30	3.17	4.45	5.60	I M
16..	28	8 53	3.34	3.84	5.02	II	54..	15	8 38	3.30	4.50	5.42	I M
17..	29	9 12	2.92	3.41	4.40	I	55..	16	9 0	3.18	4.40	5.65	I M
18..	30	9 25	3.02	3.35	4.15	II	56..	17	8 2	3.20	4.40	5.53	I M
19..	July 1	9 30	2.73	3.34	4.60	I	57..	18	8 33	3.22	4.50	5.50	III M
20..	2	9 0	2.59	3.25	4.45	I	58..	20	9 15	3.70	4.71	5.52	I
21..	4	9 15	2.78	3.22	4.15	I	59..	21	9 30	3.73	4.75	5.51	I
22..	5	9 30	2.73	3.22	4.15	III	60..	23	9 0	3.75	4.60	5.55	I
23..	6	9 0	2.60	3.30	4.65	III	61..	24	8 40	3.60	4.75	5.60	I
24..	7	9 20	2.65	3.34	4.40	I	62..	25	9 0	3.45	4.70	5.73	II
25..	8	9 10	3.12	3.77	5.00	I	63..	27	9 10	3.40	4.40	5.80	I
26..	9	9 0	3.25	3.86	5.12	II	64..	28	9 9	3.72	4.50	5.82	II
27..	11	9 30	3.40	3.83	5.01	II	65..	31	8 10	3.80	4.80	5.80	I
28..	12	10 0	3.56	3.95	4.74	I	66..	Sept. 3	8 16	3.75	4.88	5.85	II
29..	13	8 55	3.38	3.72	4.60	I	67..	4	8 8	3.68	4.88	5.85	I
30..	14	9 15	3.23	3.82	4.81	II	68..	5	9 15	3.85	4.80	5.82	I
31..	15	9 15	3.22	3.78	4.87	I M	69..	8	7 18	3.80	4.80	5.65	II
32..	16	9 5	3.22	3.91	5.00	I M	70..	10	7 35	3.72	4.85	5.62	I
33..	17	9 10	3.23	3.79	4.85	I M	71..	11	7 7	3.60	4.85	5.60	I
34..	19	8 35	3.23	3.92	5.04	I M	72..	13	7 50	3.60	4.90	5.57	I M
35..	21	9 21	3.36	4.25	5.23	I M	73..	14	7 0	3.60	4.90	5.55	I M
36..	24	9 15	3.01	3.86	5.03	I M	74..	16	8 8	3.60	4.90	5.45	I M
37..	25	9 7	2.98	3.90	5.15	I M	75..	21	8 15	3.82	4.43	5.15	II M
38..	26	8 40	2.90	3.86	5.12	I M	76..	22	7 33	3.28	4.80	5.72	II M

I = Very fair. II = Sky fair. III = Sky veiled at the horizon. M = Moonlight.

SPECTRAL TYPE

The relations (1) have been used in the determination of spectral type; magnitude-differences $m_{412}-m_{645}$ were taken and spectral type deduced from the last equation (1); the results are in the fourth column of Table II and in Fig. 2. They show that at the first observation (June 12) the Nova was of spectral type B6. In general during the month of June the spectrum changed from

TABLE II

No.	1918	$\frac{i_{645}}{i_{412}}$	Sp.	T	No.	1918	$\frac{i_{645}}{i_{412}}$	Sp.	T
1	June 12	1.28	B6	14500°	39	July 27	5.35	K4	4300°
2	13	2.05	F4	6400	40	28	4.57	K1	4500
3	14	1.61	F0	7300	41	29	5.40	K4	4200
4	15	1.96	F3	6600	42	30	3.37	G4	5000
5	16	3.19	G3	5200	43	31	4.17	G9	4700
6	17	1.79	F2	6900	44	Aug. 1	7.11	K9	3900
7	18	1.11	A2	9700	45	2	7.52	M0	3800
8	20	5.45	K4	4200	46	4	8.09	M2	3700
9	21	4.70	K1	4400	47	7	7.94	M2	3800
10	22	4.13	G8	4700	48	8	6.79	K8	3900
11	23	2.68	G0	5600	49	10	9.64	M6	3500
12	24	1.89	F3	6700	50	11	9.55	M5	3600
13	25	7.24	M0	3800	51	12	8.71	M3	3700
14	26	7.52	M0	3800	52	13	7.59	M1	3800
15	27	8.71	M3	3600	53	14	9.38	M5	3600
16	28	4.70	K1	4400	54	15	7.05	K9	3900
17	29	3.91	G7	4800	55	16	9.73	M6	3500
18	30	2.83	G1	5500	56	17	8.55	M3	3700
19	July 1	5.60	K5	4200	57	18	8.17	M2	3700
20	2	5.55	K5	4200	58	20	5.35	K4	4200
21	4	3.53	G5	5000	59	21	5.15	K3	4300
22	5	3.70	G6	4900	60	23	5.25	K3	4200
23	6	6.61	K8	4000	61	24	6.31	K7	4000
24	7	5.01	K2	4300	62	25	8.17	M2	3700
25	8	5.65	K5	4200	63	27	9.12	M4	3600
26	9	5.60	K4	4200	64	28	6.92	K9	3900
27	11	4.41	K0	4600	65	31	6.31	K7	4000
28	12	2.97	G2	5300	66	Sept. 3	6.92	K9	3900
29	13	3.08	G3	5300	67	4	7.38	M0	3800
30	14	4.29	G9	4600	68	5	6.14	K7	4000
31	15	4.57	K1	4500	69	8	5.50	K4	4200
32	16	5.15	K3	4300	70	10	5.75	K5	4100
33	17	4.45	K0	4500	71	11	6.31	K7	4000
34	19	5.30	K4	4300	72	13	6.14	K7	4000
35	21	5.60	K5	4200	73	14	6.03	K6	4100
36	24	6.43	K7	4000	74	16	5.50	K4	4200
37	25	7.38	M0	3800	75	21	3.40	G5	5000
38	26	7.73	M1	3800	76	22	9.46	M5	3600

B to M; in July it remained at about K; in August and September it was of type M.[1]

It is clearly evident that in the spectral type we find recorded the fluctuations in the brightness. At the dates June 12, 18, 24, 30, we have a return of spectrum toward the types F and A; this phenomenon may be seen also in July and August. In the following

[1] These are of course only the inferences as to spectral type; the spectroscopic observations do not show such changes.—Ed.

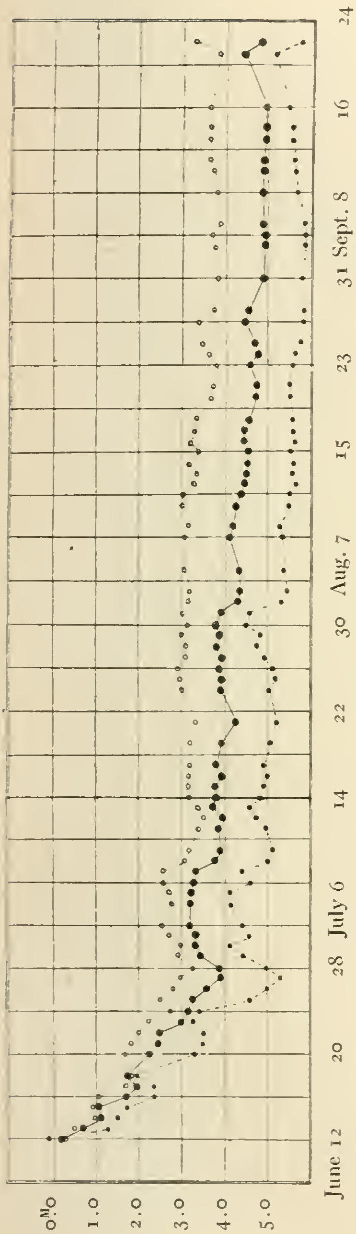

FIG. 1.—Above, empty circles: brightness at λ645 μμ; Central, large disks: brightness at λ558; Below, small disks: brightness at λ412

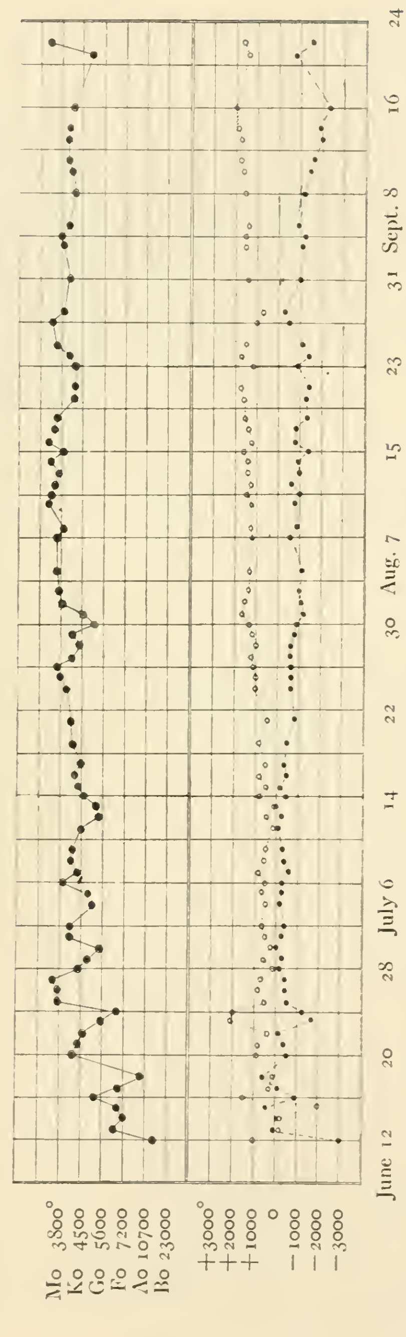

FIG. 2.—Above: Change in spectrum and temperature, $T_{(412-645)}$; Empty circles: differences, $T_{(412-645)}-T_{(558-645)}$; Black disks, below: differences, $T_{(412-645)}-T_{(412-558)}$.

table are the dates, spectral types, and amplitude of fluctuations in spectral classes:

Min.		Max.		Ampl.
June 12	B6	June 16	G3	2.7
18	A2	20	K4	3.2
24	F3	27	M3	3.0
30	G1	July 2	K5	1.4
July 30	G4	Aug. 4	M2	1.8

We see that the amplitudes diminish.

EFFECTIVE TEMPERATURE

Having the brightness in three special radiations we have determined for every observation the effective temperature of the Nova. According to Planck's law the most approximate relation between color and temperature of a body is

$$J=C\lambda^{-5}\left(e^{\frac{c}{\lambda T}}-1\right)^{-1},$$

in which J is the energy at wave-length λ, T the effective temperature, C and c are constants. For two regions of spectrum, λ_1 and λ_2, we have the ratio:

$$\frac{J_1}{J_2}=\left(\frac{\lambda_2}{\lambda_1}\right)^5\frac{e^{\frac{c}{\lambda_2 T}}-1}{e^{\frac{c}{\lambda_1 T}}-1}$$

Now if for a star i_1 and i_2 are the observed brightness at λ_1, λ_2, and if $J_1=Ki$, and $J_2=Ki_2$, we have

$$\log\left(\frac{J_1}{J_2}\right)=\log K+\log\left(\frac{i_1}{i_2}\right)=\log K+0.4(m_2-m_1).$$

Hence for a star of known temperature T_0

$$\log\left(\frac{J_1'}{J_2'}\right)=\log K+0.4(m_2'-m_1').$$

Subtracting we have:

$$\log\left(\frac{J_1}{J_2}\right)-\log\left(\frac{J_1'}{J_2'}\right)=0.4[(m_2-m_1)-(m_2'-m_1')].$$

And, according to Planck's law,

$$0.4[(m_2-m_1)-(m'_2-m'_1)] = \left[\log\left(e^{\frac{c}{\lambda_2 T}}-1\right)-\log\left(e^{\frac{c}{\lambda_1 T}}-1\right)\right]-\left[\log\left(e^{\frac{c}{\lambda_2 T_0}}-1\right)-\log\left(e^{\frac{c}{\lambda_1 T_0}}-1\right)\right].$$

This expression provides the means of determining effective temperature T of the Nova, when we know the magnitude-differences in two radiations and the magnitude-differences corresponding to the known temperature T_0.

I have determined the temperature of the Nova for each date, introducing in the expression the quantities:

$$c = 14200^{1}$$
$$T_0 = 10700°$$

The temperature T_0 corresponds to spectral type A0 according to the Potsdam results.

The values of effective temperature were deduced from the magnitude=differences $m_{412}-m_{645}$; they are in Table II. Two further sets of values were also obtained from the magnitude=differences $m_{558}-m_{645}$ and $m_{412}-m_{558}$; these temperatures are different from that of Table II. This fact is a sign that the distribution of energy in the spectrum of the Nova has changed from day to day. The temperature-differences $T_{(412-645)}-T_{(558-645)}$ and $T_{(412-645)}-T_{(412-558)}$ were formed and plotted in Fig. 2. We see that the greatest and irregular changes are during the period of greatest luminosity. Since Planck's law refers to the black body, the following conclusions may be deduced from Fig. 2:

1. Energy-distribution in the spectrum of the Nova has changed; the Nova does not follow the law of the black body.

2. Except in the first days of observation, temperature $T_{(412-645)}$ is constantly superior to $T_{(558-645)}$ and inferior to $T_{(412-558)}$.

3. The temperature-differences increase according as the brightness of the Nova diminishes, hence the difference from the black body increases.

FLORENCE
October 1918.

[1] Holborn und Valentiner. *Sitzungsberichte der Akademie Berlin*, **1906**, 811.

THE PERIOD OF *004872* V TUCANAE

BY BERNHARD H. DAWSON

The variability of the star V Tucanae was discovered by Miss Annie J. Cannon from an examination of plates taken at Arequipa and was announced in *Harvard Circular*, No. 134, where the star is designated H. V. 3021. In *Astronomische Nachrichten*, No. 4230, it was given the provisional designation 185:1907, in accordance with the custom of that periodical. The Variable Star Committee of the Astronomische Gesellschaft, in their list published in *Astronomische Nachrichten*, No. 4278, gave it the definitive name V Tucanae, and it first appears in the *Vierteljahrsschrift* of that Society in the "Katalog und Ephemeriden für 1909" in Vol. 44.

The star is *C.P.D.* $-72°$ 69, magnitude 9.9, and, so far as the writer knows, it has not been observed on the meridian. Its place according to the *C.P.D.* is

$$\alpha = 0^h\,47^m\,15^s; \quad \delta = -72°\,40'.8, \quad 1875.0$$

from which are derived

$$\alpha = 0^h\,48^m\,10^s; \quad \delta = -72°\,32'.6, \quad 1900.0$$
$$\lambda = 312°\,18'.9; \quad \beta = -64°\,9'.9, \quad 1900.0$$
$$\log. 8.308 \cos\beta = 0.55878$$

The La Plata observations are comparisons by Argelander's method, using several neighboring stars whose co-ordinates and assumed magnitudes are given in Table I. As there are no stars in the neighborhood as faint as the variable with photometrically determined magnitudes, the adopted magnitudes have been derived from the observations, with the more or less arbitrary assumption that the light-step is equal to a tenth of a magnitude in all parts of the scale. The observations themselves are given in Table II, which also contains other data explained later. Of the 148 observations 63 are well toward the middle of the eclipse, giving magnitudes of 10.48 or fainter on the scale adopted. and were assigned full weight in the determination of period; 28 others during eclipse but

not so near minimum were given half weight, and the rest were considered as of no weight in the determination and were disregarded except in drawing the light-curve. From the depth of the primary minimum it was seen that the secondary minimum would be too shallow to be accurately observed by this method, and no attempt was made to observe that part of the curve further than a few comparisons to eliminate the possibility of a period half of that here derived.

TABLE I

Comparison Stars

Designation Employed	C. P. D. Number	1900		Adopted Magnitude
		$\Delta\alpha$	$\Delta\delta$	
r	$-72°70$	$+0^m11^s$	$+13'.6$	9.69
t	73 49	$-1\ 4$	-19.4	9.79
s	72 72	$+2\ 41$	$+\ 7.6$	9.82
q	72 65	$-5\ 24$	-13.2	9.95
y	....	$+3\ 21$	$-\ 6.8$	10.08
w	72 71	$+1\ 59$	$-\ 8.6$	10.14
x	....	$+0\ 30$	$-\ 4.1$	10.31
λ	....	$+2\ 26$	-10.8	10.45
z	....	$-0\ 34$	$+\ 4.2$	11.55
τ	....	$-0\ 36$	$+\ 5.9$	11.67
ξ	....	$-0\ 42$	$-\ 0.7$	11.79
α	....	$+0\ \ 4$	$+\ 0.5$	12.25

The preliminary reduction was made by sketching a curve from the observations of each eclipse and taking the mid-point of the best determined horizontal chord as the time of minimum. These provisional minima were weighted arbitrarily, and a solution was made for preliminary values of epoch and period, with which an ephemeris was computed. Calling T the time of minimum given by this ephemeris, the quantity $t-T$ was derived for each observation, and all the observations were united upon a single plot with the arithmetical value of $t-T$ as abscissa. Through these points was drawn the curve which seemed best to represent them, and from this curve, with magnitude as argument, a value of $T-t$ was read off for each observation during eclipse and applied to the time of observation to obtain a value of the geocentric time of the corresponding minimum. The several times obtained for each minimum

TABLE II

VISUAL OBSERVATIONS

Date	2421	Mag.	Red. to Min. $0^d.001$	Geocentric Min.	Date	2421	Mag.	Red. to Min. $0^d.001$	Geocentric Min.
Oct. 19, 1917	521.622	10.04	....	Obs.	Oct. 31, 1917	533.475	9.88	....	
	.691	11.14	−14	.677		.536	9.95	....	
	.732	9.96	....	Comp.		.625	9.94	....	Obs.
Oct. 20	522.504	10.14	+50*	.679	Nov. 3	536.478	12.01	0	.483
	.535	10.72	+22			.492	11.75	− 4	Comp.
	.548	11.65	+ 5		Nov. 5	538.737	9.99	....	.485
	.578	10.81	−21	Obs.	Nov. 8	541.654	10.03	....	
	.588	10.14	−50*	.553		.666	10.08	....	
	.640	10.06	....	Comp.		.681	10.36	+35*	
Oct. 21	523.509	9.90	....	.550		.692	10.38	+34*	
	.602	10.01	....			.700	10.88	+19	
	.650	10.01	....			.710	11.55	+ 7	
	.707	10.03	....			.720	11.15	−14	
	.762	10.04	....			.728	10.93	−18	Obs.
Oct. 22	524.675	9.98	....			.733	10.48	−30	.712
Oct. 23	525.538	10.02	....			.741	10.33	−36*	Comp.
Oct. 25	527.468	9.87	....			.748	10.24	−42*	.711
	.750	10.26	+40*	Obs.		.757	10.20	−45*	
	.790	10.93	−18	.778	Nov. 9	542.488	9.87	....	
	.823	10.11	....	Comp.		.517	9.95	....	
Oct. 27	529.467	10.07	....	.776		.532	10.05	....	
	.495	10.62	+25	Obs.		.562	10.80	+21	
	.520	11.78	− 3	.518		.580	11.97	0	
	.554	10.26	+40*	Comp.		.589	11.70	− 6	
	.570	10.11	....	.518		.603	10.97	−17	Obs.
Oct. 29	531.490	9.88	....			.613	10.38	−34*	.582
	.530	9.90	....			.625	10.14	−50*	Comp.
Oct. 30	532.508	9.92	....			.646	9.97	....	.582
	.535	10.04	....			.673	10.00	....	

Nov. 12, 1917	545.552	9.95	....	
	.595	9.95	....	
Nov. 16	549.587	10.18	−46*	Obs.
	.640	10.00	....	.541
	.666	10.00	....	Comp.
	.687	10.00	....	.550
Nov. 17	550.626	10.00	....	
Jan. 5, 1918	599.553	10.05	....	
	.579	10.08	....	
Jan. 9	603.542	11.83	0	
	.554	11.56	− 7	
	.563	11.13	−14	Obs.
	.572	10.93	−18	.549
	.588	10.30	−37*	Comp.
	.596	10.22	−43*	.548
	.619	10.06	....	
	.632	10.11	....	
	.682	9.98	....	
Jan. 14	608.564	10.02	....	
	.612	10.02	....	
	.655	10.02	....	
	.702	10.06	....	
	.708	10.18	+46*	
	.725	10.28	+38*	
	.749	11.08	+15	Obs.
	.757	11.23	+12	.767
	.765	11.52	+ 8	Comp.
	.772	11.82	− 1	.773
Feb. 18	643.599	11.24	+12	Obs.
	.605	11.72	+ 4	.611
	.635	10.83	−20	Comp.
	.642	10.31	−37*	.610

Feb. 26, 1918	651.507	10.06	....	
	.512	10.08	....	
	.536	10.06	....	
	.558	10.21	....	
	.565	10.16	....	
	.583	10.21	....	
	.616	10.01	....	
March 3	656.585	10.01	....	
	.600	10.18	+46*	
	.614	10.31	+37*	
	.653	11.08	+15	
	.663	11.42	+ 9	
	.668	11.52	+ 7	
	.672	11.87	0	
	.680	11.37	−10	
	.683	11.22	−13	
	.687	11.08	−15	
	.691	10.88	−19	
	.694	10.73	−22	
	.700	10.83	−20	Obs.
	.708	10.71	−23	.673
	.713	10.61	−26	Comp.
	.732	10.31	−37*	.674
March 4	657.512	10.31	+37*	
	.528	11.12	+14	
	.539	11.52	+ 7	
	.548	11.57	− 6	
	.553	11.27	−12	
	.557	11.05	−15	
	.562	10.82	−20	Obs.
	.570	10.68	−24	.545
	.578	10.61	−26	Comp.
	.588	10.31	−37*	.545

TABLE II—*Continued*

Date	2421	Mag.	Red. to Min. 0ᵈ.001	Geocentric Min.	Date	2421	Mag.	Red. to Min. 0ᵈ.001	Geocentric Min.
July 5, 1918.......	780.711	9.76	....		Aug. 6, 1918......	812.508	10.20	+44*	
July 10..........	785.626	9.80	....			.511	10.20	+44*	
Aug. 5...........	811.645	10.01	....			.520	10.41	+32*	
	.651	10.05	....			.529	10.78	+21	
	.668	10.37	+34*			.540	10.98	+17	
	.675	10.83	+20			.548	11.28	+12	
	.691	11.96	0			.560	12.03	0	
	.696	11.77	− 3			.577	11.42	− 9	
	.704	11.28	−12	Obs.		.582	10.98	−17	Obs.
	.708	11.13	−14	.693		.589	10.88	−19	.561
	.713	10.88	−19	Comp.		.597	10.51	−29	Comp.
	.718	10.30	−38*	.693		.607	10.31	−37*	.564
	.726	10.00	....						

were then averaged, giving half weight to observations near the beginning and end of the eclipse, and the mean times thus obtained were reduced to the sun. These heliocentric times of minimum were then used to determine the elements of variation, with the resulting formula:

$$\text{Minimum} = \text{J. D. } 2421594.8368 + (0.870910 \pm 0.000009)\, E.$$

On the basis of this period the observations during eclipse were collected to a single revolution and plotted. As there was no a priori reason for supposing the eclipse unsymmetrical the variation was assumed to be symmetrical, the observations were grouped according to time from minimum, and a smooth curve was drawn through the resulting normal points.

As the period so determined did not represent all the data given in the original announcement of variability, the matter was allowed to rest at this point for some time. The minima of August 5 and 6 were observed to check the period derived, and later, through the kindness of Miss Cannon, the source of the discrepancy was made clear. The last plate mentioned in *Harvard Circular*, No. 134, was given as of August 6, 1906, whereas the correct date is August 8, 1906. The time of mid-exposure of each of the plates referred to was also communicated. These data, together with the computed times of minimum, are given in Table III. Upon their receipt the light-curve already determined was used to derive new observed epochs of minimum, giving the quantities which are included in Table II under the head "Red. to Min.," in which the quantities with asterisk (*) correspond to the observations which were given half weight. In the column "Geocentric Min." the upper of each pair of numbers is the mean of the observed times. After reduction to the sun these were combined with the plates 2790, 2791, 3901, and 4477, giving unit weight to each plate, and a solution was made for epoch and period by the method of least squares, using equations of condition of the type

$$T - 21415^{d}.427 = A + B(E - 5457)$$

The normal equations deduced are

$$-27^{d}.763 = +81A - 32B$$
$$+96994015.568 = -32A + 111370117B$$

From these are derived the values

$$A=+0^{d}00131 \text{ with weight } 81$$
$$B=+0.87091599 \text{ with weight } 111370104$$

The residuals in the equations of condition give the probable errors

$$\pm 0^{d}00612 \text{ for the unit of weight}$$
$$\pm 0.00068 \text{ in } A$$
$$\pm 0.00000058 \text{ in } B$$

and reintroducing the constants used in forming the equations of condition, we obtain the formula

$$\text{Minimum}=\text{J.D. } 2416662.8398+0^{d}8709160\ E,$$

in which the computed times of minima are subject to the probable errors

$\pm 0^{d}0032$ for $E=$ 0, first observation
± 0.0007 for $E=$ 5457, best determined minimum
± 0.0009 for $E=$ 6501, first minimum in 1920
± 0.0019 for $E=$ 8599, first minimum in 1925
± 0.0031 for $E=$ 10695, first minimum in 1930

TABLE III

Photographic Observations

Plate A. M	Date	J.D. 2410000+	Comp. Min.	Observation
2790........	1904, June 30	6662.828	.840	Variable faint
2791........	June 30	6662.874	.840	Variable faint
2798........	July 1	6663.841	.711	Variable bright
2810........	July 5	6667.862	.194	Variable bright
2889........	July 30	6692.708	.451	Variable bright
3852........	1905, Sept. 6	7095.656	.685	Variable somewhat faint
3868........	Sept. 8	7097.755	.427	Variable bright
3901........	Sept. 19	7108.743	.749	Variable faint
4477........	1906, Aug. 8	7431.833	.859	Variable faint

The representation of the observations by these elements is thoroughly satisfactory. The lower of each pair of numbers in the column "Geocentric Min." of Table II is the computed time

of minimum derived from the foregoing formula. The agreement with the plates given in Table III is not quite so close, but when it is remembered that the photographic observations were generally exposures of an hour, while the eclipse lasts barely four hours, with the marked decrease in brightness lasting only two hours, it seems remarkable that the variation was discovered at all.

With the period given above the visual observations during and near eclipse were collected and plotted, and the light-curve was drawn, again assuming symmetry. The plot obtained is

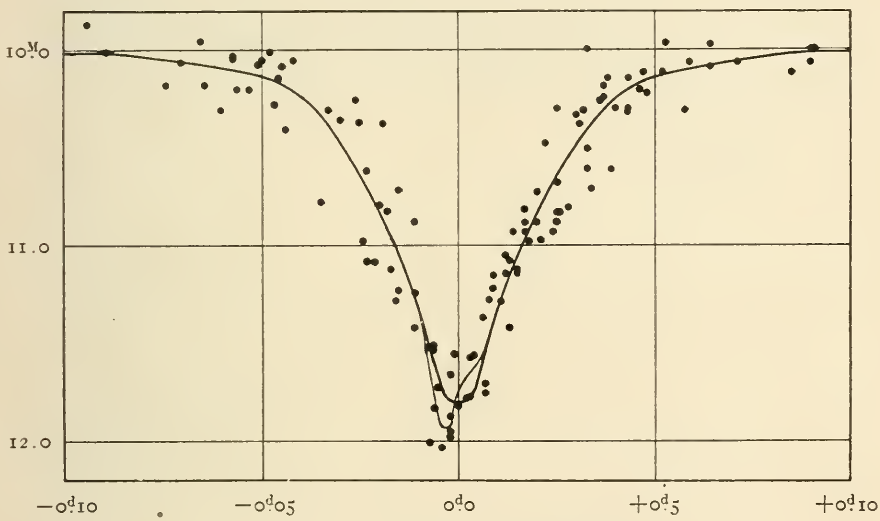

Fig. 1.—Light-curve of V Tucanae

reproduced in the accompanying figure. From the form of the light-curve it appears that the eclipse is partial, and near mid-eclipse the variation is exceedingly rapid, amounting to half a magnitude in twelve minutes.

SUMMARY

From 148 visual comparisons of V Tucanae with neighboring stars the eclipsing type of variation was confirmed, and a period of 0.8709 day was deduced.

A combination of these observations with the original Arequipa plates observed by Miss Cannon has enabled the writer to deduce

a formula for time of minimum which will be sufficiently accurate for a considerable time to come.

The period deduced from the visual observations of 1917–18 agrees with that deduced from all the observations within the probable error of the former, indicating that the period is sensibly constant.

The light-curve indicates a partial eclipse lasting 0.18 day, with a loss of light amounting to 1.8 magnitudes of the scale employed.

Lack of photometric data has made it necessary to use an arbitrary scale, rendering the light-curve unsuitable for the computation of the physical elements of the system.

La Plata, Argentina
September 1918

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